Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

I declare this is my own work.

Level 2 Certificate
FURTHER MATHEMATICS

Paper 2 Calculator

Wednesday 21 June 2023 Afternoon Time allowed: 1 hour 45 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** questions in the spaces provided.

1 Solve $\frac{8d-3}{3d-7} = \frac{5}{2}$

[3 marks]

$16d - 6 = 15d - 35$ ← Multiplying both sides by the denominators to eliminate them

$d - 6 = -35$ ← Subtracting 15d from both sides to get all d on the same side

$d =$ -29

Adding 6 to both sides to get d on its own

2 (a) The first four terms of a linear sequence are

15 18.5 22 25.5

Work out an expression for the nth term.

[2 marks]

It goes up by 3.5 between each term so it must involve 3.5n. Going backward in the sequence finds that the 0th term (the one before the 1st term) would be 15 - 3.5 = 11.5. So the nth term must be 3.5n + 11.5

Answer $3.5n + 11.5$

2 (b) A different linear sequence has nth term $318 - 9n$

Work out the value of the first **negative** term in the sequence.

[2 marks]

$318 - 9n < 0$ ← If the term is negative the nth term must be less than 0

$318 < 9n$ ← Adding 9n to both sides to get the n term positive

$35.\dot{3} < n$ ← Dividing both sides by 9 to get n on its own. n is greater than 35.3... so the first negative term must be when n is 36

$318 - 9(36)$ ← Substituting 36 for n in the nth term works out that the 36th term is -6. This is the first negative term

Answer -6

3 $$\begin{pmatrix}3 & 5\\ u & 2\end{pmatrix}\begin{pmatrix}1\\ 4\end{pmatrix} = \begin{pmatrix}t\\ 6\end{pmatrix}$$

Work out the values of t and u.

[2 marks]

$3 \times 1 + 5 \times 4 = t$

$u \times 1 + 2 \times 4 = 6$ ← $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}ax + by\\ cx + dy\end{pmatrix}$

$u = 6 - 8$ ← u × 1 = u and 2 × 4 = 8. Then subtracting 8 from both sides gets u on its own

$t =$ 23 $u =$ -2

9

4 A line passes through P (1, k) and Q (r, 6) where k and r are constants.

The midpoint of PQ has x-coordinate 5

The gradient of the line is 2

Work out the value of k.

[4 marks]

$\frac{1 + r}{2} = 5$ ← Doing the mean of the x-coordinates of P and Q must give 5 as this is the x-coordinate of the midpoint of PQ

$1 + r = 10$ ← Multiplying both sides by 2 eliminates the denominator

$r = 9$ ← Subtracting 1 from both sides finds r

$\frac{6 - k}{9 - 1} = 2$ ← Expressing the gradient in terms of k. Gradient = (change in y)/(change in x). This must be equal to the gradient of 2

$6 - k = 16$ ← Multiplying both sides by 9 - 1 eliminates the denominator

$6 = 16 + k$ ← Adding k to both sides to make it positive

$k =$ -10

Subtracting 16 from both sides gets k on its own

5 $y = 0.5x^4$

Work out the value of x for which the rate of change of y with respect to x is 6.75

[3 marks]

$2x^3 = 6.75$ ← Differentiated (to give an expression in terms of x for the rate of change of y with respect to x) by multiplying by the power then subtracting 1 from the power. Setting it equal to the 6.75

$x^3 = 3.375$ ← Dividing both sides by 2 to get x^3 on its own

$x =$ 1.5

Cube rooting both sides to get x on its own

6 The equation of a circle is $(x + 7)^2 + (y - 4)^2 = 36$

Complete these statements.

[2 marks]

The general equation of a circle with centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$

The coordinates of the centre of the circle are (-7 , 4)

The radius of the circle is 6

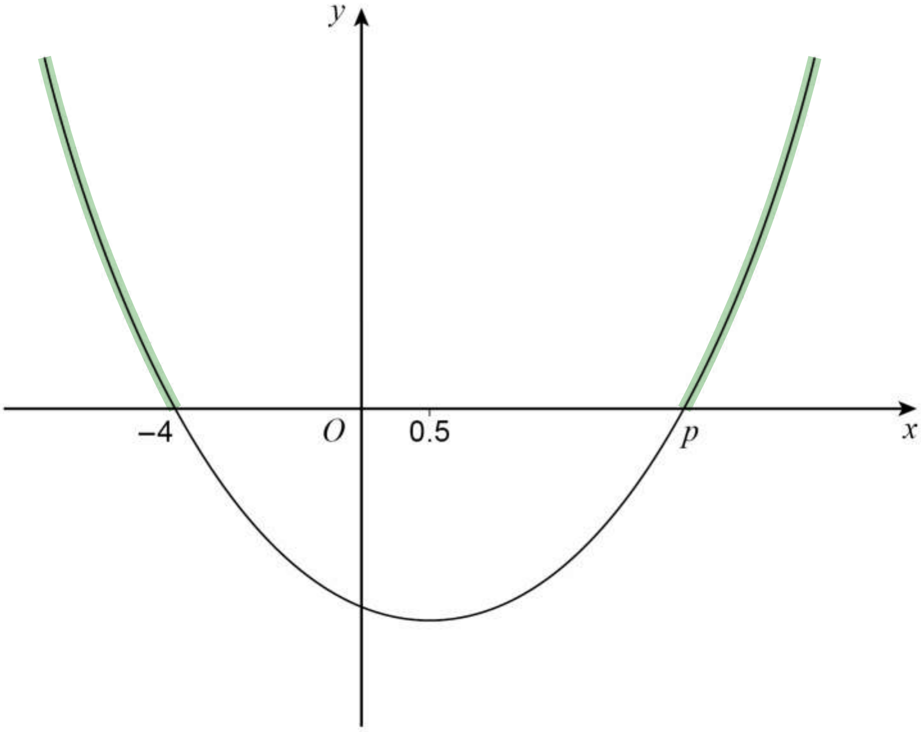
$r^2 = 36$ so $r = 6$

9

7 Here is a sketch of the curve $y = ax^2 + bx + c$ where a, b and c are constants.

The curve intersects the x-axis at (–4, 0) and (p, 0)

The turning point has x-coordinate 0.5



7 (a) Work out the value of p.

[1 mark]

0.5 - -4 ← This works out that the distance between the x-coordinate of the midpoint and the -4 is 4.5

0.5 + 4.5 ← Quadratic graphs are symmetrical so adding the 4.5 distance to 0.5 works out p

p = 5

7 (b) Solve $ax^2 + bx + c > 0$

[2 marks]

It is greater than 0 where it is highlighted in green. To the left from -4 is less than -4 in the x-direction and to the right from 5 is greater than 5 in the x-direction

Answer x < -4 or x > 5

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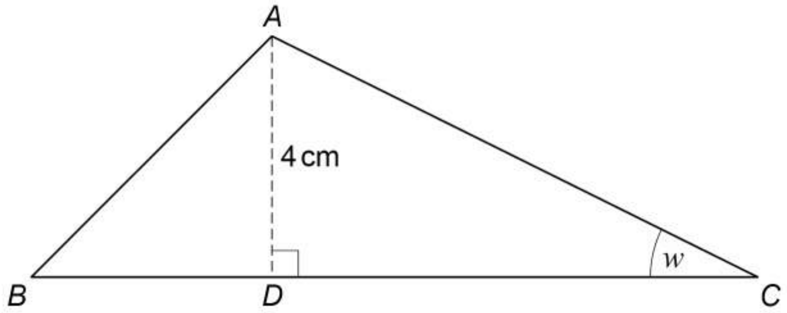
8 *ABC* is a triangle with perpendicular height *AD*.



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Area of *ABC* = 25 cm^2

BD : *DC* = 2 : 3

Work out the size of angle w.

[4 marks]

$\frac{1}{2} \times BC \times 4 = 25$ ← Expressing the area of the triangle in terms of BC then setting equal to the 25 cm^2. Area of triangle = 1/2 × base × height. BC is the base and 4 cm is the height

$12.5 \div 5$ ← Dividing both sides by 1/2 × 4 finds that BC = 12.5 cm. This is represented by 2 + 3 = 5 parts of the ratio. So dividing the 12.5 cm by 5 works out that 1 part of the ratio is worth 2.5 cm

$2.5 \times 3 = 7.5$ ← Multiplying the value of 1 part of the ratio by 3 works out that DC is 7.5 cm

$S\overset{\overset{\checkmark}{O}}{\ }H\quad C\overset{\overset{\checkmark}{A}}{\ }H\quad T\overset{\overset{\checkmark}{O}}{\ }\overset{\checkmark}{A}$ ← Using right-angled trigonometry in triangle ADC. The 4 cm is the opposite so ticking O. The 7.5 cm is the adjacent so ticking A. There are two ticks on the TOA formula triangle so this one can be used

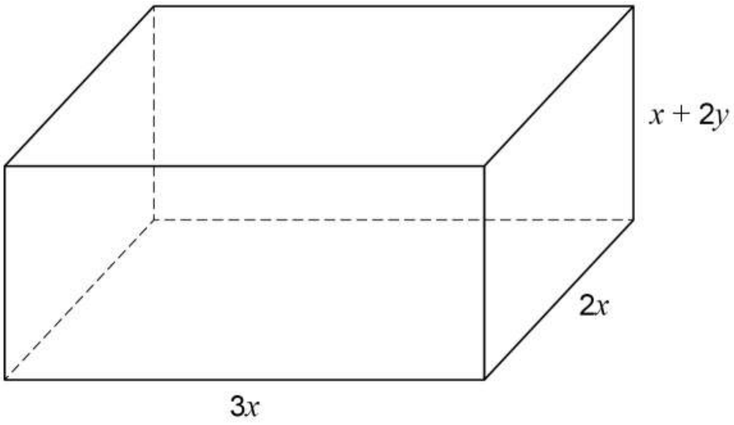
$\tan w = \frac{4}{7.5}$ ← Covering T in the TOA formula triangle finds that tan of the angle = opposite/adjacent

w = 28.1 °

Doing the inverse tan of both sides finds w

7

9 The dimensions of the cuboid are given in centimetres.



The total length of all 12 edges is 300 cm

9 (a) Show that $y = \dfrac{75 - 6x}{2}$

[2 marks]

$4(3x) + 4(2x) + 4(x + 2y)$ ← Expressing the total length of all 12 edges in terms of x and y. There are 4 lengths, 4 widths and 4 heights

$12x + 8x + 4x + 8y$ ← Expanding the brackets

$24x + 8y = 300$ ← Collecting like terms. Then setting the expression of the total length of all 12 edges equal to the 300 cm

$8y = 300 - 24x$ ← Subtracting 24x from both sides to get the y term on its own

$y = \dfrac{300 - 24x}{8}$ ← Dividing both sides by 8 to get y on its own

$y = \dfrac{75 - 6x}{2}$ ← Simplifying the fraction by dividing both the numerator and denominator by 4

9 (b) The volume of the cuboid is V cm^3

Show that $V = 450x^2 - 30x^3$

[2 marks]

$x + 2\left(\dfrac{75 - 6x}{2}\right)$	Substituting (75 - 6x)/2 for y in the height x + 2y to express the height only in terms of x
$x + 75 - 6x$	The denominator of 2 and the multiplication of 2 cancel out. Then collecting like terms gives 75 - 5x
$V = 3x \times 2x \times (75 - 5x)$	Volume of cuboid = length × width × height
$= 6x^2(75 - 5x)$	$3x \times 2x = 6x^2$
$= 450x^2 - 30x^3$	Expanding the brackets

9 (c) Use calculus to work out the maximum value of V as x varies.

[3 marks]

$900x - 90x^2$	Differentiated (to express the rate which V changes with respect to x) by multiplying each term by the power then subtracting 1 from the power
$90x(10 - x) = 0$	Factorising and setting to 0 as at a maximum point the rate which V changes with respect to x will be 0. Either 90x = 0 (so x = 0) or 10 - x = 0 (so x = 10)
$450(10)^2 - 30(10)^3$	x cannot be 0 as length must be positive. So x must be 10. Substituting 10 for x in the right side of the formula for V works out the maximum volume

Answer 15000

7

10 Line K has equation $4x - 5y = 17$

Line L passes through the points (3, 6) and (–5, 16)

Tick (✓) the correct statement about lines K and L.

The lines are parallel. ☐

The lines are perpendicular. ☑

The lines are neither parallel nor perpendicular. ☐

Show working to support your answer.

[3 marks]

$4x - 17 = 5y$ ← Adding 5y to both sides to make it positive then subtracting 17 from both sides to get the y term on its own

$y = 0.8x - 3.4$ ← Dividing both sides by 5 gets it into the form y = mx + c, where m is the gradient and c is the y-intercept. So the gradient of line K is 0.8

$\frac{16 - 6}{-5 - 3} = -1.25$ ← Gradient = (change in y)/(change in x). So the gradient of line L is -1.25

$0.8 \times -1.25 = -1$ ← The gradients are not the same so the lines are not parallel. The gradients multiply to -1 so they must be perpendicular

11 Expand and simplify fully $(2x^3 - 9)(3x^2 + 4) + x(x - 4)^2$

[4 marks]

$x(x^2 - 8x + 16) \leftarrow$ First dealing with the x(x - 4)^2. Expanding the square bracket by squaring the first term, doubling the product of the two terms, squaring the last term

$6x^5 + 8x^3 - 27x^2 - 36 + x^3 - 8x^2 + 16x \leftarrow$ Expanding all the brackets

Answer $6x^5 + 9x^3 - 35x^2 + 16x - 36$

$\uparrow$ Collecting like terms

Turn over for the next question

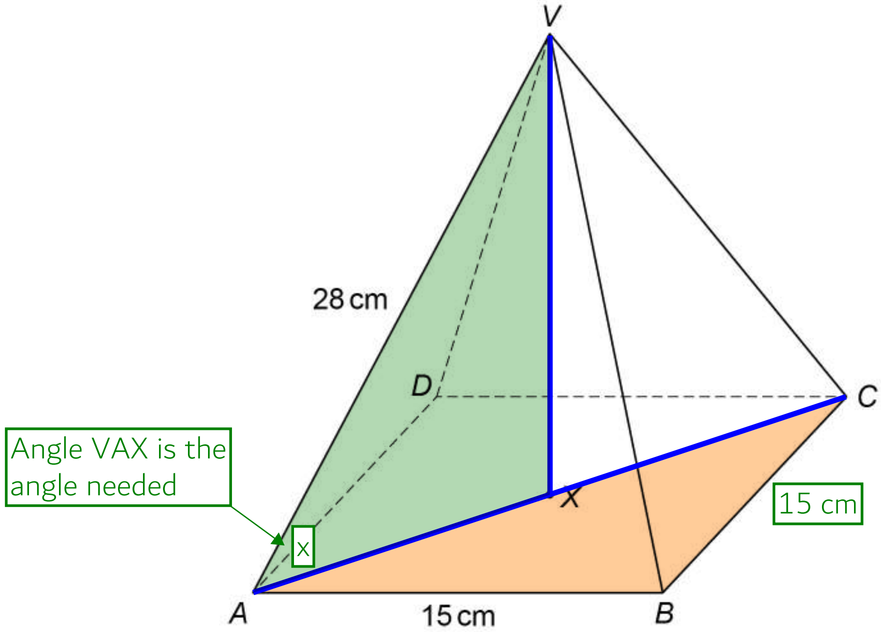
7

12 *VABCD* is a pyramid.

The square horizontal base, *ABCD*, has side length 15 cm

V is directly above the centre, *X*, of the base.

VA = 28 cm



Angle VAX is the angle needed

15 cm

Work out the size of the angle that *VA* makes with *ABCD*.

[3 marks]

$15^2 + 15^2 = AC^2$ ← Using Pythagoras' Theorem in the orange right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting 15 for a, 15 for b and AC for c

$AC = \sqrt{450}$ ← Square rooting both sides to find AC

$15\sqrt{2} \div 2$ ← Dividing AC by 2 works out AX

$S\,\frac{O\checkmark}{H}\quad C\,\frac{\overset{\checkmark}{A}\checkmark}{H}\quad T\,\frac{O\checkmark}{A}$ ← Doing right-angled trigonometry in the green triangle. AV is the hypotenuse so ticking H. AX is the adjacent so ticking A. There are two ticks on the CAH formula triangle so this one can be used

$\cos x = \frac{10.6...}{28}$ ← Covering C in the CAH formula triangle finds that cos of the angle = adjacent/hypotenuse

Doing the inverse cos of both sides finds the angle

Answer 67.7 °

13 (a) Circle the expression equivalent to $3x^{-7}$

[1 mark]

$-\frac{3}{x^7}$ $-\frac{1}{3x^7}$ $\frac{1}{3x^7}$ [✓ $\frac{3}{x^7}$]

The negative power means reciprocal and only applies to x, not the 3

13 (b) Simplify fully $\frac{12w^8}{\left(4w^3\right)^2}$

[2 marks]

$\frac{12w^8}{16w^6}$ ← Squaring both the 4 and the w^3. $(w^3)^2 = w^{3 \times 2} = w^6$

Answer $0.75w^2$

12/16 = 0.75 and $w^8/w^6 = w^{8-6} = w^2$

13 (c) $\sqrt{y} \times \sqrt[3]{y} = \sqrt[c]{y^d}$ where c and d are positive integers.

Work out the **least** possible values of c and d.

[3 marks]

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ ← $\sqrt{y} = y^{1/2}$ and $\sqrt[3]{y} = y^{1/3}$. $y^{1/2} \times y^{1/3} = y^{1/2+1/3} = y^{5/6}$

$c =$ 6 $d =$ 5

5/6 cannot be simplified to get smaller integers for the numerator and denominator. The denominator as a power is c and the numerator as a power is d

9

Do not write outside the box

14 Simplify fully $$\frac{15a^2}{a^2+6a-16} \times \frac{8-4a}{3a}$$

[4 marks]

$$\frac{15a^2}{(a+8)(a-2)} \times \frac{-4(a-2)}{3a}$$

Fully factorising both the numerators and denominators. For 8 - 4a, the a term is negative so a negative factor should be brought out

$$\frac{5a}{a+8} \times \frac{-4}{1}$$

Simplifying by cancelling out common factors to the numerators and denominators. The (a - 2) cancels out and the $15a^2$ and 3a can both be divided by 3a

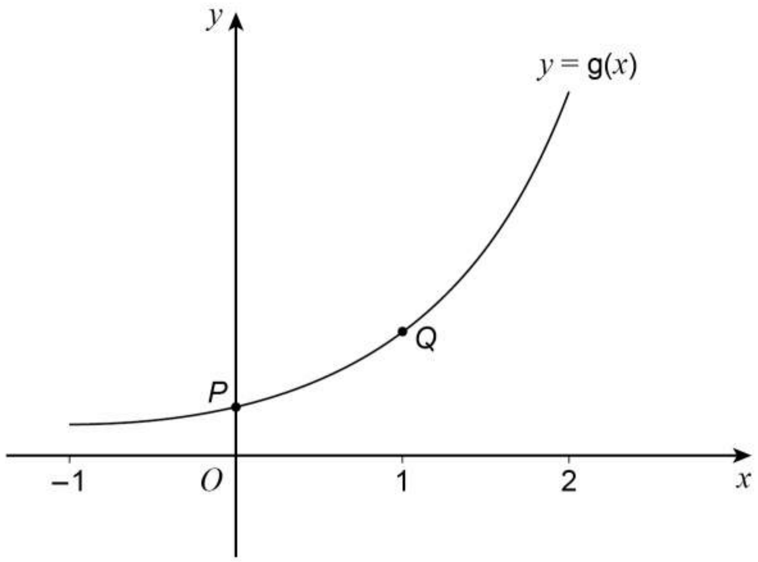
Answer $\frac{-20a}{a+8}$

Multiplying the fractions by multiplying the numerators and multiplying the denominators

15 The function g is given by $g(x) = a \times b^x$ where a and b are constants.

The domain of the function is $-1 \leqslant x \leqslant 2$

$P\left(0, \frac{1}{2}\right)$ and $Q\left(1, \frac{3}{2}\right)$ are points on the graph $y = g(x)$



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Work out the range of the function.

[4 marks]

$a \times b^0 = \frac{1}{2}$ ← Substituting 0 for x and 1/2 for g(x) using the point P

$a = \frac{1}{2}$ ← Anything to the power of 0 is 1 so b^0 = 1 and a × 1 = a. So a is 1/2

$\frac{1}{2} \times b = \frac{3}{2}$ ← Substituting 1 for x and 3/2 for g(x) using the point Q. Also substituting 1/2 for a

$b = 3$ ← Dividing both sides by 1/2 finds that b is 3

$\frac{1}{2} \times 3^{-1} = \frac{1}{6}$ ← So g(x) = 1/2 × 3^x. Substituting in -1 for x as this is where g(x) is the lowest

$\frac{1}{2} \times 3^2 = \frac{9}{2}$ ← Substituting in 2 for x as this is where g(x) is the greatest

Answer $\frac{1}{6} \leq g(x) \leq \frac{9}{2}$

1/6 is the lowest and 9/2 is the greatest g(x) can be within the domain

8

Turn over ►

16 $(2x - 3)$ is a factor of $6x^3 - 25x^2 + 28x - 6$

Solve $6x^3 - 25x^2 + 28x - 6 = 0$

Give all solutions as **exact** values.

[4 marks]

$$\begin{array}{r l}
 & 3x^2 - 8x + 2 \\
2x - 3 & \overline{)\,6x^3 - 25x^2 + 28x - 6} \\
 & \underline{6x^3 - 9x^2} \\
 & \quad -16x^2 + 28x \\
 & \quad \underline{-16x^2 + 24x} \\
 & \qquad\qquad 4x - 6
\end{array}$$

Dividing the expression by 2x - 3 using algebraic division to work out the other factor. So the equation can be written as (2x - 3)(3x² - 8x + 2) = 0

$$x = \frac{--8 \pm \sqrt{(-8)^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3x² - 8x + 2 could equal 0. This is in the form ax² + bx + c = 0 so can be solved using the quadratic formula

When 2x - 3 = 0, x = 3/2

Answer $x = \dfrac{3}{2}$ $\quad x = \dfrac{4 + \sqrt{10}}{3}$ $\quad x = \dfrac{4 - \sqrt{10}}{3}$

17 The function h is given by $h(x) = ax(3x^2 - 2) + 5x$ where a is a **positive** constant.

h is an **increasing** function for all values of x.

Work out the possible values of a.

Give your answer as an inequality.

[4 marks]

$3ax^3 - 2ax + 5x$ ← Expanding the brackets

$9ax^2 - 2a + 5 > 0$ ← Differentiating to express the gradient by multiplying each term by the power then subtracting 1 from the power. 2ax = $2ax^1$ so when differentiated it becomes 1 × $2ax^0$ = 2a. The gradient must be greater than 0 as it is an increasing function for all values of x

$-2a + 5 > 0$ ← The lowest $9ax^2$ can be is 0. So the -2a + 5 must be greater than 0 otherwise the gradient could be negative or 0

$5 > 2a$ ← Adding 2a to both sides to make it positive

Answer $0 < a < 2.5$

Dividing both sides by 2 finds that a is less than 2.5. a must also be greater than 0 as it is positive

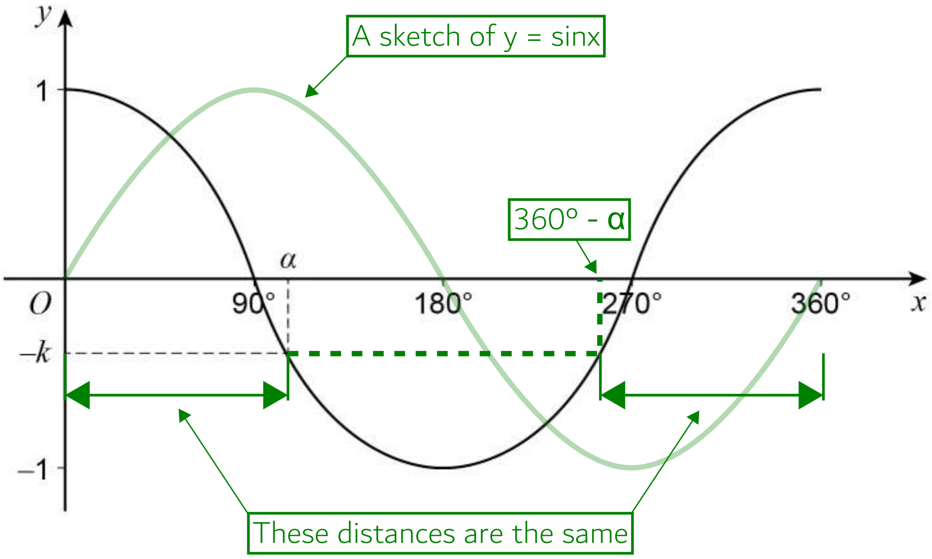
Turn over for the next question

8

18 Here is a sketch of $y = \cos x$ for values of x from 0° to 360°

α is an obtuse angle measured in degrees.

$\cos \alpha = -k$ where k is a positive constant.



18 (a) Tick (✓) **two** boxes that show expressions for x where $\cos x = -k$

[2 marks]

☐ $180° - \alpha$ ☐ $180° + \alpha$

☐ $270° - \alpha$ ☐ $270° + \alpha$

☑ $360° - \alpha$ ☑ $360° + \alpha$

The graph repeats every 360°

18 (b) Circle the expression for x where $\sin x = -k$

[1 mark]

α [✓ $90° + \alpha$] $180° - \alpha$ $180° + \alpha$

The sin curve is the same as a cos curve but 90° to the right

19 In these simultaneous equations, k is a positive constant.

$$3x + 4y = k$$

$$y = 2kx$$

Solve the simultaneous equations.

Give the answers in their simplest form in terms of k.

[3 marks]

$3x + 8kx = k$ ← Substituting 2kx for y in the 1st equation. 4(2kx) = 8kx

$x(3 + 8k) = k$ ← Bringing x out as a factor to get x out of the first two terms

Dividing both sides by 3 + 8k makes x the subject. There are no common factors to the numerator and denominator so it cannot be simplified

$x = \dfrac{k}{3 + 8k}$ $y = \dfrac{2k^2}{3 + 8k}$

y = 2kx so multiplying whatever x is in terms of k by 2k to express y. There are no common factors to the numerator and denominator so it cannot be simplified

6

20 Show that

$2\sin^3 x + 2\sin x\cos^2 x + 5\tan x\cos x$ simplifies to $p\sin x$ where p is a constant.

[3 marks]

$\sin^2 x + \cos^2 x \equiv 1$ ← Writing down one of the trig identities which involves $\cos^2$x

$\cos^2 x \equiv 1 - \sin^2 x$ ← Rearranging to get $\cos^2$x on its own by subtracting $\sin^2$x from both sides

$2\sin x(1 - \sin^2 x)$ ← Substituting 1 - $\sin^2$x for $\cos^2$x in the second term, 2sinx $\cos^2$x

$2\sin x - 2\sin^3 x$ ← Expanding the brackets. This is a simplified expression for the second term in term of sinx

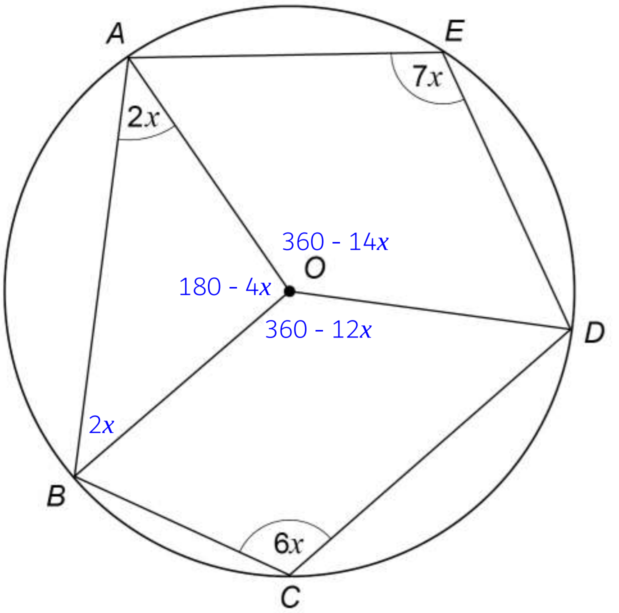
$5 \times \dfrac{\sin x}{\cos x} \times \cos x$ ← tanx ≡ sinx/cosx. So replacing tanx with sinx/cosx in the third term, 5tanx cosx

$5\sin x$ ← Simplifying by cancelling out the cosx

$2\sin^3 x + 2\sin x - 2\sin^3 x + 5\sin x$ ← Now writing the whole expression in terms of sinx

$7\sin x$ ← Collecting like terms

21 *A*, *B*, *C*, *D* and *E* are points on a circle, centre *O*.



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Work out the value of x.

[4 marks]

Triangle AOB is isosceles as it has two equal sides (OA and OB are both radii). Its base angles are equal so angle ABO is 2x. There are 180° in total in a triangle so subtracting both angle BAO and ABO from 180° leaves angle AOB, which is (180 - 4x)°.

The angle at the centre is double the angle at the circumference so reflex angle AOD is 14x. There are 360° in total around a point so subtracting 14x from 360° works out that angle AOD is (360 - 14x)°.

The angle at the centre is double the angle at the circumference so the reflex angle BOD is 12x. There are 360° in total around a point so subtracting 12x from 360° works out that angle BOD is (360 - 12x)°.

$180 - 4x + 360 - 14x + 360 - 12x$ ← Adding angles AOB, AOD and BOD

$900 - 30x = 360$ ← Simplifying by collecting like terms. This must be equal to 360° as there are 360° around a point

$540 = 30x$ ← Adding 30x to both sides to make it positive. Then subtracting 360 from both sides to get the x term on its own

$x =$ 18

Dividing both sides by 30 gets x on its own

7

Turn over ►

22 Five-digit integers are made using

1 2 7 8 9

For each integer, all the digits are used exactly once.

The integers are

greater than 40 000 **and** odd.

How many different integers can be made?

You **must** show your working.

[3 marks]

There will be 5 digits. The 1st digit must be 7, 8 or 9 to be greater than 40000. The 5th digit must be 1, 7 or 9 to be odd

$2 \times 3 \times 2 \times 1 = 12$

First assuming that the 1st digit is 7. The 5th digit could be 1 or 9 so there are 2 possibilities for the 5th digit. There are 3 remaining numbers for the 2nd digit. There are 2 remaining numbers for the 3rd digit. There is 1 remaining number for the 4th digit. Using the product rule for counting, there are 12 possibilities when starting with 7

$3 \times 3 \times 2 \times 1 = 18$

Next assuming that the 1st digit is 8. The 5th digit could be 1, 7 or 9 so there are 3 possibilities for the 5th digit. There are 3 remaining numbers for the 2nd digit. There are 2 remaining numbers for the 3rd digit. There is 1 remaining number for the 4th digit. Using the product rule for counting, there are 18 possibilities when starting with 8

$2 \times 3 \times 2 \times 1 = 12$

Next assuming that the 1st digit is 9. The 5th digit could be 1 or 7 so there are 2 possibilities for the 5th digit. There are 3 remaining numbers for the 2nd digit. There are 2 remaining numbers for the 3rd digit. There is 1 remaining number for the 4th digit. Using the product rule for counting, there are 12 possibilities when starting with 9

$12 + 18 + 12$

Adding together all the numbers of possibilities

Answer 42

END OF QUESTIONS

3