OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Monday 3 June 2024 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

H

* J 5 6 0 0 5 *

* 1 3 9 5 1 3 7 1 3 8 *

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Work out.

$1.2 \div 0.03$

$$3\overline{)1\,{}^{1}2\,0}\quad 0\,4\,0$$

Multiplying both the 1.2 and 0.03 by 100 gets an equivalent division which does not involve dividing by a decimal

..... 40 ..... **[2]**

2 Kai has these four number cards.

Kai takes two of the cards at random without replacement and finds the positive difference between the two numbers.

(a) Complete the table to show all of the possible differences.

Difference = largest - smallest

First card

Second card: Difference	0	2	5	9
0		2	5	9
2	2		3	7
5	5	3		4
9	9	7	4	

9 - 2 = 7

9 - 5 = 4

[2]

(b) Find the probability that Kai takes two cards with a difference that is an even number or a factor of 10.

Factors of 10: 1, 10, 2, 5. Even digits: 0, 2, 4, 6, 8. The differences highlighted in green are even or a factor of 10. This is 6 out of the 12 possible differences

(b) ..... $\frac{6}{12}$ ..... **[2]**

3 **(a)** Ryan makes a journey of 200 miles from his home to the coast.

$\frac{1}{10}$ of the journey is on roads with a speed limit of 40 miles per hour.

40% of the journey is on roads with a speed limit of 50 miles per hour.

The remainder of the journey takes a time of 1 hour 30 minutes.

Ryan leaves home at 08 50 and does not exceed the speed limits on the journey.

Find the earliest time that Ryan could arrive at the coast.

You must show your working.

$\begin{matrix} s & \overset{d}{} & t \end{matrix}$ ← Writing the formula triangle for speed, distance, time

$200 \div 10$ ← This works out that 1/10 of the journey is 20 miles

$\frac{20}{40} = \frac{1}{2}$ ← Covering t in the formula triangle finds that time = distance/speed. So putting the 20 miles over the 40 miles per hour. This simplifies to 1/2 hour

$2\overline{)6\ 0}$ → $3\ 0$ ← There are 60 minutes in an hour. Dividing 60 minutes by 2 works out that 1/2 hour is 30 minutes

20×4 ← 10% is 1/10 as a fraction. So multiplying the 20 miles (which was 1/10 of the journey) by 4 works out that 40% of the journey is 80 miles

$\frac{80}{50} = \frac{8}{5}$ ← Covering t in the formula triangle finds that time = distance/speed. So putting the 80 miles over the 50 miles per hour. This simplifies to 8/5 hour

$5\overline{)6\ ^{1}0}$ → $1\ 2$ ← There are 60 minutes in an hour. Dividing 60 minutes by 5 works out that 1/5 hour is 12 minutes

$\begin{matrix} 12 \\ \times\ 8 \\ \hline 96 \\ {\scriptstyle 1} \end{matrix}$ ← Multiplying the 12 minutes by 8 works out that 8/5 hours is 96 minutes

$\begin{matrix} 8: & 50 \\ +\ 0: & 30 \\ +\ 0: & 96 \\ +\ 1: & 30 \\ \hline 9: & {\scriptstyle 2}06 \end{matrix}$ ← Adding the 30 minutes, the 96 minutes and the 1 hour and 30 minutes to the time of 08 50. This gives 9 hours and 206 minutes

60, 120, 180, 240 ← Listing out the 60 times table works out that 3 lots of 60 go into the 206 minutes with a remainder of 26. So 206 minutes is 3 hours and 26 minutes

Adding the 3 hours to the 9 and leaving the remaining 26 minutes

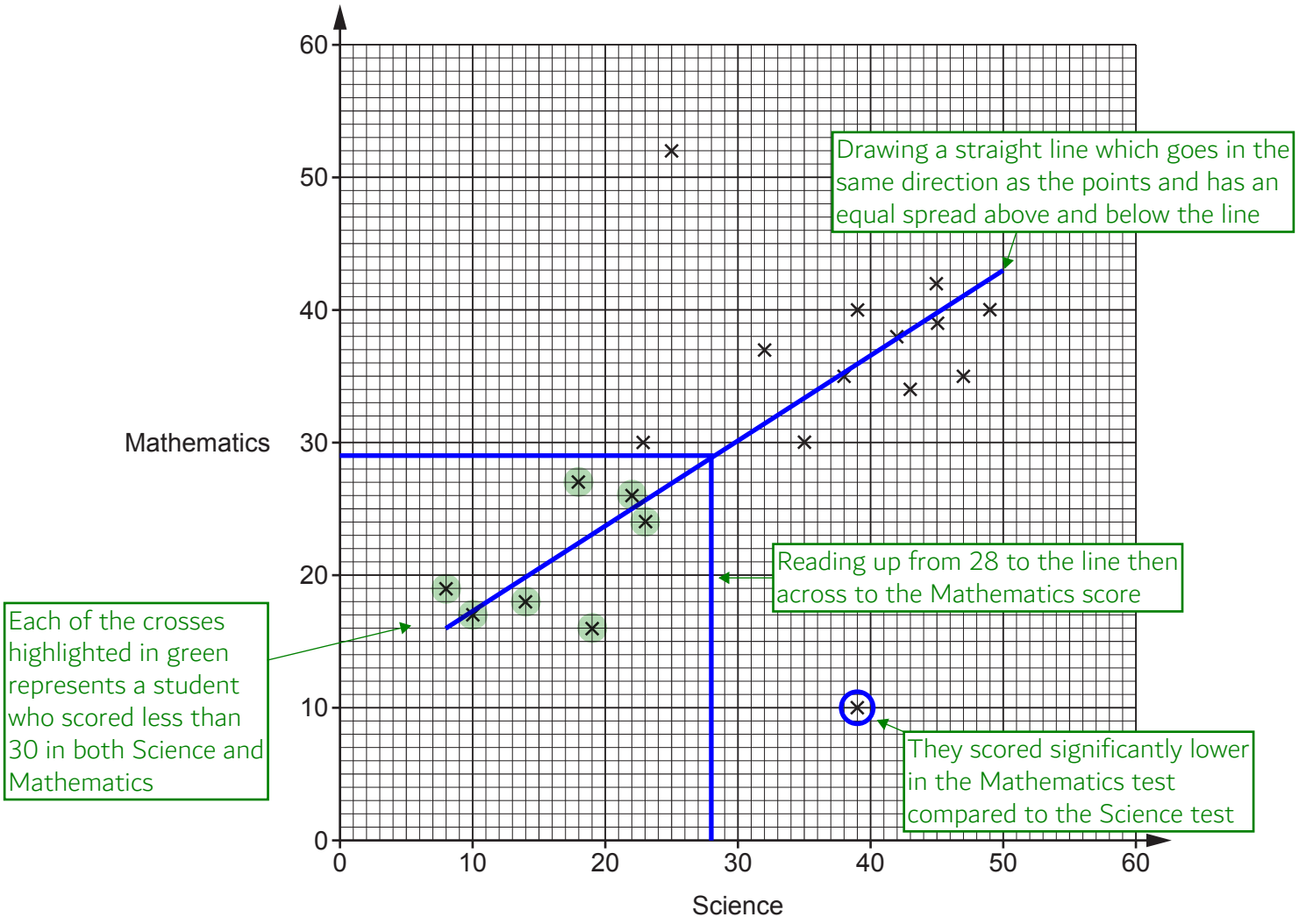
(a) 12 26 ..... **[6]**

(b) Write down an assumption you have made when working out the answer to part **(a)**.

Ryan constantly drives at the speed limit

Ryan would probably not be able to constantly drive at the speed limit as there will be traffic and other delays which would slow Ryan down. However this was assumed in order to work out the earliest possible time ..... **[1]**

4 The scatter diagram shows the test scores for 20 pupils in Science and Mathematics.



(a) Describe the type of correlation shown in the scatter diagram.

As the scores in Science increase, so does the scores in Mathematics. This is positive correlation

(a) Positive **[1]**

(b) One pupil took the Science test but was then ill during the Mathematics test and had to leave early.

On the scatter diagram, circle the point that is most likely to represent this pupil. **[1]**

(c) By drawing a line of best fit, estimate the test score in Mathematics for a pupil who scored 28 in the Science test.

(c) 29 **[2]**

(d) Explain why using the scatter diagram to estimate the test score in Mathematics for a pupil who scored 60 in Science may be unreliable.

It is outside the range of the data

The trend might not continue in a straight line **[1]**

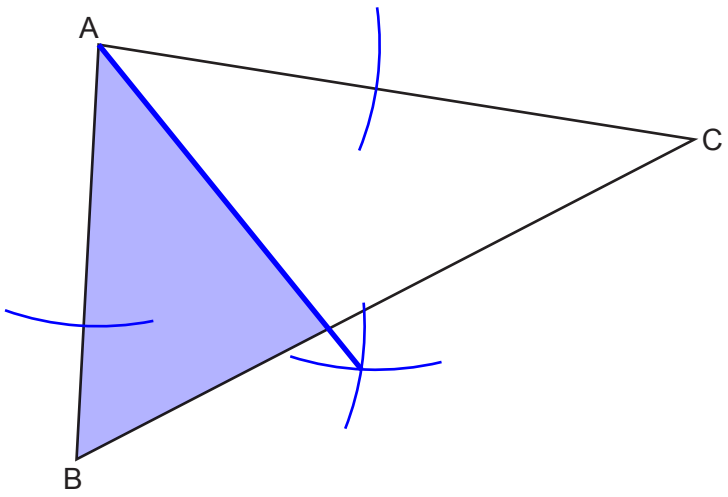
(e) Find the percentage of the 20 pupils who scored less than 30 marks in **both** Science and Mathematics.

100 ÷ 20 ← Dividing 100% by the 20 students works out that each student is 5%

5 × 7 ← There are 7 students who scored less than 30 marks in both Science and Mathematics. Multiplying 5% by 7 works out that this is 35% of the students

(e) ..... 35 ..... % **[3]**

5 Triangle ABC is drawn below.



Constructing an angle bisector of angle BAC. Scribing an arc from A on AC and on AB using a compass with the same radius. Scribing an arc from the first arc and an arc from the second arc using a compass with the same radius to form a cross. Drawing a straight line through the cross. Everything to the left of the line is closer to AB than AC

Using a ruler and compasses only, construct and shade the region which is closer to AB than AC. **[3]**

6 A bronze ornament has a volume of $198\,cm^3$.
The density of bronze is $8.9\,g/cm^3$.

By rounding each value correct to one significant figure, work out an estimate for the mass of the bronze ornament.

$\frac{m}{d\ v}$ ← Writing the formula triangle for density, mass, volume

9×200 ← Covering m in the formula triangle finds that mass = density × volume. The density rounds to 9 g/cm³ and the volume rounds to 200 cm³ to one significant figure

9 × 2 = 18 so 9 × 200 = 1800

1800

.....g **[3]**

7 A bottle contains $1\frac{3}{4}$ litres of cordial.
To make orange squash, 1 part of this cordial is mixed with 7 parts of water.
Cups that can hold $\frac{1}{6}$ of a litre are completely filled with this orange squash.

Work out the maximum number of cups that can be filled from the bottle of cordial.
You must show your working.

$1\frac{3}{4} \times 8$ ← 1 + 7 = 8 parts in total in the orange squash. So there is 8 times as much orange squash as cordial as 8 is 8 × 1. Multiplying the amount of cordial by 8 works out how much orange squash there is

$8 + 6$ ← 1 × 8 = 8 and 3/4 × 8 = 6 (as 8 ÷ 4 = 2 then 2 × 3 = 6). Adding these together works out that $1\frac{3}{4} \times 8 = 14$ litres of orange squash

$14 \times 6 = 84$ ← Dividing the 14 litres of orange squash by the 1/6 of a litre in each cup works out that the maximum number of cups is 84. To divide by a fraction: keep the first number, change the division to a multiply, flip the second fraction. So 14 ÷ 1/6 becomes 14 × 6

84

..... **[6]**

8 **(a)** y is directly proportional to x.

Write down the percentage increase in y when x is increased by 100%.

Whatever x is multiplied or divided by, y is multiplied or divided by the same amount. So if x is increased by a percentage, y will be increased by the same percentage

(a) ..... 100 ..... % **[1]**

(b) z is inversely proportional to x.

Write down the percentage decrease in z when x is increased by 100%.

x is multiplied by 2 so z will be divided by 2. This is a decrease of 50%

(b) ..... 50 ..... % **[1]**

9 The following kinematics formulas may be used in this question.

$v = u + at$

$v^2 = u^2 + 2as$

A particle has an initial velocity of 0 m/s.
The particle accelerates uniformly at 3 m/s^2 for 4 seconds.

Find the distance travelled by the particle in the 4 seconds.

$v = 0 + 3 \times 4 = 12$ ← First using v = u + at. v is the final velocity, u is the initial velocity, a is the acceleration, t is the time. Substituting 0 m/s for u, 3 m/s^2 for a, 4 seconds for t. So the final velocity, v, is 12 m/s

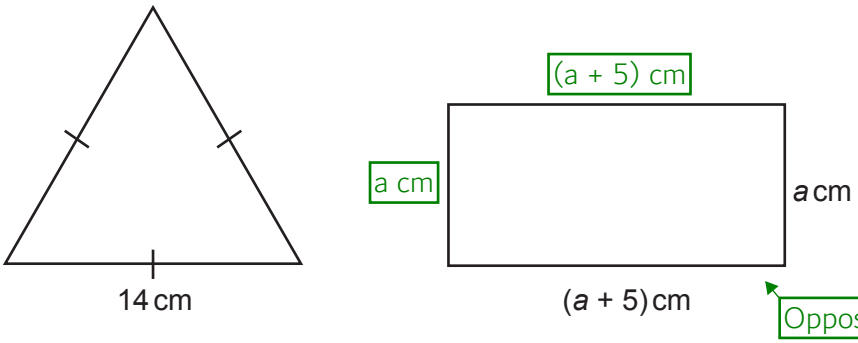
$12^2 = 0^2 + 2 \times 3 \times s$ ← Now using $v^2 = u^2 + 2as$. v is the final velocity, u is the initial velocity, a is the acceleration, s is the distance travelled. Substituting 12 m/s for v, 0 m/s for u, 3 m/s^2 for a

$144 = 6s$ ← $12^2 = 144$ and $0^2 + 2 \times 3 \times s = 0 + 6s = 6s$

$6 \overline{)\, 1\ {}^{1}4\ {}^{2}4}$ ← Dividing both sides by 6 eliminates the 6 on the right and gets s, the distance travelled, on its own

..... 24 ..... m **[4]**

10 The diagram shows an equilateral triangle and a rectangle.



Not to scale

Opposite sides of the rectangle are equal

The equilateral triangle has the same perimeter as the rectangle.

Find the value of a.

$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline 42 \\ {\scriptstyle 1} \end{array}$$

Multiplying the 14 cm by the 3 equal sides works out that the perimeter of the equilateral triangle is 42 cm. This is also the perimeter of the rectangle

$a + 5 + a + 5 + a + a$ ← Expressing the perimeter of the rectangle in terms of a by adding all of the outside sides

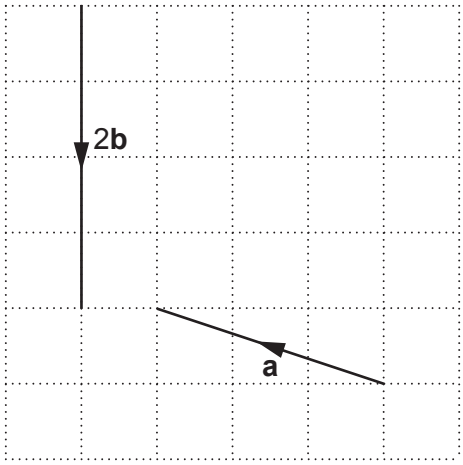
$4a + 10 = 42$ ← Simplifying the expression of the perimeter of the rectangle by collecting like terms. Setting equal to the 42 cm as this is the value of the perimeter of the rectangle

$4a = 32$ ← Subtracting 10 from both sides of the equation eliminates the +10 on the left and gets the a term on its own

Dividing both sides by 4 eliminates the 4 on the left and gets a on its own

$a =$ 8 ..... **[4]**

11 Vector **a** and vector 2**b** are drawn on this grid.



(a) Write vector **a** as a column vector.

a goes 3 to the left (-3 in the x-direction) and 1 up (1 in the y-direction)

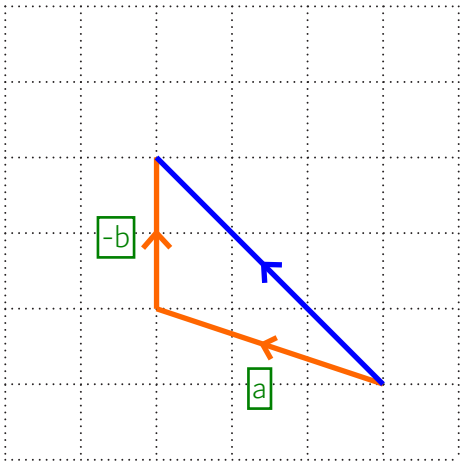
(a) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ (x-component: -3; y-component: 1) **[2]**

(b) Write a column vector that is different in direction but has the same length as vector **a**.

This is a in the opposite direction

(b) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ **[1]**

(c) On the grid below, draw the vector **a** – **b**.



[3]

Drawing a then -b at the end of it (shown in orange). Joining up the start of a and the end of -b gives the vector a - b (shown in blue). -b is half the length of 2b and in the opposite direction

12 **(a)** Sasha invests £1000 at a rate of 5% per year compound interest.
Sasha says

After one year, my investment will get £50 in interest and will be worth £1050. Therefore, after two years, my investment will get another £50 in interest and will be worth £1100.

Is Sasha correct?
Give a reason for your answer.

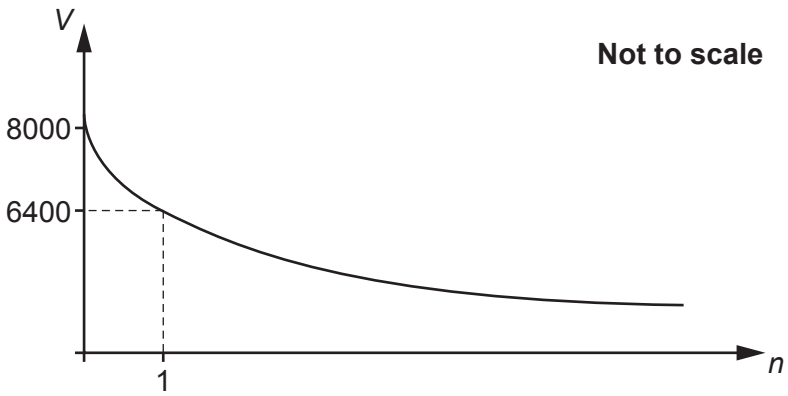
No because 5% of £1050 is not £50

Compound interest means that it increases by 5% of the previous amount each year **[1]**

(b) Sasha buys a car.
The value, £V, of the car after n years is given by the formula

$V = a \times b^n$.

The graph shows some information about the value of the car.



Find the value of a and the value of b.

$8000 = a \times b^0$ ← When n = 0, V = 8000. Substituting in these values

Anything to the power of 0 is 1 and a × 1 = a. So a = 8000

$6400 = 8000 \times b^1$ ← When n = 1, V = 6400. Substituting in these values

b^1 = b. Dividing both sides by 8000 gets b on its own

(b) $a =$ 8000

$b =$ $\frac{6400}{8000}$ **[4]**

13 Solve the inequality.

$x^2 - 100 \geqslant 0$.

$x^2 \geq 100$ ← Adding 100 to both sides eliminates the -100 on the left and gets the x^2 on its own

Doing the positive and negative square root of both sides eliminates the power of 2 and gets x on its own. When doing the negative square root, the inequality symbol must flip. The answer cannot be written as one inequality so writing them separately

$x \geq 10$
$x \leq -10$ ..... **[3]**

14 Here are the first four terms of a sequence.

$\frac{2}{5}$ $\frac{4}{9}$ $\frac{6}{13}$ $\frac{8}{17}$

Find the *n*th term of the sequence.

For the numerator: it goes up by 2 between each term so must involve 2n. Going backward in the sequence finds that the 0th term (the one before the 1st term) would be 0, so the nth term must be 2n + 0, or just 2n.

For the denominator: it goes up by 4 between each term so must involve 4n. Going backward in the sequence finds that the 0th term (the one before the 1st term) would be 1, so the nth term must be 4n + 1

$\frac{2n}{4n + 1}$ ..... **[3]**

15 Expand and simplify.

$(x+3)(4x+1)(x-2)$

$4x^2 + x + 12x + 3$ ← Expanding the first two brackets

$(4x^2 + 13x + 3)(x - 2)$ ← Simplifying by collecting like terms and writing multiplied by the third bracket

$4x^3 - 8x^2 + 13x^2 - 26x + 3x - 6$ ← Expanding these two brackets

Simplifying by collecting like terms

$4x^3 + 5x^2 - 23x - 6$ ..... **[3]**

16 Two prisms, A and B, are mathematically similar.

The ratio of the volume of prism A to the volume of prism B is 8 : 27.
The height of prism A is 6 cm.

Work out the height of prism B.

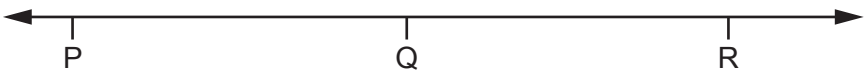
2 : 3 ← The unit of volume is cm^3 and the unit of height is cm. So cube rooting both sides of the ratio of the volumes gives the ratio of the heights

6 ÷ 2 ← The 6 cm is represented by 2 parts of the ratio of the heights. Dividing the 6 cm by 2 works out that 1 part of the ratio is worth 3 cm

3 × 3 ← Multiplying the value of 1 part of the ratio by the 3 parts representing the height of B works out that the height of B is 9 cm

9 ..... cm **[3]**

17 The diagram shows a number line.



$P = 1.\dot{2}$ and $Q = 1\frac{2}{3}$.

Q is the midpoint of PR.

Find the value of R.
Give your answer as a mixed number in its simplest form.
You must show your working.

$10P = 12.\dot{2}$ ← There is one recurring digit in P. So multiplying by ten once allows it to be written with the recurring digit in the same decimal place

$9P = 11$ ← Subtracting P from 10P cancels out the recurring digit

$P = \frac{11}{9}$ ← Dividing both sides by 9 gets P on its own and expresses P as a fraction

$\frac{5}{3}$ ← Expressing Q as an improper fraction by multiplying the 1 by the 3 and adding the result to the 2

$\frac{15}{9} - \frac{11}{9}$ ← Multiplying both the numerator and denominator of 5/3 by 3 so that the denominator is the same as 11/9. Subtracting P from Q works out the difference between them. To subtract fraction when the denominator is the same the numerators can be subtracted so the difference between P and Q is 4/9

$\frac{15}{9} + \frac{4}{9}$ ← As Q is it the midpoint of PR, the difference between Q and R must be the same as the difference between P and Q. So adding this difference to Q works out R

$\frac{19}{9}$ ← To add fractions when the denominators are the same the numerators can be added. So R is 19/9

Dividing the 19 by the 9 gives 2 with a remainder of 1.
The 2 is the whole number and the 1 is left in the fraction → $2\frac{1}{9}$

..... **[5]**

18 A sphere has radius x cm.
A cone has radius R cm and height $2R$ cm.

The volume of the sphere is equal to the volume of the cone.

Write R in terms of x.

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

$\frac{4}{3}\pi x^3 = \frac{1}{3}\pi R^2 \times 2R$ ← Setting the volume of the sphere equal to the volume of the cone. Substituting x for the radius of the sphere, R for the radius of the cone and 2R for the height of the cone

$4x^3 = 2R^3$ ← Multiplying both sides of the equation by 3 to eliminate the denominators. Dividing both sides by π cancels it out. $R^2 \times 2R = 2R^3$

$2x^3 = R^3$ ← Dividing both sides by 2 eliminates the 2 on the right

Cube rooting both sides eliminates the power of 3 to get R on its own

$R = \sqrt[3]{2}x$ ..... **[4]**

19 **(a)** Describe fully the **single** transformation that is equivalent to:

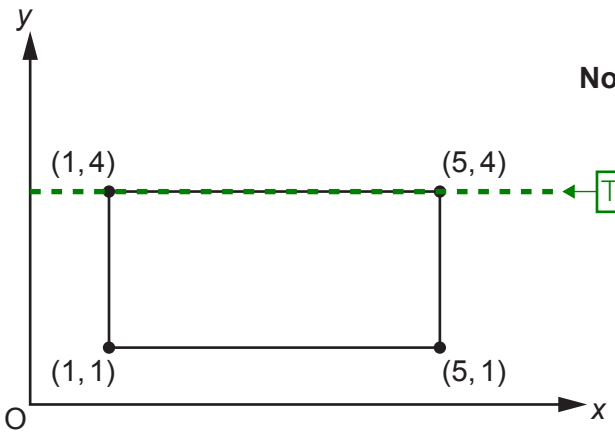
- a rotation of 20° anticlockwise about the origin, followed by
- a rotation of 70° clockwise about the origin.

20° anticlockwise could be considered as -20° clockwise. -20° + 70° = 50° clockwise

Rotation of 50° clockwise about the origin

..... **[2]**

(b) The diagram shows the coordinates of the vertices of a rectangle.



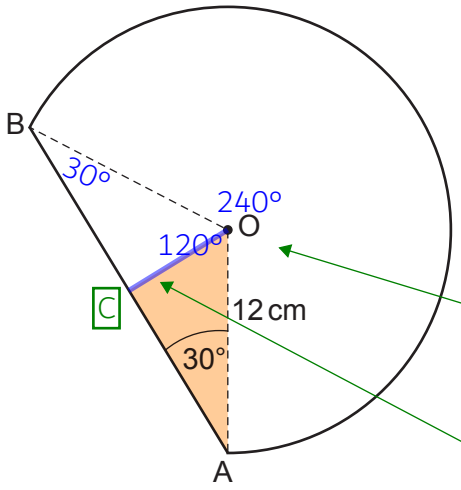
The rectangle is reflected in the line $y = 4$.

Write down the coordinates of the vertices of the rectangle that are invariant.

(b) (1, 4) (5, 4) **[1]**

These vertices will not change when reflected on the line y = 4 as they are on the line

20 The shape below is part of a circle, centre O and radius 12 cm.
Angle OAB = 30°.



Not to scale

OB = OA as they are both radii. So triangle BOA is isosceles as two of its sides are the same. The base angles of an isosceles triangle are equal so angle ABO is also 30°. Angles in a triangle add up to 180° and 180 - 30 - 30 = 120 so the minor angle AOB is 120°. Angles around a point add up to 360° and 360 - 120 = 240 so the major angle AOB is 240°

Drawing a line here forms the orange right-angled triangle

Work out the perimeter of the shape.
Give your answer in its simplest terms in the form $a\sqrt{b} + k\pi$.
You must show your working.

S O H C A H T O A

Doing right-angled trigonometry in the orange right-angled triangle to find AC. Ticking H as the 12 cm is the hypotenuse. Ticking A as AC is the adjacent. So the CAH formula triangle can be used

$\cos 30 \times 12$

Covering A in the CAH formula triangle finds that adjacent = cos of the angle × hypotenuse

0	30	45	60	90
4	3	2	1	0

Listing the angles of 0°, 30°, 45°, 60°, 90°. Listing 4, 3, 2, 1, 0 under these angles. Square rooting the 3 and putting it over 2 finds that $\cos 30 = \sqrt{3}/2$

$\frac{\sqrt{3}}{2} \times 12 \times 2$

Substituting $\sqrt{3}/2$ for cos30 expresses AC. OC is the line of symmetry for the isosceles triangle BOA. So BC = AC. Multiplying AC by 2 expresses AB

$12\sqrt{3}$

This is length AB. The 2 and the denominator of 2 cancel out

$\frac{240}{360} = \frac{24}{36} = \frac{2}{3}$

There are 360° around the centre of a circle. The sector has 240° out of the 360°. Expressing this as a fraction and simplifying it

$\frac{2}{3}\pi \times 24$

Circumference = π × diameter. The diameter is double the radius of 12 cm so is 24 cm. Doing 2/3 of the circumference of the circle works out the arc length of the sector

2/3 of 24 is 16. So the arc length of the sector is 16π cm. Adding this to AB expresses the perimeter

$12\sqrt{3} + 16\pi$ ..... **[7]**

21 Work out the coordinates of the intersection of the graphs of $y = 5 - 2x$ and $y = 3x^2$.
You must show your working.

$3x^2 = 5 - 2x$ ← Both 5 - 2x and $3x^2$ are equal to y. So $3x^2$ must be equal to 5 - 2x

$3x^2 + 2x - 5 = 0$ ← Rearranging into the quadratic form $ax^2 + bx + c = 0$ by adding 2x to both sides and subtracting 5 from both sides

$3x^2 + 5x - 3x - 5$ ← The left side could be factorised to solve for x. Multiplying a by c gives -15. Two numbers which multiply to this -15 and add to b (which is 2) are 5 and -3. Splitting the middle x-term into these numbers of x

$x(3x + 5) - 1(3x + 5)$ ← Factorising the left two terms and the right two terms separately. A factor must be brought out so, as the -3x is negative, bringing out -1 as a factor for the right two terms

$(x - 1)(3x + 5) = 0$ ← Bringing the x and -1 together into a bracket and writing the 3x + 5 once. The left side is now factorised. Setting it equal to 0 again

$x = 1$ ← One of the two brackets must be 0 to multiply to 0. When x - 1 = 0, x = 1

$y = 5 - 2(1) = 3$ ← Substituting 1 for x in y = 5 - 2x finds that y = 3

$x = -\frac{5}{3}$ ← One of the two brackets must be 0 to multiply to 0. When 3x + 5 = 0, x = -5/3

$y = 5 - 2(-\frac{5}{3})$ ← Substituting -5/3 for x in y = 5 - 2x to find y

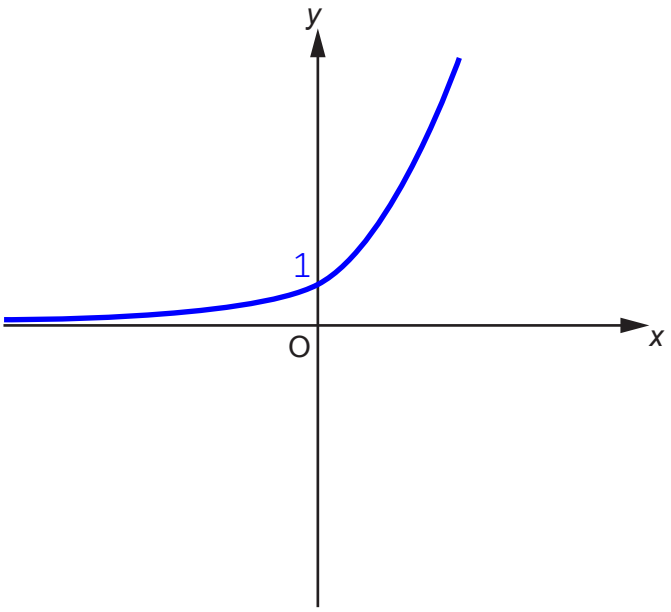
$= \frac{15}{3} + \frac{10}{3}$ ← Expressing 5/1 as 15/3 so that the denominator is 3 and is the same as the other fraction. -2(-5/3) = 10/3

(.....1....., ...3.....) and (.....$-\frac{5}{3}$....., ...$\frac{25}{3}$.....) **[6]**

When x = 1, y = 3

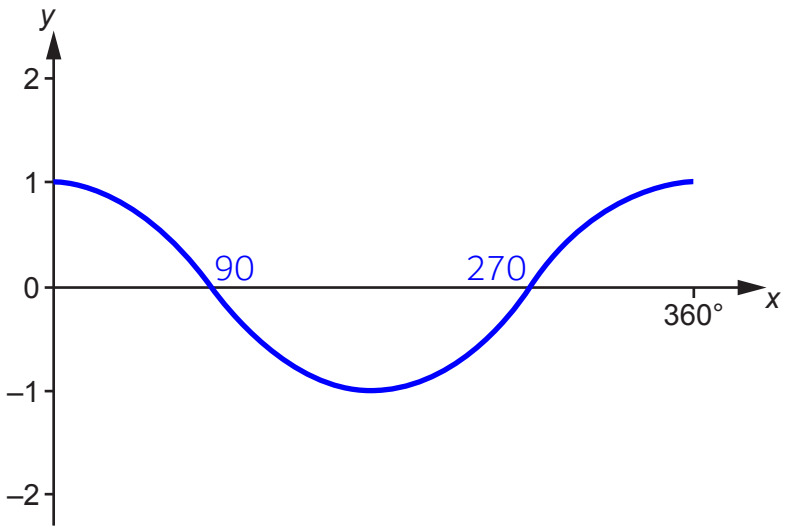
When x = -5/3, y = 25/3

22 **(a)** Sketch the graph of $y = 8^x$.
Indicate any values where the graph crosses the axes.



An exponential curve. $8^0 = 1$ so it crosses the y-axis at (0, 1) **[2]**

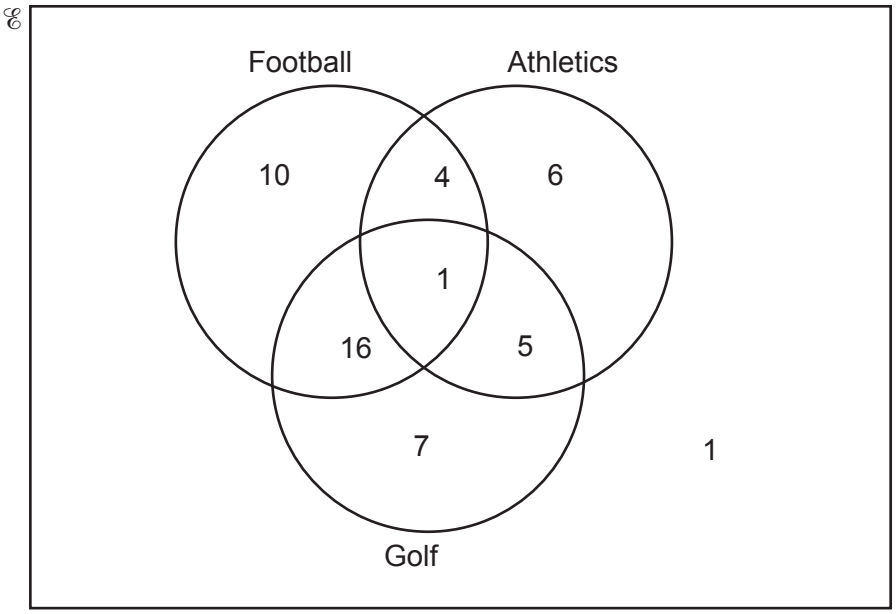
(b) Sketch the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$.
Indicate any values where the graph crosses the x-axis.



[2]

This graph needs to be memorised to answer this question

23 In a survey, 50 people are asked which sports they watch.
The results are shown on the Venn diagram.



(a) One person is chosen at random from those that watch athletics.

Find the probability that this person watches **only one** other sport.

4 of the people who watch athletics also watch football only, 5 of the people who watch athletics also watch golf only. 4 + 5 = 9 of the people who watch athletics that watch only one other sport. 4 + 1 + 5 + 6 = 16 people who watch athletics. So 9 out of the 16 people who watch athletics watch only one other sport

(a) $\frac{9}{16}$ **[2]**

(b) Two of the 50 people are chosen at random.

Show that the probability that one of them watches **only** football and the other watches **only** golf is $\frac{2}{35}$. **[3]**

$\frac{10}{50} \times \frac{7}{49} + \frac{7}{50} \times \frac{10}{49}$ ← Football only AND golf only OR golf only AND football only. AND means to multiply the probabilities. OR means to add the probabilities. After the first person is picked there are only 49 people left to pick from

$\frac{1}{5} \times \frac{1}{7} + \frac{1}{5} \times \frac{1}{7}$ ← Simplifying by cancelling out common factors of the numerators and denominators in each pair of fractions which are multiplied

$\frac{1}{35} + \frac{1}{35}$ ← To multiply fractions: multiply the numerators and multiply the denominators

$\frac{2}{35}$ ← To add fractions with the same denominators: add the numerators and the denominator stays the same

END OF QUESTION PAPER