Friday 8 November 2024 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

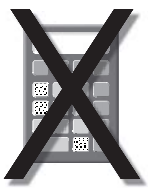
First name(s) ____________________

Last name ____________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined page at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.



Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Work out.

$$1\frac{5}{6}-\frac{2}{3}\div\frac{3}{4}$$

$\frac{2}{3} \times \frac{4}{3}$ ← The order of operations (BIDMAS) needs to be followed so doing the division first. To divide fractions: keep the 1st fraction, change the division to a multiplication, flip the 2nd fraction. Then to multiply fractions: multiply the numerators and multiply the denominators. So it becomes 8/9

$\frac{11}{6} - \frac{8}{9}$ ← Converting the mixed fraction to an improper fraction by multiplying the 1 by the 6 then adding the result to the numerator

$\frac{99}{54} - \frac{48}{54}$ ← To subtract fractions the denominators need to be the same. Multiplying both the numerator and denominator of the 1st fraction by 9 and multiplying both the numerator and denominator of the 2nd fraction by 6 makes both denominators 54

The numerators are subtracted and the denominator stays the same →

$\frac{51}{54}$ .... **[4]**

2 **(a)** Work out the size of an exterior angle of a regular hexagon.

$6 \overline{)3\ {}^{3}6\ 0}$ = 0 6 0 ← The exterior angles of any polygon add up to 360°. Dividing this 360° by the 6 exterior angles of a hexagon works out that each exterior angle is 60°

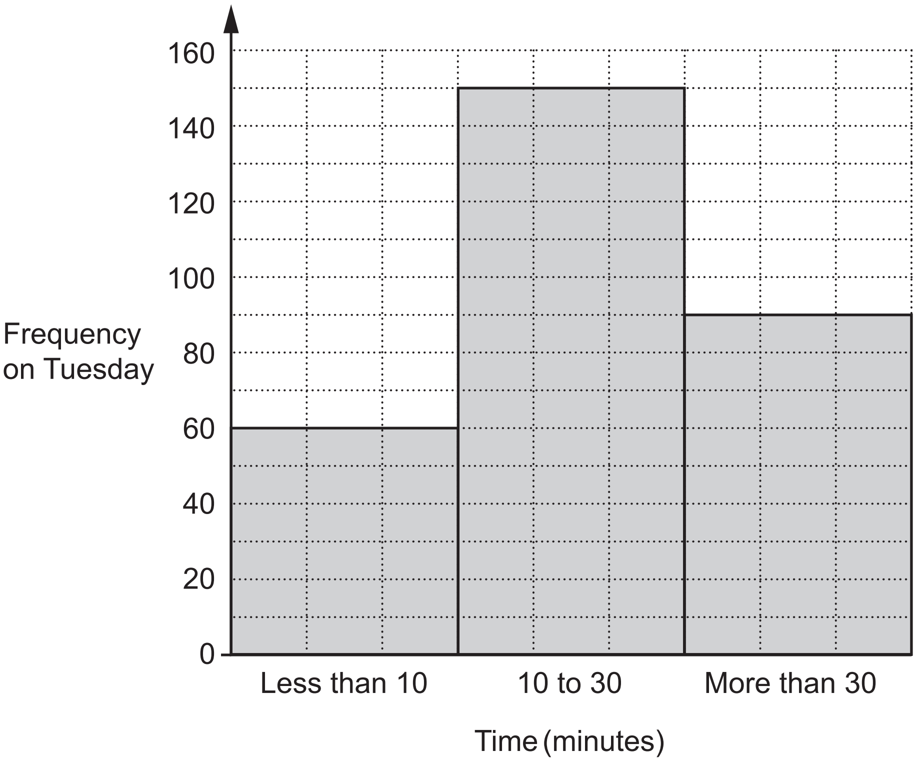
(a) ....60°.......... **[2]**

(b) Use your answer to part **(a)** to write down the size of an interior angle of a regular hexagon.

The interior and exterior angle of a polygon form a straight line. The angles around a point on a straight line add up to 180°. So subtracting the exterior angle from 180° leaves the interior angle. 180 - 60 = 120

(b) ....120°.......... **[1]**

3 The graph shows the time, in minutes, taken by some pupils to travel to school on **Tuesday**.



(a) Find the percentage of these pupils that took more than 30 minutes to travel to school.

60
+ 150
+ 90
300

Adding the 60 who took less than 10 minutes, the 150 who took 10 to 30 minutes and the 90 who took more than 30 minutes works out that there were 300 pupils

$\frac{90}{300} = \frac{30}{100}$

90/300 of the pupils took more than 30 minutes. Dividing both the numerator and denominator by 3 simplifies it to a fraction with 100 as the denominator

Percentage is out of 100 → **(a)** ....30............ % **[3]**

(b) On **Tuesday** the number of pupils taking 10 to 30 minutes to travel to school was 25% less than on Monday.

Find the number of pupils taking 10 to 30 minutes to travel to school on **Monday**.

100 - 25 ← Subtracting the 25% from 100% works out that Tuesday has decreased to 75% of Monday

150 ÷ 75 ← Dividing the 150 who took 10 to 30 minutes on Tuesday by 75 works out that 1% of Monday is 2 pupils

2 × 100 ← Multiplying the value of 1% by 100 works out that 100% of Monday is 200 pupils

(b) ....200............ **[3]**

4 An electrician charges £30 per visit plus £22 per hour.

Write an expression for the cost, in £, charged by the electrician for one visit lasting *n* hours.

Multiplying the £22 by the number of hours n. Then adding this to the £30

£ .... $30 + 22n$ .... **[2]**

5 Anika has a shelf 79.6 cm long.
She has many books, each of width 3.4 cm.
Anika puts two paperweights, each of width 5 cm, and the maximum possible number of books on the shelf.

Work out the amount of space on the shelf that is left over.
You must show your working.

5×2 ← Multiplying the width of each paperweight by 2 works out that the width of both paperweights combined is 10 cm

$79.6 - 10$ ← Subtracting the 10 cm width of both paperweights from the 79.6 cm length of the shelf works out that there is 69.6 cm left for the books

$34 \overline{)6\ {}^{6}9\ {}^{1}6}$ = 0 2 0 r16 ← Dividing the 69.6 cm left for the books by the 3.4 cm width of each book works out how many books can fit and the remainder, which is to do with the amount of space left over. Multiplying both the 69.6 and 3.4 by 10 eliminates the decimals to make the division easier

34, 68 ← Listing the 34 times table can help with the division

The remainder of 16 needs to be divided by 10 to give the space left over as the 79.6 and 3.4 were multiplied by 10 (note that the 20 represents the number of books and this would not need to be divided by 10)

.... 1.6 .... cm **[5]**

6 Jack has ten cards numbered 11 to 20.
He picks a card at random.

Jack says,

In these ten cards, there are two multiples of 5 and five even numbers.
Therefore, the probability that I pick a card that is a multiple of 5 or an even number is

$\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$.

Describe the error in Jack's method and give the correct answer.

The error is 20 is counted twice ← As it is both an even number and a multiple of 5

The correct answer is $\frac{6}{10}$ **[2]**

The numbers 12, 14, 15, 16, 18, 20 are multiples of 5 or even numbers. This is 6 out of the 10 numbers

7 Felix makes craft figures at a constant rate.
He can make 5 craft figures in 40 minutes.

(a) Find the number of craft figures Felix can make in 4 hours.

4 × 60 ← 1 hour = 60 minutes. So multiplying the 4 hours by 60 converts it to 240 minutes

240 ÷ 40 ← Dividing the 240 minutes by the 40 minutes works out that the 240 minutes is 6 lots of the 40 minutes

5 × 6 ← It is 6 lots of the 40 minutes so it will also be 6 lots of the 5 craft figures

(a) 30 **[3]**

(b) Darcie makes craft figures 10% quicker than Felix. ← This is referring to the time being 10% less, not the speed being 10% more

Work out how long Darcie takes to make 15 craft figures.

15 ÷ 5 ← Dividing the 15 craft figures by the 5 craft figures works out that the 15 craft figures is 3 lots of the 5 craft figures

40 × 3 ← It is 3 times as many craft figures so will take 3 times the time. So it would take 120 minutes before reducing it by 10%

120 ÷ 10 ← This finds that 10% of the 120 minutes is 12 minutes

120 - 12 ← This reduces the 120 minutes by 10%

(b) 108 minutes **[3]**

8 Here is a question and an incorrect answer.

> Question:
>
> Expand the brackets and simplify fully.
>
> $3(a + 2b) + a$
>
> Answer:
>
> $a4 + 6 \times b$

Explain why the answer is **not** correct.

The 4 should be before the a and the 6 × b should be 6b

$3a + 6b + a$ ← Expanding the brackets. $3 \times a = 3a$ and $3 \times 2b = 6b$

$4a + 6b$ ← Simplifying fully by collecting like terms. $3a + a = 4a$

[2]

9 Solve.

$3x + 12 = 9 - 7x$

$10x + 12 = 9$ ← Adding 7x to both sides gets all the x on the same side

$10x = -3$ ← Subtracting 12 from both sides gets the x term on its own

Dividing both sides by 10 gets x on its own

$x =$ -0.3 **[3]**

10 A straight line has equation $y = 4x + 9$.

(a) Write down the gradient of the line.

The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept

(a) ....4............................. **[1]**

(b) Casey says the graph of $y = 4x + 9$ passes through the point (3, 23).

Is Casey correct?
Show how you decide.

$y = 4 \times 3 + 9$ ← Substituting the x-coordinate into the equation works out what the y-coordinate should be

$= 21$ ← The y-coordinate should be 21 when x is 3

No ← It goes through the point (3, 21), not (3, 23)

.... because ....

.... **[2]**

11 A bag only contains green, red, blue and yellow discs.

Orla carries out an experiment.
She picks one disc at a time from the bag, records its colour and then returns the disc to the bag.
When she has finished the experiment, Orla works out the relative frequency of each colour.
Some of her results are shown in the table.

Colour	Green	Red
Relative frequency	0.35	0.25

The relative frequency of the yellow discs was three times the relative frequency of the blue discs.

In total, there are 2000 discs in the bag.

(a) Use this information to find an estimate for the **total** number of green and yellow discs that are in the bag.
You must show your working.

$0.35 + 0.25 = 0.60$

Adding the relative frequencies for green and red works out that the relative frequency of green and red combined was 0.6

$1.0 - 0.6 = 0.4$

Subtracting the relative frequency for green and red combined from 1 works out that the relative frequency of yellow and blue discs combined was 0.4

$0.4 \div 4 = 0.1$

The ratio of yellow to blue is 3 : 1. There are 4 parts in total which represent the combined relative frequency of 0.4. So dividing the 0.4 by the 4 parts works out that 1 part of the ratio is worth 0.1

$0.1 \times 3 = 0.3$

Yellow is represented by 3 parts of the ratio. So multiplying the value of 1 part of the ratio by 3 works out that the relative frequency for yellow was 0.3

$0.35 + 0.30 = 0.65$

Adding the relative frequencies of green and yellow works out that the relative frequency of green and yellow combined was 0.65

$0.65 \times 2000 = 1300.00$

Multiplying the relative frequency of green and yellow combined by the 2000 total discs estimates that there are a total of 1300 green and yellow counters

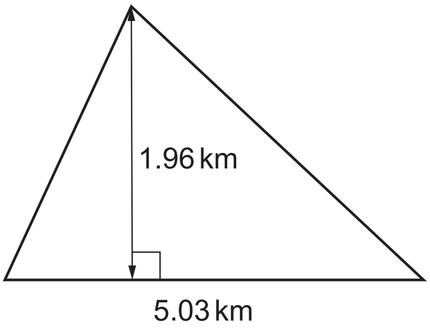
(a) ....1300........ **[5]**

(b) Explain why your estimate may **not** be reliable.

Not many discs may have been picked

The more discs picked, the more reliable the estimated relative frequencies of the discs in the bag .... **[1]**

12 A housing estate is built on a triangular piece of land.



Not to scale

There are 3951 people living on the estate.

Work out an **estimate** of the population density of the estate in people per km^2.

$\frac{1}{2} \times 5 \times 2$ ← Area of triangle = 1/2 × base × height. The base is 5 km to 1 significant figure and the height is 2 km to 1 significant figure. So the area of the land is roughly 5 km^2

$5 \overline{)4\ {}^{4}0\ 0\ 0}$ = 0 8 0 0 ← Per means to divide. People per km^2 means to divide the number of people by the area in km^2. 3951 people is 4000 people to 1 significant figure

..........800.......... people per km^2 **[4]**

13 Write $\frac{4}{11}$ as a recurring decimal.

$11 \overline{)4.{}^{4}0\ {}^{7}0}$ = 0. 3 6 ← Dividing the numerator by the denominator converts the fraction to a decimal. The remainder of 4 repeats so the digits 3 and 6 must be recurring

..........$0.\dot{3}\dot{6}$.......... **[2]**

14 The expected value of a painting, £*P*, is given by the formula

$P = 2500 \times 1.2^n$

where *n* is the number of years after it was bought and $0 \leqslant n \leqslant 4$.

(a) Write down the value of the painting when it was bought.

$1.2^0 = 1$ (as anything to the power of 0 is 1) and $2500 \times 1 = 2500$

(a) £ .... 2500 .... **[1]**

(b) Write down the annual percentage increase in the expected value of the painting.

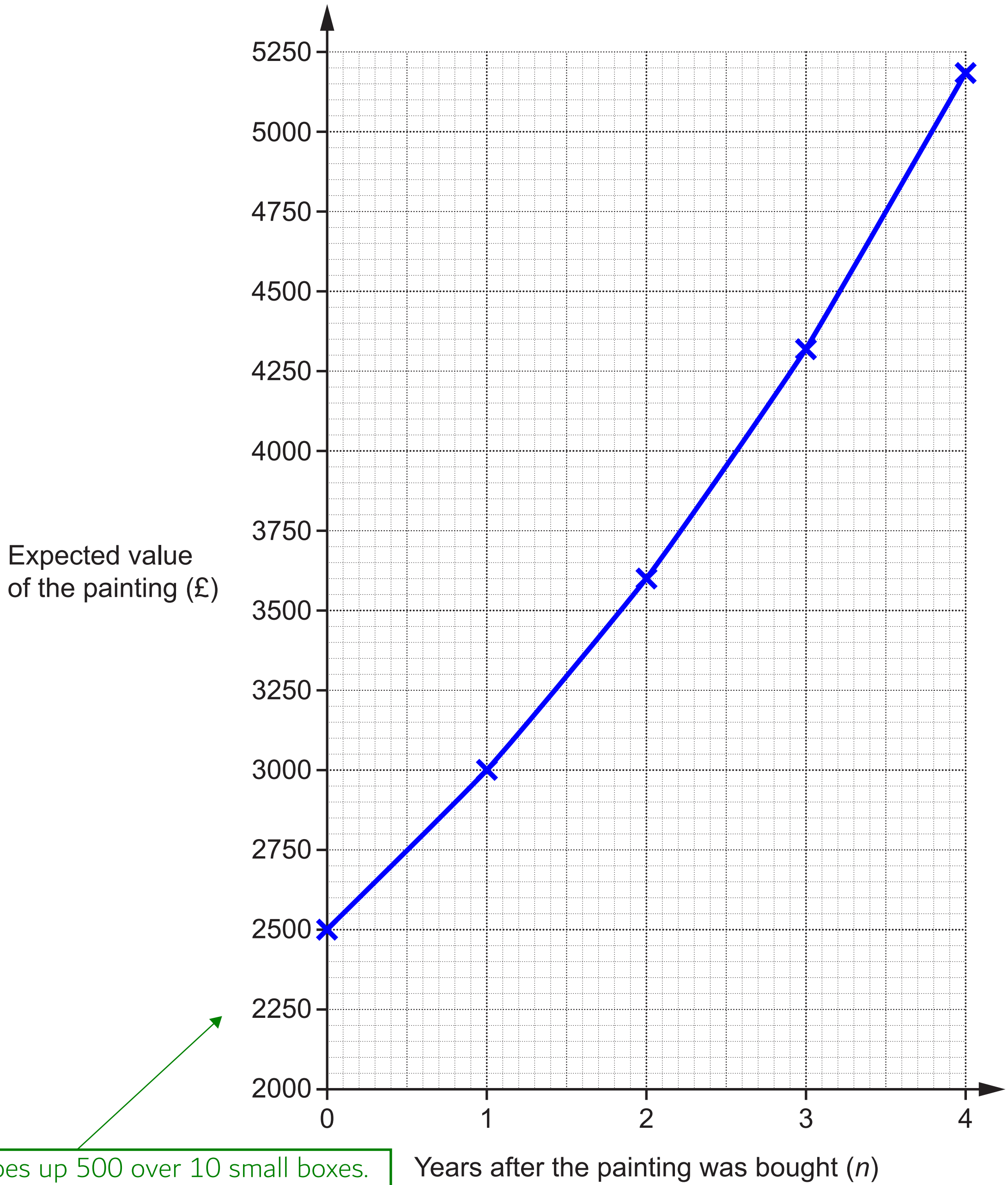
Multiplying the 1.2 by 100 converts it to 120%, which is an increase of 20% compared to 100%

(b) .... 20 .... % **[1]**

(c) The table shows the expected value of the painting *n* years after it was bought.

Years after the painting is bought (*n*)	1	2	3	4
Expected value of the painting (£)	3000	3600	4320	5184

On the page opposite, draw a suitable graph to show the expected value of the painting *n* years after it was bought, where $0 \leqslant n \leqslant 4$.



This scale goes up 500 over 10 small boxes.
500 ÷ 10 = 50, so each small box is worth 50

[3]

(d) An art collector correctly works out 2500×1.2^{10} as 15479.

They say,

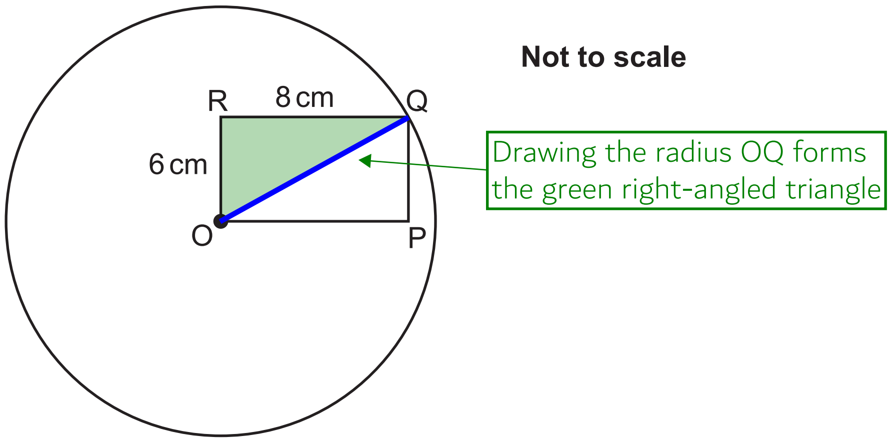
The expected value of the painting 10 years after it was bought is £15479.

What assumption has the art collector made.

The formula works works after 4 years

It stated that n is the number of years after it was bought and $0 \leq n \leq 4$. 10 years is more than 4 years **[1]**

15 (a) The diagram shows a rectangle, OPQR, and a circle, centre O, which passes through Q.
OR = 6 cm and RQ = 8 cm.



Not to scale

Find the circumference of the circle.
Give your answer in terms of π.

$6^2 + 8^2 = r^2$ ← Using Pythagoras' Theorem in the green right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side

$36 + 64 = 100$ ← $6^2 = 36$ and $8^2 = 64$. Adding these works out that $r^2 = 100$

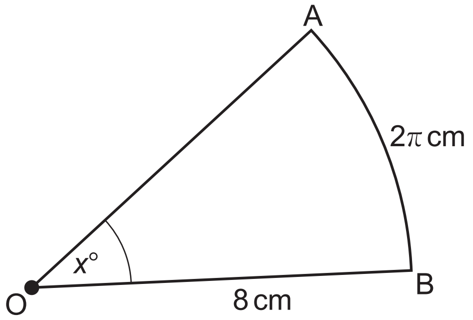
$r = \sqrt{100}$ ← Square rooting both sides of $r^2 = 100$ works out that the radius of the circle is 10 cm

10×2 ← The diameter is double the radius so is 20 cm

Circumference = π × diameter

(a) ....20π........ cm **[4]**

(b) AOB is a sector of a circle, centre O and radius 8 cm.
Angle AOB = x°.
The arc, AB, has length 2π cm.



Not to scale

Find the area of the sector.
Give your answer in terms of π.

8×2 ← The diameter is double the radius so is 16 cm

$\frac{x}{360} \times \pi \times 16 = 2\pi$ ← Circumference = π × diameter. Doing x/360 of the circumference will give the arc length AB

$360 \times 2 = 720$ ← Multiplying both sides by 360 gives x × π × 16 = 720π

$720 \div 16 = 045$ ← Dividing both sides by 16 gives x × π = 45π. Then dividing both sides by π finds that x = 45

16, 32, 48, 64, 80 ← Listing the 16 times table helps to divide by 16

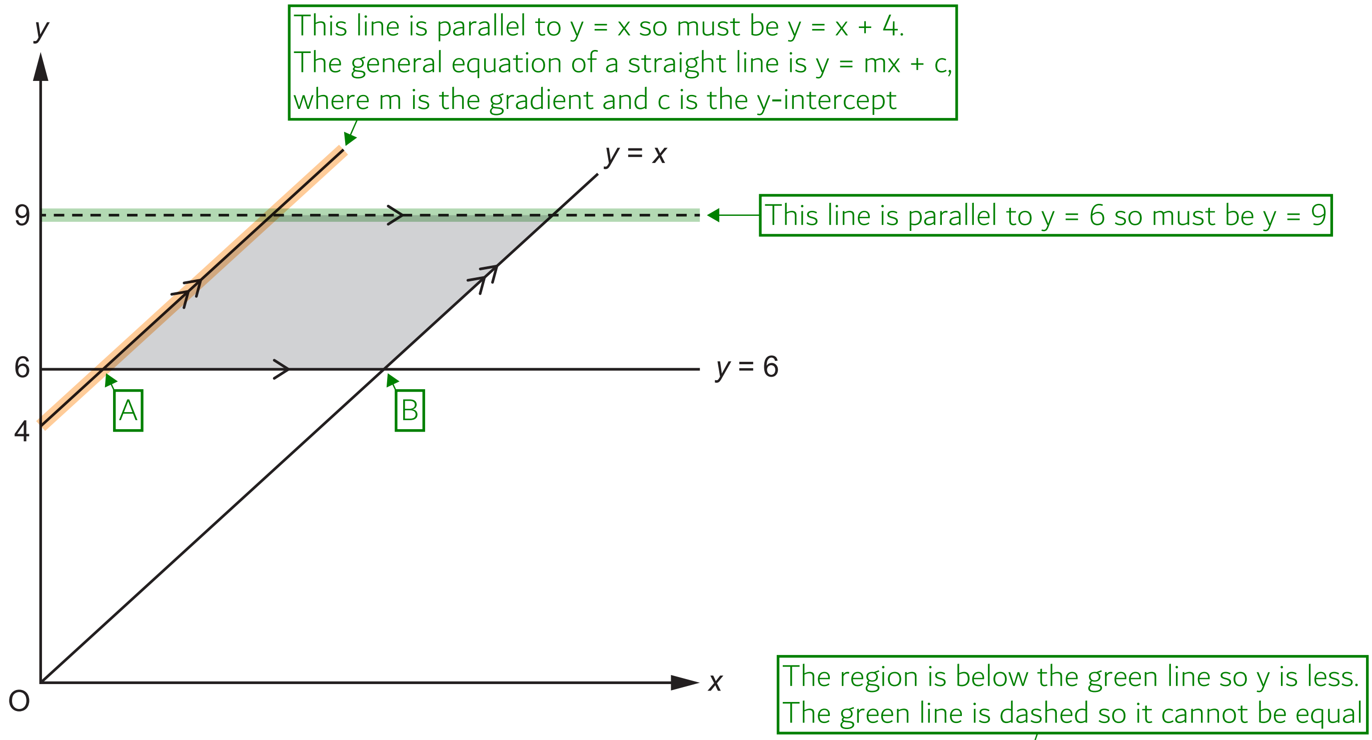
$\frac{45}{360}$ ← Expressing the fraction of the circle which the sector is. Dividing both the numerator and denominator by 45 simplifies it to 1/8

$\frac{1}{8} \times \pi \times 8^2$ ← Doing 1/8 of the area of the circle. Area of circle = π × radius²

$1/8 \times 8^2 = 8$ then $8 \times \pi = 8\pi$

(b) ....8π........ cm² **[4]**

16 In the diagram below, the shaded region is a parallelogram.
The parallelogram can be identified by four inequalities.
Two of the inequalities are $y \geqslant 6$ and $y \geqslant x$.



(a) Write down the other **two** inequalities that identify the parallelogram.

(a) .... $y < 9$ ....

The region is below the orange line so y is less. The orange line is solid so it can be equal → .... $y \leq x + 4$ ....

[3]

(b) Work out the area of the parallelogram.
You must show your working.

$6 = x + 4$ ← Doing simultaneous equations with y = 6 and y = x + 4 to work out the x-coordinate of point A. Substituting 6 for y in y = x + 4

$2 = x$ ← Subtracting 4 from both sides finds that the x-coordinate of point A is 2

4×3 ← Area of parallelogram = base × height. The base is 4 as this is the distance between the x-coordinate of A and the x-coordinate of B. The x-coordinate of B is 6 as it is on the lines y = 6 (so its y-coordinate is 6) and y = x (so the x-coordinate is the same as the y-coordinate). The height of the parallelogram is 3 as this it the vertical distance between y = 6 and y = 9

(b) .... 12 .... square units **[4]**

17 A farmer grows pumpkins.
The farmer records the masses, *m* kilograms, of 80 of their pumpkins.
The table shows the results.

Mass (*m* kg)	$0 < m \leqslant 5$	$5 < m \leqslant 10$	$10 < m \leqslant 15$	$15 < m \leqslant 20$	$20 < m \leqslant 25$
Frequency	10	22	28	14	6

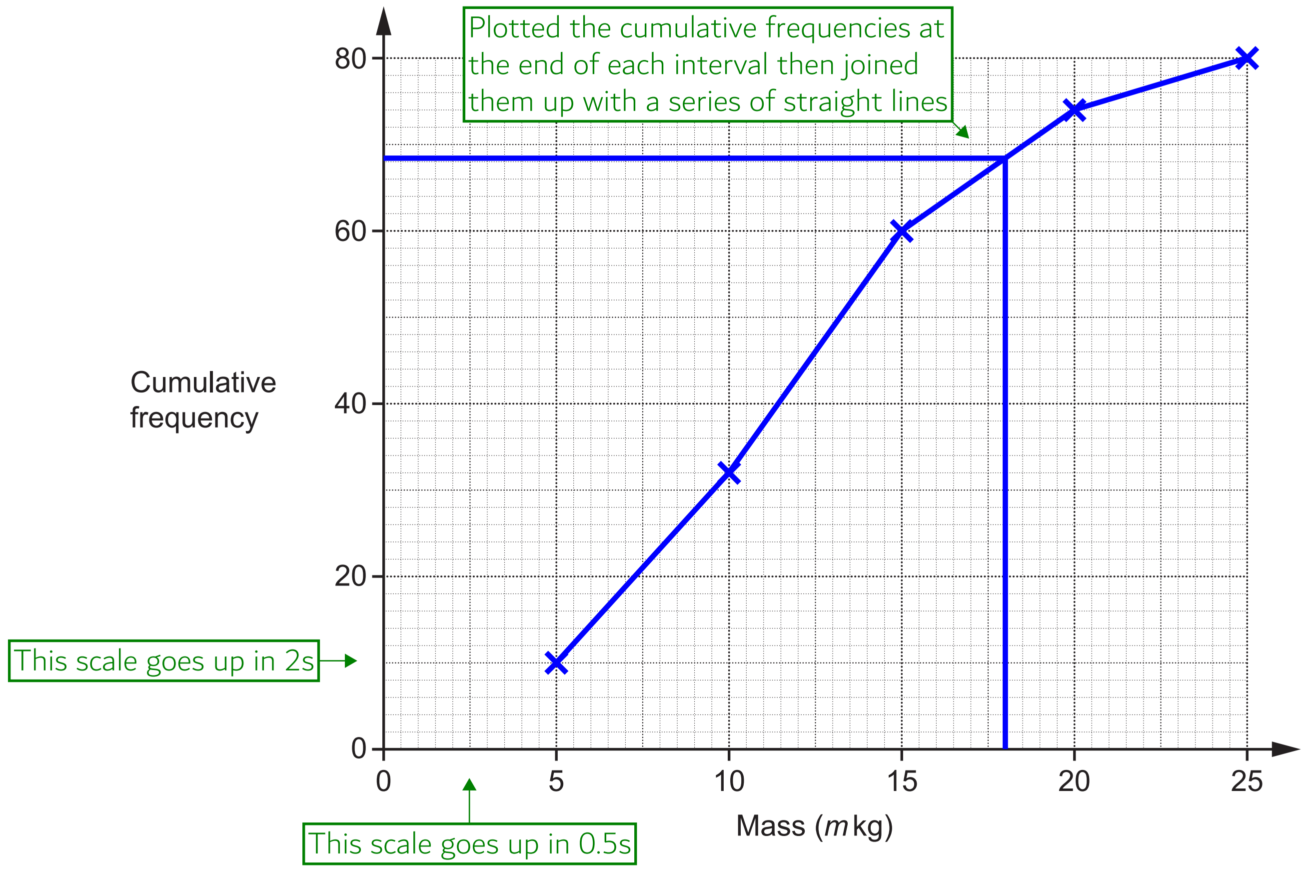
(a) Complete the cumulative frequency table.

Adding the frequencies as they go. 32 + 28 = 60, then 60 + 14 = 74, then 74 + 6 = 80

Mass (*m* kg)	$m \leqslant 5$	$m \leqslant 10$	$m \leqslant 15$	$m \leqslant 20$	$m \leqslant 25$
Cumulative frequency	10	32	60	74	80

[1]

(b) Draw the cumulative frequency graph to represent these results.



[3]

(c) Write down the upper quartile of the mass of the 80 pumpkins.

The upper quartile is 3/4 of the way through the 80 so is roughly the 60th, which is 15 kg

(c) ....15........ kg **[1]**

(d) The farmer picks a pumpkin at random.

Find an estimate for the probability that the pumpkin has a mass greater than 18 kg.

Reading up from 18 kg to the line then across estimates that 68 have a mass of 18 kg or less. So 12 would have a mass greater than 18 kg. This can be given as a fraction of the 80

(d) ....$\frac{12}{80}$........ **[2]**

18 Solve.

$$\frac{x^2-5}{x-4}=4x$$

You must show your working.

$x^2 - 5 = 4x(x - 4)$ ← Multiplying both sides by x - 4 to eliminate the denominator on the left

$= 4x^2 - 16x$ ← Expanding the brackets on the right

$0 = 3x^2 - 16x + 5$ ← Subtracting x^2 from both sides and adding 5 to both sides to put it into the quadratic form

$= (3x - 1)(x - 5)$ ← Factorising the right. There must be double brackets. Putting 3x in the 1st bracket and x in the 2nd bracket as this is the only way to get $3x^2$ when expanding. The only way to get the +5 when expanding while making a negative x term is using -5 and -1. To expand to get -16x, the -1 must go in the 1st bracket and the -5 must go in the 2nd bracket

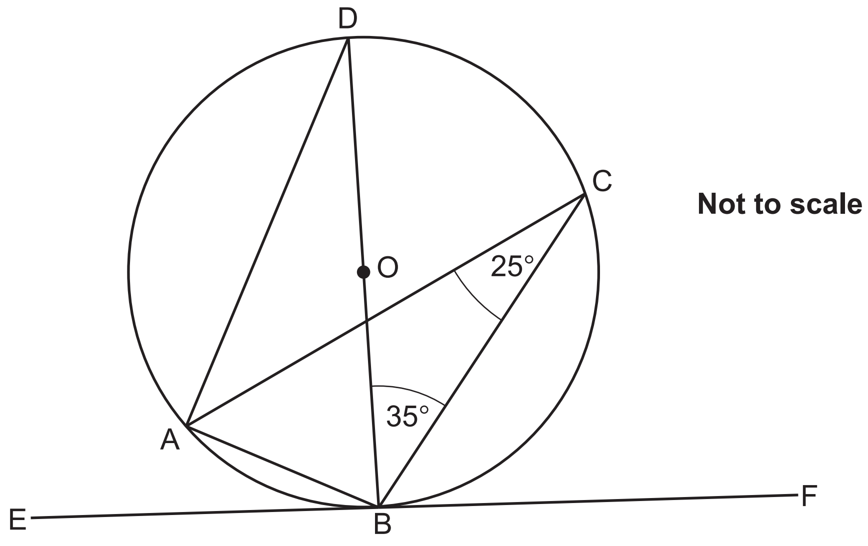
$3x - 1 = 0$ ← One of the two brackets must be 0. 3x - 1 could be 0

$3x = 1$ ← Adding 1 to both sides get the x term on its own. Then dividing both sides by 3 gets x on its own and finds that x = 1/3

$x - 5 = 0$ ← One of the two brackets must be 0. x - 5 could be 0. Then adding 5 to both sides gets x on its own and finds that x = 5

x = ...$\frac{1}{3}$.......... or x = ...5.......... **[6]**

19 A, B, C and D are points on the circumference of a circle, centre O.
BD is a diameter of the circle.
EBF is a tangent to the circle.



(a) Give a reason why angle BAD = 90°.

Angle in a semicircle is 90° ← BD is a diameter so angle BAD is an angle in a semicircle .... **[1]**

(b) Write down **one** other angle that is 90°.
Give a reason for your answer.

Angle ...OBF... because the angle between a radius and tangent is 90°

OB is a radius which meets the tangent EBF .... **[2]**

(c) Write down the value of angle CAD.

As angles in the same segment from a common chord are equal

(c) ...35...° **[1]**

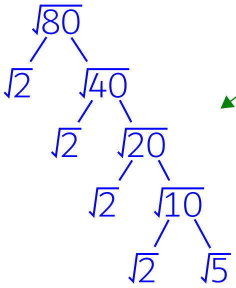
(d) Write down the value of angle EBA.

Due to the alternate segment theorem

(d) ...25...° **[1]**

20 Simplify.

$\sqrt{160} \div \sqrt{2}$



$\sqrt{160} \div \sqrt{2} = \sqrt{80}$. Then doing a factor tree

So $\sqrt{80} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{5} = 2 \times 2 \times \sqrt{5} = 4\sqrt{5}$

$4\sqrt{5}$ .... **[2]**

21 **(a)** Work out.

$\left(\frac{1}{8}\right)^{\frac{1}{3}}$

The 1/3 as a power means to cube root. $\sqrt[3]{1} = 1$ and $\sqrt[3]{8} = 2$

(a) $\frac{1}{2}$ .... **[1]**

(b) $2^x \times 4^y = 16$

Show that $y = 2 - \frac{x}{2}$. **[4]**

$2^x \times (2^2)^y = 2^4$ ← Expressing 4 and 16 as powers of 2

$2^x \times 2^{2y} = 2^4$ ← $(a^x)^w = a^{xw}$. So $(2^2)^y = 2^{2y}$

$2^{x+2y} = 2^4$ ← $a^x \times a^w = a^{x+w}$. So $2^x \times 2^{2y} = 2^{x+2y}$

$x + 2y = 4$ ← Both sides are powers of 2. So the power on both sides must be equal

$2y = 4 - x$ ← Subtracting x from both sides get the y term on its own

$y = 2 - \frac{x}{2}$ ← Dividing both sides by 2 gets y on its own

22 A sequence has nth term $2n^2 + 1$.

Prove algebraically that the sum of any two consecutive terms in this sequence is always a multiple of 4. **[6]**

$2n^2 + 1 + 2(n + 1)^2 + 1$ ← $2n^2 + 1$ expresses a term in the sequence. $2(n + 1)^2 + 1$ expresses the next term in the sequence. Adding these together expresses the sum

$(n + 1)(n + 1)$ ← Writing out the $(n + 1)^2$

$n^2 + n + n + 1$ ← Expanding the brackets

$2(n^2 + 2n + 1)$ ← Collecting like terms then putting the expanded and simplified $(n + 1)^2$ back into the $2(n + 1)^2$

$2n^2 + 1 + 2n^2 + 4n + 2 + 1$ ← This is the original expression with all the brackets expanded

$4n^2 + 4n + 4$ ← Collecting like terms

$4(n^2 + n + 1)$ ← Bringing 4 out as a factor

So multiple of 4 ← As $n^2 + n + 1$ must be a whole number and it is multiplied by 4

END OF QUESTION PAPER