AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE
MATHEMATICS

H

Higher Tier Paper 3 Calculator

Tuesday 12 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

Answer **all** questions in the spaces provided

1 Circle the decimal that is closest in value to $\frac{11}{20}$

[1 mark]

[✓ 0.56]	0.6	0.525	0.5
-0.01	-0.05	0.025	0.05

Subtracting each decimal from 11/20 gives these distances. -0.01 has the smallest magnitude so 0.56 is closest

2 Circle the list of **all** the integers that satisfy $-2 < x \leqslant 4$

[1 mark]

–2, –1, 0, 1, 2, 3

–1, 0, 1, 2, 3

–2, –1, 0, 1, 2, 3, 4

[✓ –1, 0, 1, 2, 3, 4]

The integers which are greater than -2 and less than or equal to 4

3 Circle the largest number.

[1 mark]

[✓ $3.2\dot{7}$]	3.27	3.277	$3.20\dot{7}$
3.2777...	3.2700	3.2770	3.2077...

Writing each number truncated to 4 decimal places is enough to work out which is largest

4 What is the size of an exterior angle of a regular decagon?

Circle your answer.

[1 mark]

18° [✓ 36°] 144° 162°

The sum of the exterior angles in any polygon is 360 degrees. So, as decagons have 10 sides and 10 exterior angles and the shape is regular meaning all of them are the same, dividing 360 by 10 works out that each exterior angle is 36 degrees

5 a is a common factor of 72 and 120

b is a common multiple of 6 and 9

Work out the highest possible value of $\frac{a}{b}$

[4 marks]

$72=2^3\times3^2$

$120=2^3\times3\times5$

Expressing both 72 and 120 as a product of prime factors using the calculator

$2^3\times3$

The highest common factor is the lowest power of each prime multiplied together

Newer calculators can work out the highest common factor of two numbers without doing this method

Answer $\frac{24}{18}$

Expressing the highest common factor of 72 and 120 over the lowest common multiple of 6 and 9. The LCM of 6 and 9 is found by counting up in 9s until a multiple of 6 is reached

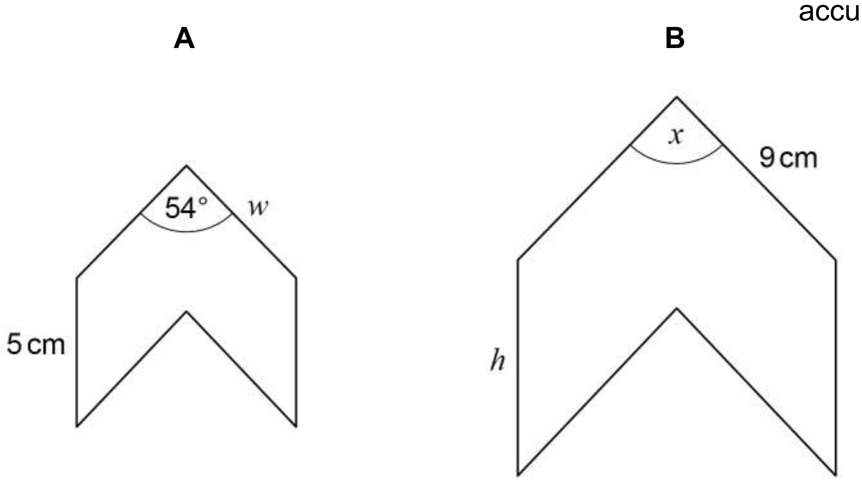
Turn over for the next question

8

6 A and B are similar shapes.

B is an enlargement of A with scale factor 1.5

Not drawn accurately



Work out the values of x, h and w.

[3 marks]

5×1.5 ← Multiplying the 5 cm by the scale factor works out h

$9 \div 1.5$ ← Dividing the 9 cm by the scale factor works out w

x is the same as the 54 degree angle as the angles in similar shapes are the same

x = 54 degrees

h = 7.5 cm

w = 6 cm

7 Investment A Save £150 per month for 2 years.
2.5% interest is added to the total amount saved.

Investment B Invest £3500
Compound interest is added at 3% per year.

After 2 years, how much **more** is investment B worth than investment A?

[4 marks]

$150 \times 12 \times 2$ ←

There are 12 months in a year and there are 2 years so multiplying the £150 by 12 then by 2 works out that £3600 is saved before the interest for investment A

$3600 \times \frac{100+2.5}{100} = 3690$ ←

Adding the 2.5% to 100% expresses the percentage that investment A increases to. Putting this over 100 converts it to a fraction, which increases the £3600 by 2.5% when it is multiplied. So investment A is worth £3690 after 2 years

$3500 \times \left(\frac{100+3}{100}\right)^2$ ←

Adding the 3% to 100% expresses the percentage that investment B increases to each year. Putting this over 100 converts it to a fraction, which increases the £3500 by 3% when it is multiplied. Raising it to the power of 2 as it is increased by 3% twice. So investment B is worth £3713.15

$3713.15 - 3690$ ←

Subtracting the value of investment A from the value of investment B works out the difference and so how much more investment B is worth than investment A

Answer £ 23.15

Turn over for the next question

7

8 (a) Show that the lines $y = 3x + 7$ and $2y - 6x = 8$ are parallel.

Do **not** use a graphical method.

[3 marks]

$2y = 6x + 8$ ← Adding 6x to both sides of the second equation to get the y term on its own

$y = 3x + 4$ ← Dividing both sides by 2 to get y on its own

Both lines have gradient of 3 ← Both equations are now in the form y = mx +c, where m is the gradient. Parallel lines have the same gradient

8 (b) Is the point (–5, –6) above, below or on the line $y = 3x + 7$?

Tick **one** box.

☑ Above ☐ Below ☐ On the line

You **must** show your working.

Do **not** use a graphical method.

[2 marks]

$y = 3(-5) + 7 = -8$ ← Substituting the x-coordinate of the point into the equation to find what y should be on the line. It should be -8 and -6 is above this

9 The cost of a ticket increases by 10% to £19.25

Work out the original cost.

[3 marks]

100+10 ← Adding the 10% to 100% works out that the cost of the ticket had increased to 110% of the original cost

19.25÷110 ← Dividing the £19.25 by 110 works out that 1% of the original cost is £0.175

0.175×100 ← Multiplying the value of 1% of the original cost by 100 works out that 100% of the original cost is £17.50

Answer £ 17.50

10 The nth term of a sequence is $12n - 5$

Work out the numbers in the sequence that

have two digits

and

are **not** prime.

[3 marks]

19,31,43,55,67,79,91 ← Using table mode on the calculator. Set f(x) = 12x - 5. Start: 1. End: 30. Step: 1. This lists out the sequence up to the 30th term. Writing down the terms which have two digits

Entering each of the above terms into the calculator and formatting them as product of prime factors. If it comes back as itself it must be prime. 55 = 5 x 11 and 91 = 7 x 13 so 55 and 91 are not prime

Answer 55, 91

11

11 $\mathbf{a} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$

11 (a) Work out $\mathbf{a} + \mathbf{b} + \mathbf{c}$

[2 marks]

$6+-1+-4$ ← Adding the x-components of a, b and c works out that the x-component of a + b + c = 1

$-10+2+7$ ← Adding the y-components of a, b and c works out that the y-component of a + b + c = -1

Answer $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ← x-component ← y-component

11 (b) Show that $\mathbf{a} + 2\mathbf{c}$ is parallel to $\mathbf{b}$

[2 marks]

$6+2(-4)=-2$ ← Working out that the x-component of a + 2c = -2

$-10+2(7)=4$ ← Working out that the y-component of a + 2c = 4

$\begin{pmatrix} -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ← Expressing the column vector as 2 lots of vector b

Do not write outside the box

12

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

A force of 40 Newtons is applied to an area of 3.2 square metres.

Work out the pressure.

Give the units of your answer.

[2 marks]

$\frac{40}{3.2}$ ← Putting the force over the area

Answer 12.5 N/m^2

The force in Newtons was divided by the area in square metres so the unit is Newtons per square metre

13 Tick **all** the statements that are true for any rhombus.

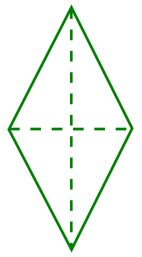
[1 mark]

☑ The diagonals are lines of symmetry

☑ The diagonals bisect each other

☑ The diagonals are perpendicular

☐ The diagonals are equal in length



The diagonals are shown in dashed lines

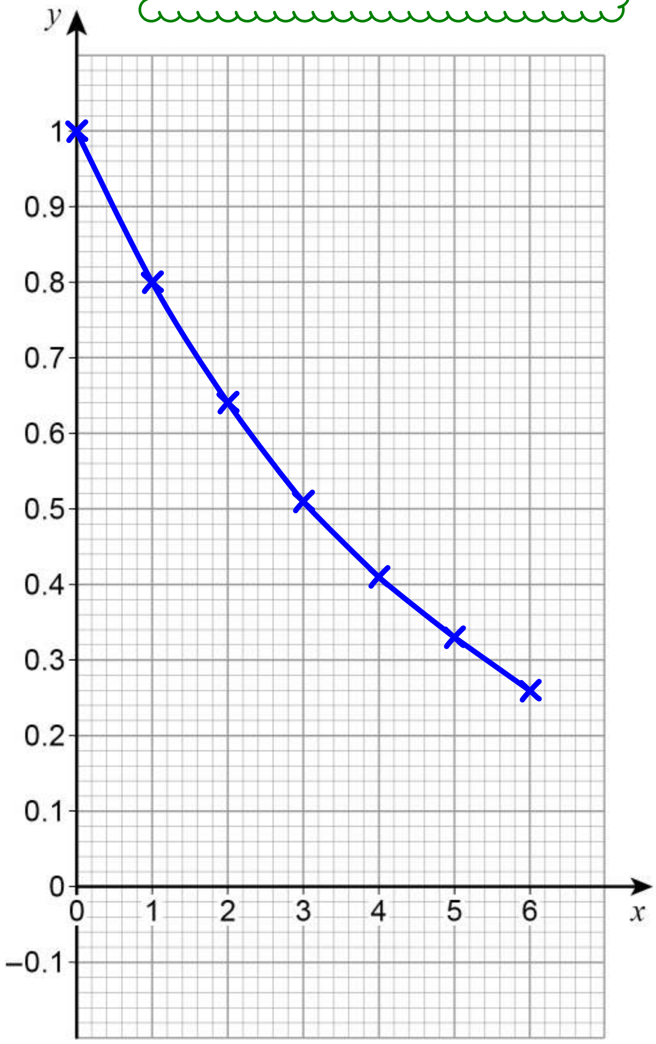
Turn over for the next question

14 Draw the graph of $y = 0.8^x$ for values of x from 0 to 6

[3 marks]

x	0	1	2	3	4	5	6
y							

No need to fill in the table of values



Using table mode in the calculator. Set f(x) = 0.8x. Start: 0. End: 6. Step: 1. This gives the table of values on the calculator. The points can be plotted to the nearest half a box by rounding each of the values to 2 decimal places

15 Amy has x beads.

Billy has three more beads than Amy.

Carly has four times as many beads as Billy.

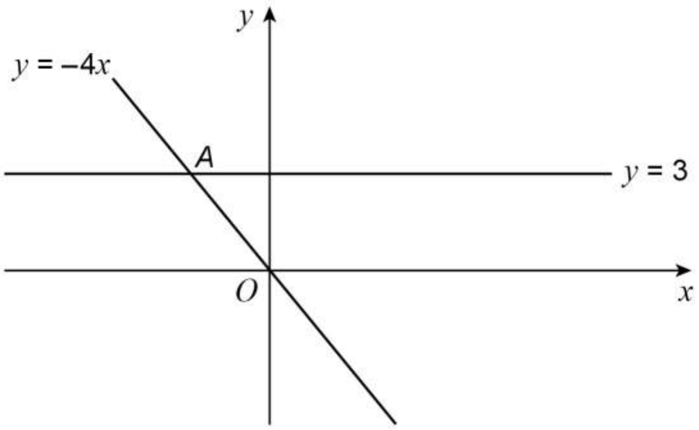
Circle the expression for the number of beads that Carly has.

[1 mark]

$4x + 3$ $3x + 4$ [✓ $4(x + 3)$] $x + 12$

Billy has x + 3. Multiplying this by 4 expresses how many Carly has

16 Two straight lines intersect at point A.



Not drawn accurately

Circle the coordinates of A.

[1 mark]

[✓ $(-\frac{3}{4}, 3)$] $(-4, 3)$ $(-12, 3)$ $(-\frac{4}{3}, 3)$

The y-coordinate must be 3 as it is on the line y = 3. Doing simultaneous equations, which finds the coordinates of intersection, gives -4x = 3 as both sides are equal to y. Dividing both sides by -4 gives x = -3/4

17 Here are two methods to make a 4-digit code.

Codes can have repeated digits.

Method A

For the first two digits use an odd number between 30 and 100

For the last two digits use a multiple of 11

Method B

Use four digits in the order even odd even odd

Do **not** use the digit zero

Which method gives the **greater** number of possible codes?

You **must** show your working.

[3 marks]

$35 \times 9 = 315$

There are 5 odds in each of the 30s, 40s, 50s, 60s, 70s, 80s and 90s. 5 x 7 = 35 so there are 35 odd numbers between 30 and 100. The 2 digit multiples of 11 are 11, 22, 33, 44, 55, 66, 77, 88 and 99. There are 9 of these. Using the product rule for counting works out the total number of possible codes by multiplying the number of possibilities for the first two and last two digits

$4 \times 5 \times 4 \times 5 = 400$

The even digits are 0, 2, 4, 6, 8 but 0 can't be used so there are 4 even digits which can be used. The odd digits are 1, 3, 5, 7, 9 so there are 5 odd digits which can be used. Using the product rule for counting works out the total number of possible codes by multiplying the number of possibilities for each digit

Answer B

Method A has 315 possible codes and method B has 400 possible codes. 400 is greater than 315

18 Show that, for $x \neq 0$

$$\frac{x+4}{3x} - \frac{5}{2x}$$

can be written in the form $\frac{ax+b}{cx}$ where a, b and c are integers.

[3 marks]

$$\frac{2x+8}{6x} - \frac{15}{6x}$$

Multiplying both the numerator and denominator of the first fraction by 2 and both the numerator and denominator of the second fraction by 3 gives a common denominator so that they can be subtracted

Answer $\frac{2x-7}{6x}$

Subtracting the numerators and the denominator stays the same

19 The equation of a straight line is $3x + 2y = 24$

Circle the point where the line crosses the x-axis.

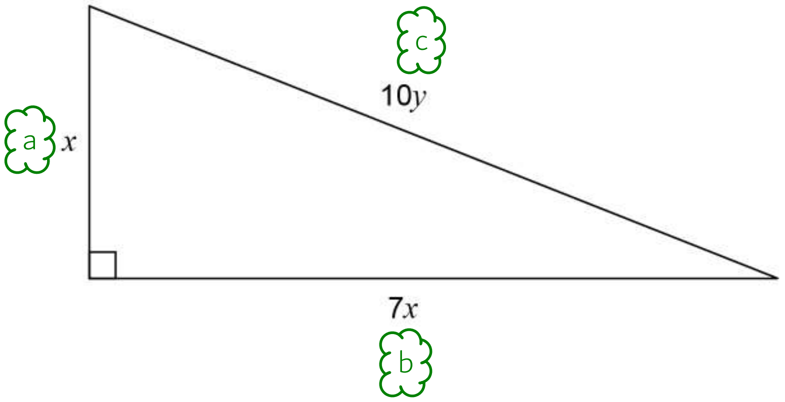
[1 mark]

(0, 8) (12, 0) (0, 12) [✓ (8, 0)]

y = 0 when the line crosses the x-axis. 3x + 2(0) = 3x = 24. Dividing both sides by 3 finds that x = 8

7

20 All dimensions are in centimetres.



Not drawn accurately

Use Pythagoras' theorem to work out the exact value of $\frac{x}{y}$

[3 marks]

$x^2+(7x)^2=(10y)^2$

Using Pythagoras' theorem. $a^2 + b^2 = c^2$, where c is the longest side and a and b are the shorter sides. a = x, b = 7x, c = 10y

$50x^2=100y^2$

$(7x)^2 = 49x^2$. Adding the x^2 gives $50x^2$. $(10y)^2 = 100y^2$

$\frac{x^2}{y^2}=\frac{100}{50}$

Dividing both sides by y^2 and by 50 to get the x and y on the same side and all the values on the other side

$=2$

Dividing the 100 by the 50 gives 2

Answer $\sqrt{2}$

Square rooting both sides gives x/y. Ignoring the negative solution as both x and y must be positive as they are lengths so they cannot be divided to give a negative value

21 The mass of an ornament is m grams.

The height of the ornament is h centimetres.

m is directly proportional to the cube of h.

$m = 1600$ when $h = 8$

21 (a) Work out an equation connecting m and h.

[3 marks]

$m = kh^3$

m ∝ h^3. The right side of the proportion can be multiplied by anything and still be directly proportional. Using k to represent what it is multiplied by to convert it into an equation

$k = \frac{m}{h^3} = \frac{1600}{8^3}$

Rearranging to make k the subject then substituting in the values of m and h given, which must satisfy the equation, to find k

Answer $m = \frac{25}{8}h^3$

Substituting the value of k back into the original equation

21 (b) Work out the mass of an ornament of height 12 centimetres.

[2 marks]

$\frac{25}{8} \times 12^3$

Substituting in 12 for h in the equation found in part (a)

Answer 5400 grams

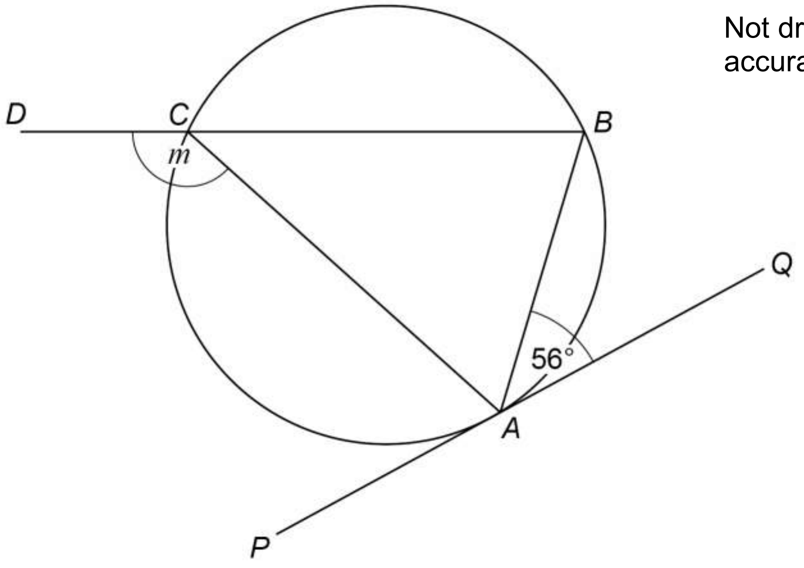
Turn over for the next question

8

22 A, B and C are points on a circle.

DCB is a straight line.

PAQ is a tangent to the circle.



Not drawn accurately

Sam is trying to work out the size of angle m.

Here is his working.

angle ACB = 56°	angles in the same segment are equal
m = 180° – 56°	angles at a point on a straight line add up to 180°
m = 124°	

Make a criticism of his working.

[1 mark]

Reason on the first line is incorrect ← Should have stated the alternate segment theorem

23 A sequence of numbers is formed by the iterative process

$$u_{n+1} = \frac{3}{u_n + 1}, \qquad u_1 = 4$$

Work out the values of u_2 and u_3

[2 marks]

In the calculator, enter 4 then press =/exe. Enter 3/(Ans + 1) then press =/exe to get u_2. Press =/exe again to get u_3. This basically took u_1 and substituted it in for u_n in the equation to find u_2. Then took u_2 and substituted it in for u_n in the equation to find u_3

$u_2 =$ 0.6

$u_3 =$ 1.875

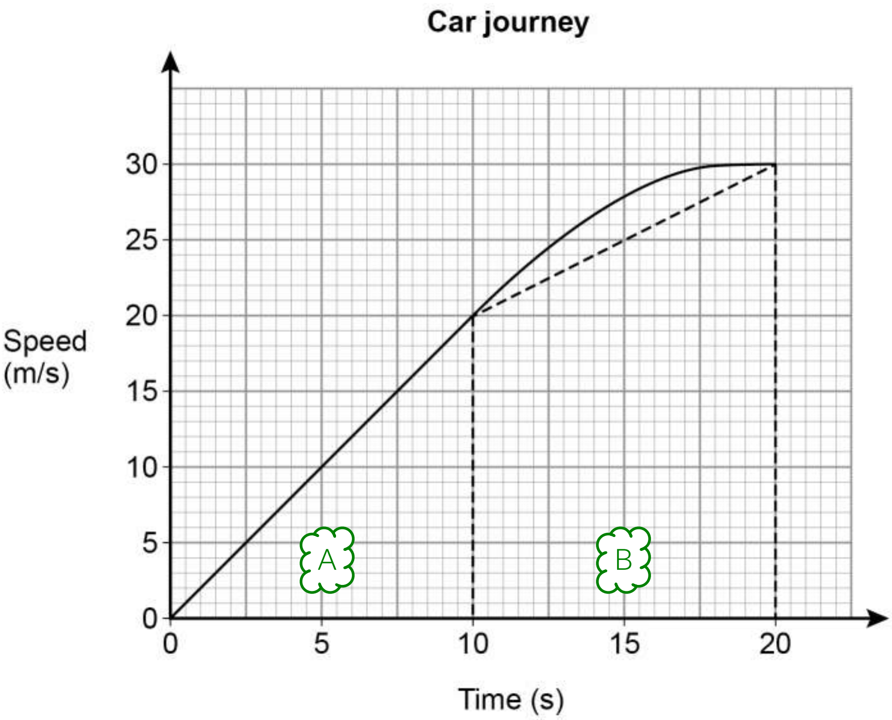
Turn over for the next question

3

24 The speed-time graph shows 20 seconds of a car journey.

Harry wants to estimate the distance the car travels in this time.

He uses a triangle and a trapezium, as shown, to estimate the area under the graph.



24 (a) Complete Harry's method to estimate the distance the car travels.

[3 marks]

$\frac{1}{2}\times10\times20=100$ ← Area of triangle A. Area of triangle = 1/2 x base x height

$\frac{1}{2}(20+30)\times10$ ← Area of trapezium B. Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them

$100+250$ ← Adding the area of shapes A and B works out an estimate of the total area under the graph, which is an estimate of the distance

Answer 350 m

24 (b) For this journey, which of these is true for Harry's method?

Tick **one** box.

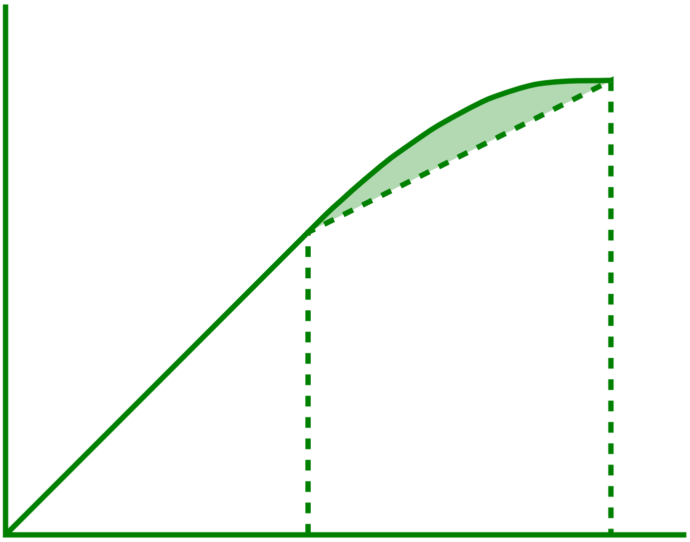
[1 mark]

☐ It works out an overestimate of the distance

☑ It works out an underestimate of the distance

☐ It could work out an overestimate or an underestimate of the distance

Turn over for the next question



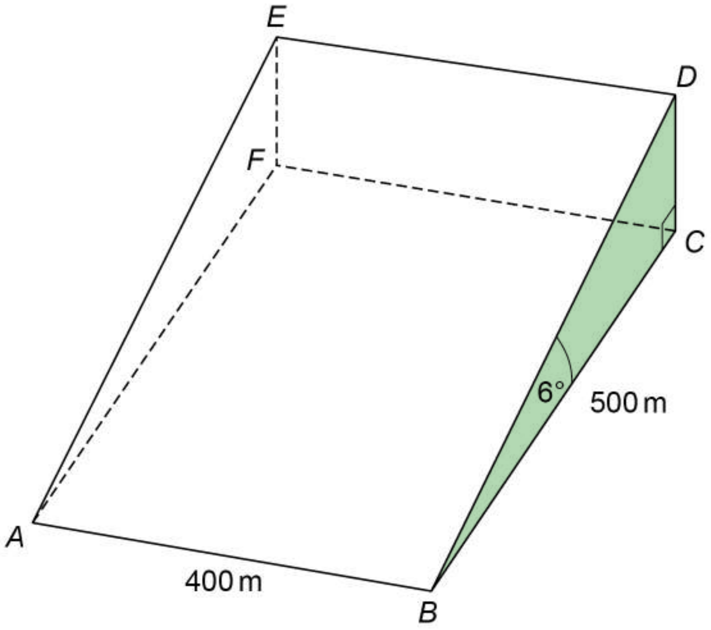
The highlighted area of the area under the graph was missed out so the area is not as much as it should be

4

25 *ABCDEF* is a triangular prism which represents part of a hill.

ABCF is the horizontal rectangular base.

D is vertically above *C*.



25 (a) Work out the height *CD*.

[2 marks]

$S\overset{\checkmark}{O}H\,C\overset{\checkmark}{A}H\,T\overset{\checkmark}{O}\overset{\checkmark}{A}$

Right angled trigonometry can be used in the highlighted triangle. Ticking A as we have the adjacent and ticking O as we are looking for the opposite. There are two ticks on the TOA formula triangle so this one can be used

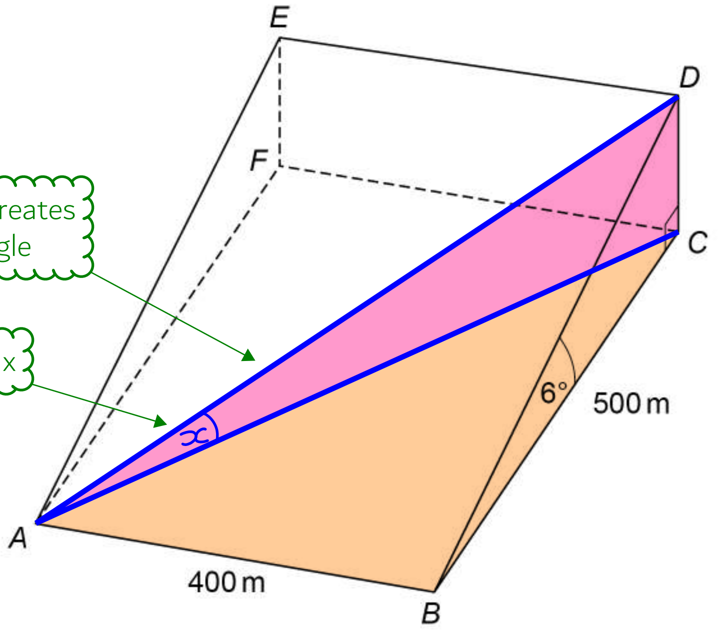
$\tan 6 \times 500$

From the formula triangle, opposite = (tan of the angle) x adjacent

Answer 52.6 m

Writing the answer to 1 decimal place but storing the exact value as A on the calculator

25 (b) Jamil walks in a straight line from *A* to *D*.



Drawing on the angle creates a right-angled triangle

Labelling angle DAC as x

Work out the size of angle *DAC*.

You **must** show your working.

[4 marks]

$400^2+500^2=AC^2$ ← Doing Pythagoras' Theorem on the orange right-angled triangle can find side AC. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting 400 m for a, 500 m for b and AC for c

$AC=\sqrt{410000}$ ← Square rooting both sides finds to undo the power of 2 on AC

$=640.3...$ ← Storing the exact value of AC as B on the calculator

$S\frac{\acute{O}}{H}\;C\frac{\acute{A}}{H}\;T\frac{\acute{O}}{\acute{A}}$ ← Using right-angled trigonometry on the pink right-angled triangle. Ticking A as we have the adjacent and ticking O as we have the opposite. There are two ticks on the TOA formula triangle so this one can be used

$\tan x=\frac{52.5...}{640.3...}$ ← Covering t in the formula triangle finds that tan of the angle = opposite/adjacent. Using the exact stored values of CD and AC

$x=\tan^{-1}(0.08...)$ ← Doing the inverse tan of both sides gets x on its own

Answer 4.7 degrees

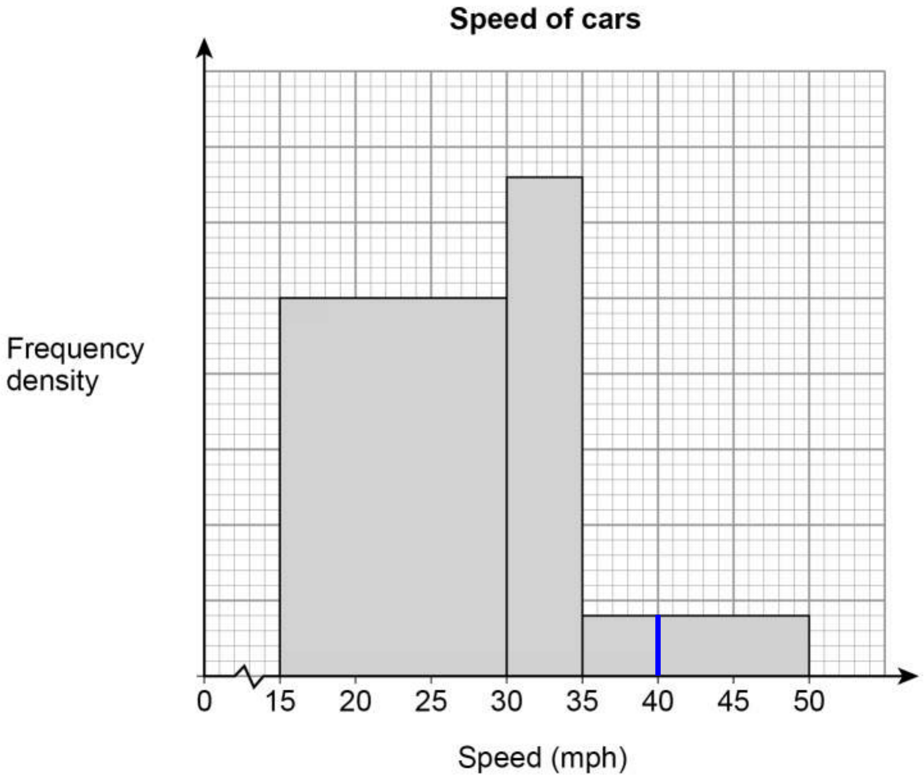
6

Turn over ►

.CG Maths.

26 The histogram shows information about the speed of cars as they pass a checkpoint.

The scale on the frequency density axis is missing.



The histogram shows information about 480 cars.

26 (a) How many cars does the first bar represent?

[4 marks]

$15 \times 25x + 5 \times 33x + 15 \times 4x$ ← Frequency is the area of each rectangle so is base x height. Let x be the height of one small box. Multiplying the base of each rectangle by the height in terms of x expresses all the frequencies. Adding all of these expressions together represents the total frequency

$600x = 480$ ← Simplifying the expression for the total frequency by ignoring x, putting it in the calculator, putting x back in, and setting it equal to the 480

$x = 0.8$ ← Dividing both sides by 480 finds x, the height of one small box

$15 \times 25 \times 0.8$ ← Substituting the value of x back into the expression of the area of the first rectangle

Answer 300

26 (b) Cars with a speed greater than 40 mph are over the speed limit.

Use the histogram to estimate the number of cars that are over the speed limit.

[2 marks]

$10\times4\times0.8$ ←

Frequency is the area of each rectangle so is base x height. Splitting the last bar gives a bar from 40 mph to 50 mph. Expressing the area of this bar. The base is 10 and the height is 4 small boxes, each of which is worth 0.8

Answer 32

Turn over for the next question

6

27 A bag contains 30 discs.

10 are red and 20 are blue.

> One disc is taken out at random and replaced by **two** of the other colour.
>
> Another disc is then taken out at random and replaced by **two** of the other colour.
>
> Another disc is then taken out at random.

Work out the probability that all three discs taken out are **red**.

[3 marks]

$\frac{10}{30} \times \frac{9}{31} \times \frac{8}{32}$ ←

Red AND red AND red. AND means to multiply the probabilities. There is one fewer red each time but one more in total each time, as one is taken out and two are added in

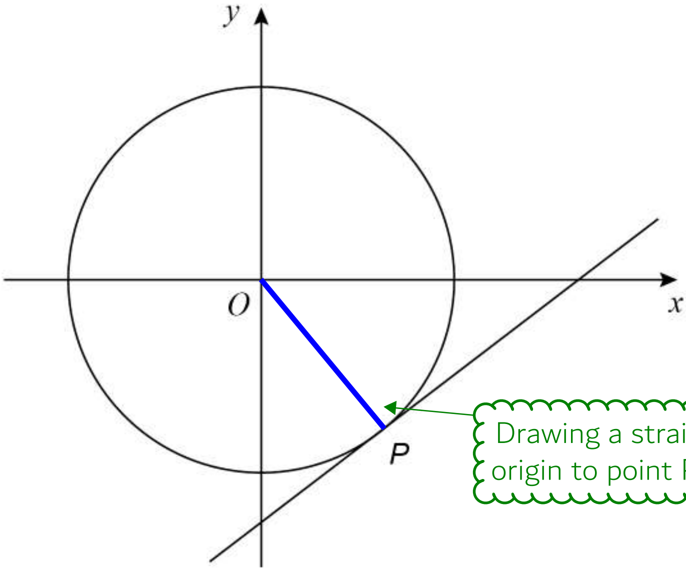
Answer $\frac{3}{124}$

Do not write outside the box

28 *P* is a point on the circle with equation $x^2 + y^2 = 80$

P has x-coordinate 4 and is below the x-axis.

Not drawn accurately



Drawing a straight line from the origin to point P creates a radius

Work out the equation of the tangent to the circle at *P*.

[5 marks]

$4^2+y^2=80$ ← Substituting in the x-coordinate of P into the equation of the circle

$y^2=64$ ← Subtracting 4^2 from both sides to get the y^2 on its own

$y=-8$ ← Doing the positive and negative square root of both sides undoes the power of 2. The positive solution is ignored as P is below the x-axis so y must be negative. So the y-coordinate of P is -2

$\frac{-8-0}{4-0}=-2$ ← Gradient = (change in y)/(change in x). Using point O and P. y changes from 0 to -8 so change in y is -8 -0. x changes from 0 to 4 so change in x is 4 - 0. So the gradient of the radius is -2

$-8=\frac{1}{2}(4)+c$ ← The gradient of the tangent is the negative reciprocal of -2, as the tangent is perpendicular to the radius, so is 1/2. Substituting in the x and y-coordinates of P and 1/2 for m in the general equation of a straight line, y = mx + c, where m is the gradient and c is the y-intercept

$c=-10$ ← Rearranging to find c by subtracting 1/2 (4) from both sides

Substituting in the values of m and c into the general equation of a straight line, y = mx + c, where m is the gradient and c is the y-intercept

Answer $y=\frac{1}{2}x-10$

END OF QUESTIONS

8