OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Monday 10 June 2024 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

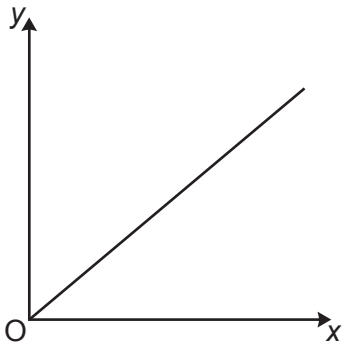
Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

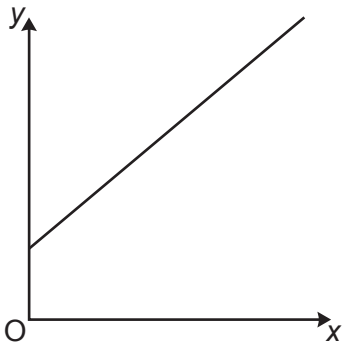
If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Below are six graphs, numbered 1 to 6, that show different relationships between x and y.

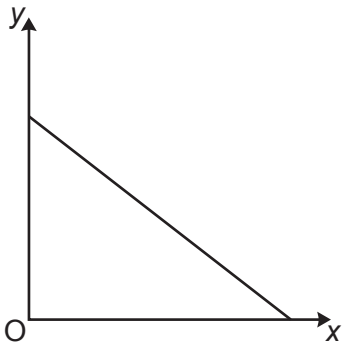
Graph 1



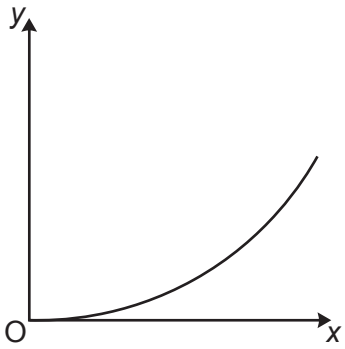
Graph 2



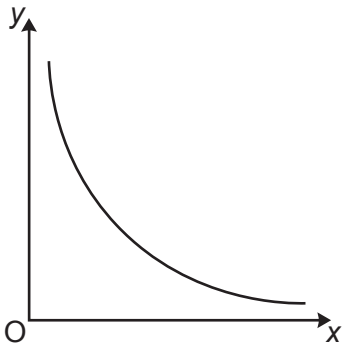
Graph 3



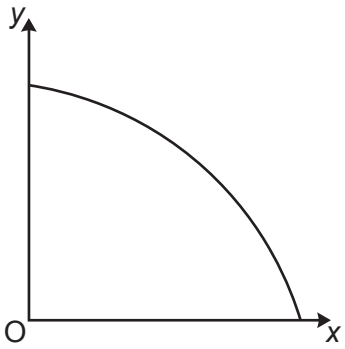
Graph 4



Graph 5



Graph 6



For each description below, write down the number of the graph that best represents the relationship between x and y.

x: temperature in °C
y: temperature in °F, where $y = 1.8x + 32$.

Graph ..... 2 ..... A

x: average speed when running 200 m
y: time taken to run 200 m.

Graph ..... 5 ..... B

x: mass of a solid object made from clay
y: volume of the same solid object.

Graph ..... 1 C

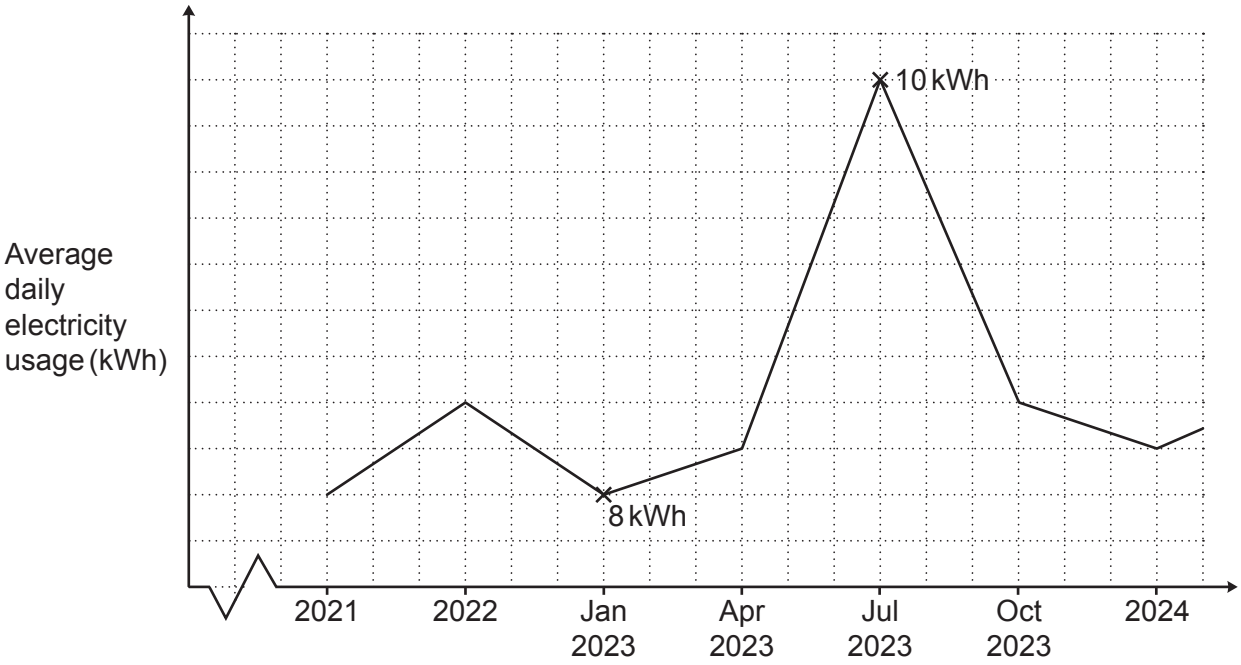
[3]

A: The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept. The gradient of y = 1.8x + 32 is 1.8, which is positive, so it must be slanting upwards. The y-intercept is 32, which is positive, so it must cross the y-axis above the x-axis. Only graph 2 meets these criteria.

B: Doubling the speed will halve the time. This is inverse proportion so must be graph 5.

C: The mass is directly proportional to the volume as doubling the mass must also double the volume. For it to be directly proportional it must be a straight line with a positive gradient which goes through the origin. So it must be graph 1

2 The graph shows a household's average daily electricity usage, in kilowatt hours (kWh).



Give **two different** reasons why this graph is misleading.

Reason 1: The horizontal scale does not go up by the same amount over each division

.....

Reason 2: There is no scale on the vertical axis

..... **[2]**

3 The word MATHEMATICS is spelt using tiles.

M	A	T	H	E	M	A	T	I	C	S

The tiles are put into an empty bag.

One at a time, 99 children each take a tile at random from the bag.
They record the letter and then put the tile back in the bag.

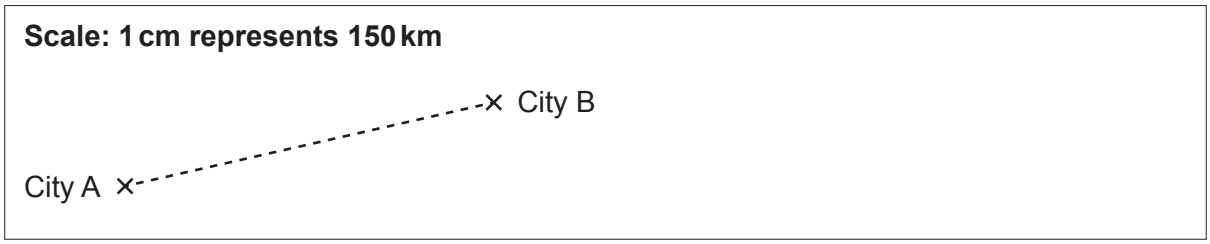
Work out how many times the letter M is expected to be taken from the bag.

$\frac{2}{11} \times 99$ ← 2 out of the 11 letters are the letter M. So the probability of picking the letter M is 2/11. Doing this fraction of the 99 works out that M is expected 18 times

18 ..... **[3]**

4 1 cm on a map represents 150 km in real life.

(a) Below is part of the map showing City A and City B.



Heidi is planning to walk from City A to City B for charity.

Heidi measures the straight-line distance on the map from City A to City B accurately as 5 cm. Heidi says she will walk from City A to City B at an average speed of 2.5 km/h for 10 hours per day.

(i) Use this information to work out how many days Heidi needs to complete the walk from City A to City B.

s d t ← Writing the formula triangle for speed, distance, time

5 × 150 ← Multiplying the 5 cm by the 150 km each 1 cm represents works out that the straight-line distance from City A to City B is 750 km

750 ÷ 2.5 ← Covering t in the formula triangle finds that time = distance ÷ speed. The unit of speed involves hours so dividing the 750 km by the 2.5 km/h works out that it should take 300 hours

300 ÷ 10 ← Dividing the 300 hours by the 10 hours per day works out that it should take 30 days

(a)(i) ..... 30 ..... days **[4]**

(ii) Explain why the information used in part **(i)** is likely to give an underestimate for the number of days Heidi needs to complete the walk.

Will not be able to walk in a straight line

There will be things in the way

..... **[1]**

(b) Yoshi writes the scale of the map, 1 cm represents 150 km, as the ratio 1 : 150.

Explain Yoshi's error and write the scale correctly in the form 1 : *n*.

150 × 1000
150000 × 100

There are 1000 m in 1 km so multiplying the 150 km by 1000 converts it into 150000 m.
There are 100 cm in 1 m so multiplying the 150000 m by 100 converts it to 15000000 cm

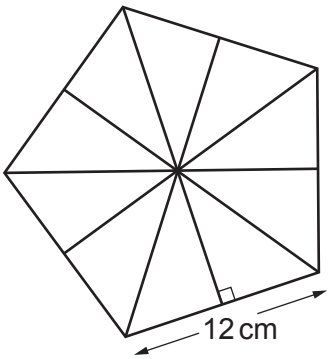
Yoshi's error is ..... the units are different .....

.....

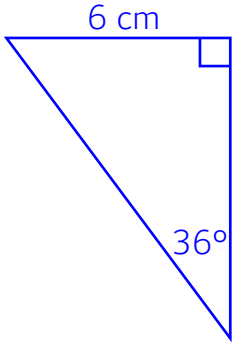
The correct answer is 1 : ..... 15000000 ..... **[2]**

1 cm represents 15000000 cm

5 The diagram shows a regular pentagon made using ten congruent right-angled triangles.
The length of one side of the pentagon is 12 cm.



Not to scale



Drawing one of the right-angled triangles

(a) Show that the area of the pentagon is 247.75 cm^2, correct to **2** decimal places. **[6]**

$12 \div 2 = 6$ ← Dividing the length of one side of the pentagon by 2 works out that the base of the right-angled triangle is 6 cm

$360 \div 10 = 36$ ← There are 360° around the centre point of the pentagon. So dividing 360° by the 10 angles around the centre works out that one of the angles around the centre is 36°

O A O
S H C H T A ← Doing right-angled trigonometry on one of the right-angled triangles to find its height. The 6 cm is the opposite so ticking O. The height is the adjacent so ticking A. There are two ticks on the TOA formula triangle so this one can be used

$\frac{6}{\tan 36}$ ← Covering A in the TOA formula triangle finds that adjacent = opposite/(tan of the angle). So the height of each right-angled triangle is 8.2... cm

$\frac{1}{2} \times 6 \times 8.2...$ ← Area of triangle = 1/2 × base × height. 6 cm is the base and using the exact value of the height for 8.2... cm. So the area of one of the right-angled triangles is 24.7... cm^2

$24.7... \times 10 = 247.75$ ← Multiplying the area of one of the right-angled triangles by 10 (as there are 10 of them which are all congruent) shows that the area of the pentagon is 247.75 cm^2 to 2 decimal places

(b) The regular pentagon is the base of a pyramid.

The pyramid has volume 450 cm^3.
The perpendicular height of the pyramid is h cm.

Calculate the value of h.

[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height.]

$\frac{1}{3} \times 247.7... \times h = 450$ ← Substituting in the exact value of the area of the pentagon for the area of the base and h as the perpendicular height to express the volume of the pyramid in terms of h. This must be equal to the 450 cm^3

Dividing both sides by 1/3 × 247.7... eliminates it on the left and gets h on its own

(b) $h =$ 5.45 **[3]**

6 **(a)** Two numbers, A and B, are written as the product of their prime factors.

$A = 2 \times 3 \times 7^2$
$B = 2^3 \times 7$

Find the lowest common multiple (LCM) of A and B.
Give your answer as an ordinary number.

$2^3 \times 3 \times 7^2$ ← The lowest common multiple is the highest power of each prime factor in both lists multiplied together

Newer models of Casio calculators can work out the lowest common multiple without having to do this method

(a) ..... 1176 ..... **[2]**

(b) A number, R, is written as the product of its prime factors.

$R = 2 \times 3^2 \times 5 \times k$, where k is a prime number.

The highest common factor (HCF) of R and another number, P, is 26.

Find the value of k.

2×13 ← Expressing 26 as a product of prime factors using the calculator

The highest common factor is the lowest power of each prime factor in both R and P multiplied together. So R must have at least one 13 as a prime factor so that the highest common factor is 26. k must be 13 as it cannot be a higher power of 13 if k is prime

(b) $k =$ ..... 13 ..... **[2]**

7 Two bags of fruit contain only apples and bananas.
In bag X, the ratio of apples to bananas is 5 : 7.
In bag Y, $\frac{5}{12}$ of the fruit are apples.

(a) Finley says

Bag X and bag Y contain the same number of apples.

Tick the correct statement.

- [ ] Finley is definitely correct
- [✓] Finley might be correct, or might not be correct
- [ ] Finley is definitely not correct

Show how you decided.

$\frac{5}{12}$ of both bags are apples. ← As 5 + 7 = 12 parts in total in the ratio. 5 out of these 12 parts represent the apples

There might or might not be the same total amount of fruit in both bags.

5/12 of the same total amount of fruit will give the same number of apples in both bags. 5/12 of a different total amount of fruit will give a different number of apples. It is not possible to know the total amount of fruit

[3]

(b) Finley adds 4 apples to bag X.
The ratio of apples to bananas is now 11 : 14.

How many bananas are in bag X?

10 : 14, 11 : 14 ← The number of bananas in bag X is the same before and after the 4 apples are added. So making the ratio before and after have the same number of parts representing the bananas so that they are compatible. 14 is the lowest common multiple of 7 and 14 so multiplying both sides of 5 : 7 to get 10 : 14. The number of apples could have been 10 and increased to 11 for the ratios to work, however this is only adding 1 apple and not adding 4 apples, so it does not work with 14 being the number of bananas

40 : 56, 44 : 56 ← 4 times as many apples needed to be added than the 1 apple which was added in the previous ratios (assuming that the number of parts is the numbers of fruit). So multiplying both ratios by 4 will work as this shows that the number of apples would have been 40 and increased to 44 for the ratios to work. This is an increase of 4 apples

There are 56 bananas according to the ratios (assuming that the number of parts is the numbers of fruit)

(b) 56 ..... **[3]**

8 **(a)** Complete this table for $y = x^3 - 3x^2$.

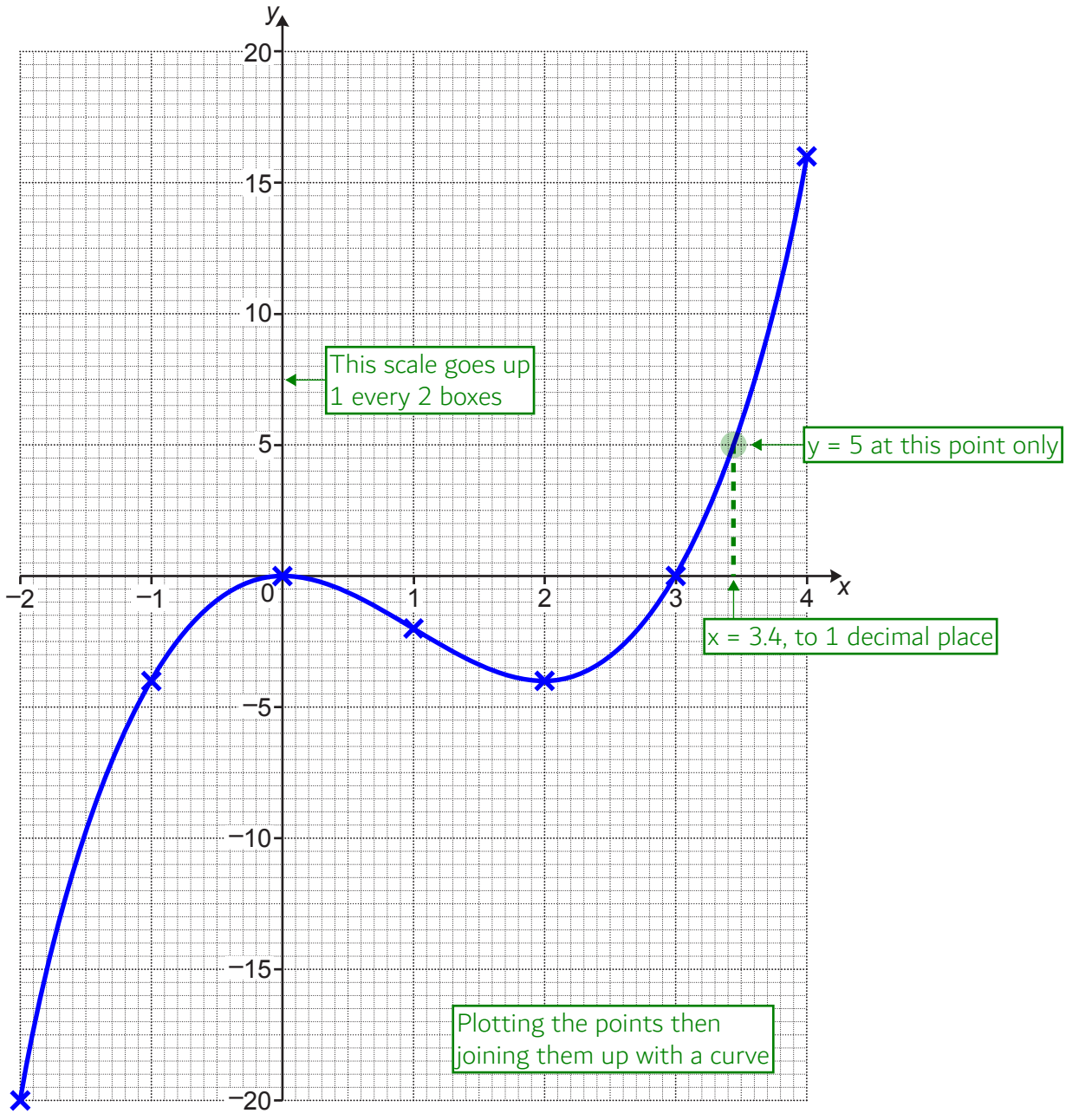
Using table mode. f(x) = x^3 - $3x^2$. Start: -2. End: 4. Step: 1

x	−2	−1	0	1	2	3	4
y	−20	-4	0	−2	−4	0	16

Plotting (4, 16) for this pair of values

[2]

(b) Draw the graph of $y = x^3 - 3x^2$ for values of x from −2 to 4.



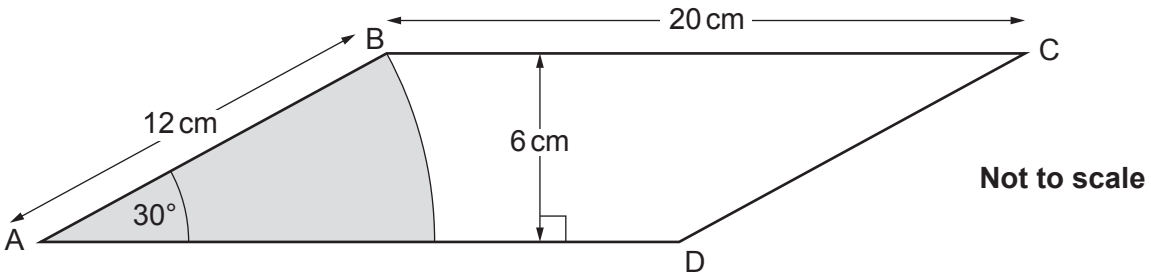
[3]

(c) Use the graph to solve the equation $x^3 - 3x^2 = 5$.
Give your answer to **1** decimal place.

y has been replaced with 5. So it is basically asking what x is when y = 5

(c) 3.4 **[1]**

9 The diagram shows a shaded sector inside a parallelogram.
The sector has an angle of 30°.
The parallelogram, ABCD, has length BC = 20 cm and AB = 12 cm.
The perpendicular distance between BC and AD is 6 cm.



(a) Show that the area of the sector is 37.7 cm^2, correct to **3** significant figures. **[3]**

$\frac{30}{360} \times \pi \times 12^2 = 37.69...$

Area of circle = π × radius^2. The sector is 30/360 of the whole circle as there are 360° around the centre of a circle and the sector has 30° out of these. So the area of the sector is 12π cm^2, which as a decimal is 37.69... cm^2, which is 37.7 cm^2 correct to 3 significant figures

(b) Work out the percentage of the parallelogram that is **not** shaded.

$20 \times 6 = 120$

Area of parallelogram = base × height. The base is 20 cm and the height is 6 cm. So the area of the parallelogram is 120 cm^2

$120 - 12\pi$

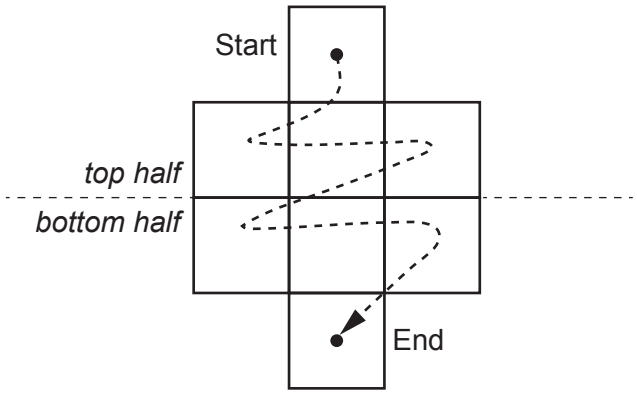
Subtracting the exact area of the sector from the area of the parallelogram works out that 82.3... cm^2 is not shaded

$\frac{82.3...}{120} \times 100$

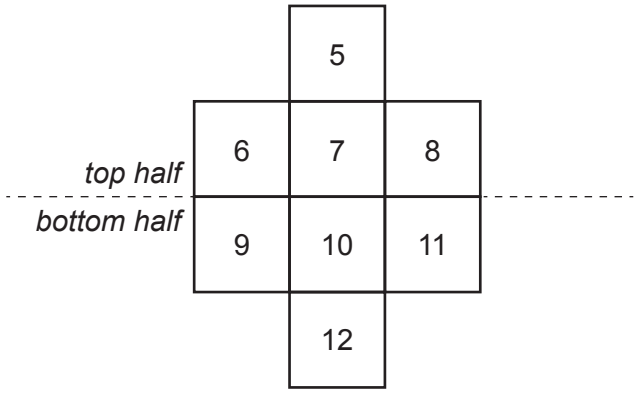
Putting the area not shaded over the area of the parallelogram expresses the fraction of the parallelogram that is not shaded. Multiplying this by 100 converts it into a percentage

(b) ..... 68.6 ..... % **[4]**

10 Eight consecutive numbers are written in ascending order in this grid, starting from the top and working left to right.



(a) Kareem writes the numbers 5 to 12 in the grid.



Show that for Kareem's grid, the sum of the numbers in the top half of the grid is 16 less than the sum of the numbers in the bottom half of the grid. **[1]**

$5 + 6 + 7 + 8 = 26$ ← The sum means to add together. So the sum of the top half of the grid is 26

$9 + 10 + 11 + 12 = 42$ ← The sum means to add together. So the sum of the bottom half of the grid is 42

$42 - 26 = 16$ ← Difference = largest - smallest. So the difference between the sum of the top half of the grid and the sum of the bottom half of the grid is 16. So this shows that the sum of the numbers in the top half of the grid is 16 less than the sum of the numbers in the bottom half of the grid

(b) **Use algebra** to prove that for **any** set of eight consecutive numbers written in this grid in the same way, the sum of the numbers in the top half of the grid is 16 less than the sum of the numbers in the bottom half of the grid. **[5]**

$x + (x + 1) + (x + 2) + (x + 3)$ ← Let x be the 1st number. (x + 1) is the 2nd number. (x + 2) is the 3rd number. (x + 3) is the 4th number. Adding these expressions together to express the sum of the first four consecutive numbers which are in the top half of the grid

$4x + 6$ ← The brackets can be ignored in this case. Collecting like terms simplifies the expression of the sum of the first four consecutive numbers which are in the top half of the grid to 4x + 6

$(x + 4) + (x + 5) + (x + 6) + (x + 7)$ ← (x + 4) is the 5th number. (x + 5) is the 6th number. (x + 6) is the 7th number. (x + 7) is the 8th number. Adding these expressions together to express the sum of the last four consecutive numbers which are in the bottom half of the grid. The brackets can be ignored in this case. Collecting like terms simplifies the expression to 4x + 22

$(4x + 22) - (4x + 6)$ ← Difference = largest - smallest. So subtracting the expression of the sum of the top half of the grid from the expression of the sum of the bottom half of the grid expresses the difference and therefore how much less the top half it that the bottom half

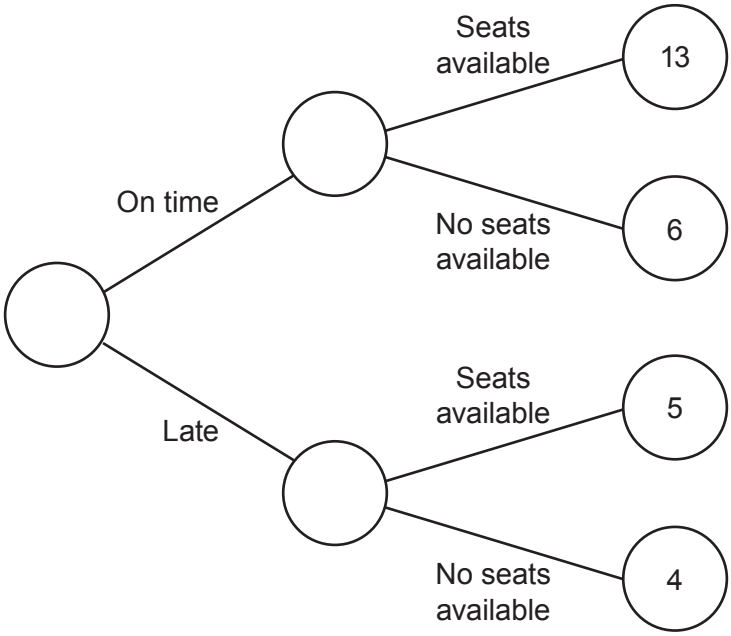
$4x + 22 - 4x - 6$ ← Subtracting everything in the second bracket

16 ← Collecting like terms gives 16

11 Jack travels to work each day by train.

He records whether
- the train is on time or late
- there are seats available or no seats available.

Jack's results are shown on this partly completed frequency tree.



(a) Find the relative frequency of there being no seats available on Jack's train journey.

$13 + 6 + 5 + 4 = 28$ ← Adding all of the frequencies at the end of the branches works out that there were 28 days

$6 + 4 = 10$ ← Adding the frequencies for no seats available works out that on 10 days there were no seats available

$\frac{10}{28}$

(a) ..... **[2]**

On 10 out of the 28 days there were no seats available

(b) Jack says

If the train is late, travellers are less likely to find seats available than if the train was on time.

Does Jack's data suggest he is correct?
Show how you decide.

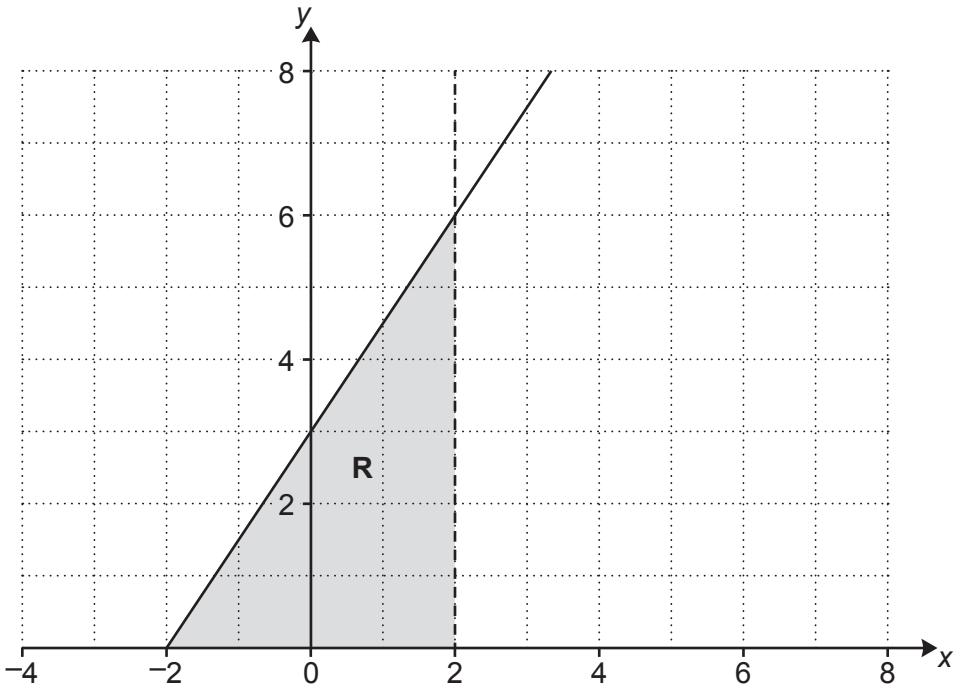
$\frac{13}{13 + 6} \times 100 = 68.4...\%$ ← Adding the 13 and 6 expresses the total number of days it was on time. Putting the 13 over this expresses the fraction of times it was on time and there were seats available. Multiplying this by 100 converts it into a percentage to make it easy to compare

$\frac{5}{5 + 4} \times 100 = 55.\dot{5}\%$ ← Adding the 5 and 4 expresses the total number of days it was late. Putting the 5 over this expresses the fraction of times it was late and there were seats available. Multiplying this by 100 converts it into a percentage to make it easy to compare

[3]

Yes ← Jack is correct as the percentage of times there were seats available when the train is late is less than the percentage of times there were seats available when the train is on time

12 The region **R** is shown on this grid.



The region **R** is defined by three inequalities.
The first inequality is given below.

Complete the second inequality and write down the third inequality needed to define region **R**.

$y \geqslant 0$

$2y$ ..... $\leq$ ..... $3x + 6$

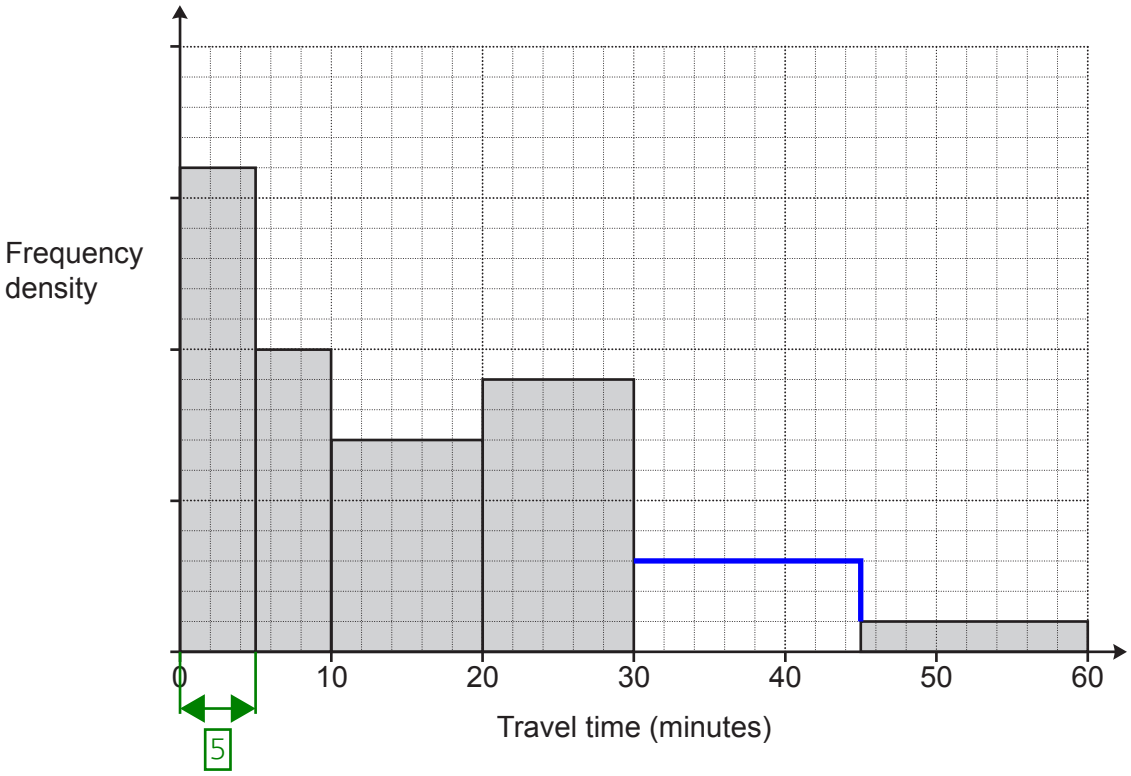
..... $x < 2$ ..... **[3]**

The x-axis is the line of y = 0. The vertical dashed line is the line of x = 2. So the slanted like must be 2y = 3x + 6.

As the region is below the line 2y = 3x + 6, y would be less and 2y is also less. It can be equal as the line is not dashed.

As the region is the left of the line x = 2, x is less. It cannot be equal as the line is dashed

13 A group of students record the time taken to travel to school.
All students in the group took less than an hour to travel to school.
Some of their results are recorded on this histogram.



16 students took less than 5 minutes to travel to school.

(a) How many students took less than 20 minutes to travel to school?

See next page for the method

(a) .....40..... students **[4]**

(b) 9 students took between 30 and 45 minutes to travel to school.

Add these students' results to the histogram. **[2]**

15H = 9 ← Frequency is the area of each bar on a histogram. Let H be the height of the 5th bar. Area of rectangle = base × height, so as the base is 15 the area of the 5th bar is 15H which must be equal to the 9

H = 0.6 ← Dividing both sides by 15 finds that the height of the 5th bar is 0.6

0.6 ÷ 0.2 = 3 ← Dividing the height of the 5th bar by the 0.2 which each small box represents works out that the 5th bar must be 3 small boxes high

$5h = 16$ ← Frequency is the area of each bar on a histogram. Let h be the height of the 1st bar. Area of rectangle = base × height, so as the base is 5 the area of the 1st bar is 5h which must be equal to the 16

$h = 3.2$ ← Dividing both sides by 5 finds that the height of the 1st bar must be 3.2

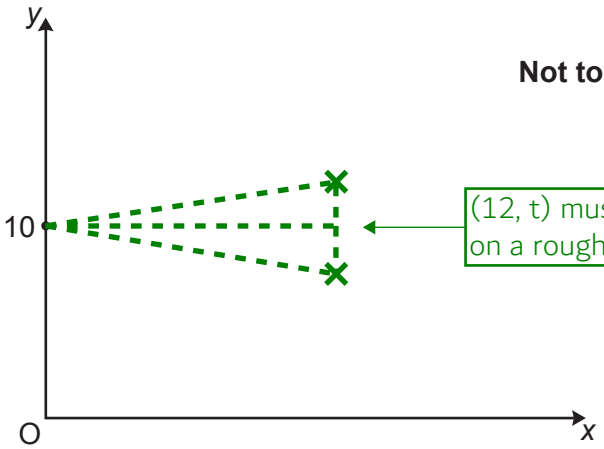
$3.2 \div 16 = 0.2$ ← Dividing the height of the 1st bar by the 16 small boxes along its height works out that each small box on the vertical scale is worth 0.2

$5 \times 10 \times 0.2 = 10$ ← Frequency is the area of each bar on a histogram. Area of rectangle = base × height. The base of the 2nd bar is 5. It is 10 small boxes high so 10 × 0.2 expresses the height of the 2nd bar. So the frequency of the 2nd bar is 10

$10 \times 7 \times 0.2 = 14$ ← Frequency is the area of each bar on a histogram. Area of rectangle = base × height. The base of the 3rd bar is 10. It is 7 small boxes high so 7 × 0.2 expresses the height of the 3rd bar. So the frequency of the 3rd bar is 14

$16 + 10 + 14$ ← Adding the frequencies of the first 3 bars works out that 40 students took less than 20 minutes

14 The sketch shows the coordinate axes and the point (0, 10).



(12, t) must have a y-coordinate above or below 10. Drawing on a rough location creates two right-angled triangles

The distance from the point (0, 10) to a point (12, *t*) is 12.5 units.

Work out the **two** possible values of *t*.
You must show your working and you may use the sketch to help.

$12^2 + y^2 = 12.5^2$ ← Let y be the vertical distance between the y-coordinate 10 and the y-coordinate t. Pythagoras' Theorem can be used to find y. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. The distance in the x-direction between (0, 10) and (12, t) is 12 so this can be a. The distance of 12.5 must be c. y can be b

$y^2 = 12.25$ ← Subtracting 12^2 from both sides eliminates it on the left and gets y^2 on its own

$y = \pm 3.5$ ← Doing the positive and negative square root eliminates the power of 2 and gets y on its own. y can be negative as this means it is going downwards

$10 + 3.5$ ← Adding 3.5 to the y-coordinate 10 works out a possible value of t

$10 - 3.5$ ← Subtracting 3.5 from the y-coordinate 10 works out a possible value of t

t = ... 13.5 ..... and ..... 6.5 ..... **[5]**

15 y is inversely proportional to x^3.
$y = 4$ when $x = 12$.

Find a formula linking x and y.

$y = \frac{k}{x^3}$ ← Writing the proportion as an equation, where k is a constant to be found

$k = 4 \times 12^3$ ← Multiplying both sides by x^3 gets k on its own. Substituting 4 for y and 12 for x. So k = 6912

Putting the value of k back into the original equation

$y = \frac{6912}{x^3}$

..... **[3]**

16 Write $\frac{9x^7 \times 2\sqrt{x}}{3x^4}$ in the form kx^m.

$\frac{18x^{7.5}}{3x^4}$ ← First simplifying the numerator. Multiplication can be done in any order so doing the 9 × 2 = 18 first. $\sqrt{x} = x^{0.5}$ and $a^w \times a^y = a^{w+y}$ so $x^7 \times x^{0.5} = x^{7.5}$

18/3 = 6 and $x^{7.5}/x^4 = x^{3.5}$ as $a^w/a^y = a^{w-y}$

$6x^{3.5}$

..... **[4]**

17 (a) Without using a calculator, show that $0.\dot{1}\dot{8}$ can be written as $\frac{2}{11}$. **[3]**

$x = 0.\dot{1}\dot{8}$ ← Let x be the recurring decimal

$100x = 18.\dot{1}\dot{8}$ ← There are two recurring digits so multiplying by ten twice gives another decimal with the recurring digits in the same decimal places

$99x = 18$ ← Subtracting x from 100x cancels out the recurring digits

$x = \frac{18}{99}$ ← Dividing both sides by 99 gives x as a fraction

$= \frac{2}{11}$ ← Simplifying the fraction by dividing both the numerator and denominator by 9

(b) Explain how $\frac{2}{11} = 0.\dot{1}\dot{8}$ can be used to find $\frac{10}{11}$ as a decimal and write down its value.

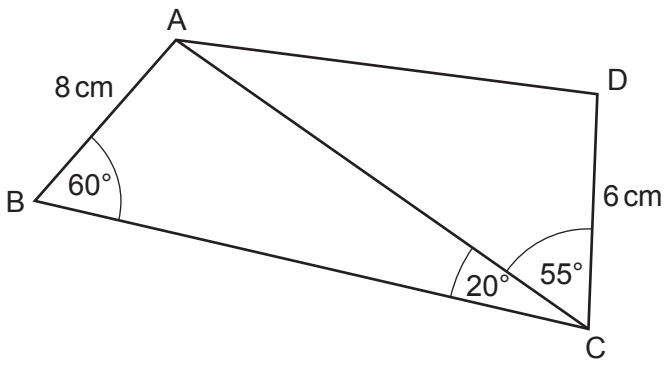
$0.\dot{1}\dot{8} \times 5$ ← As 10/11 is 5 times 2/11

$\frac{10}{11} =$ $0.\dot{9}\dot{0}$ **[2]**

The recurring decimal can be multiplied by 5 using the calculator. Alternatively putting in 10/11 and formatting it as a decimal

18 The diagram shows two triangles ABC and ADC.

Not to scale



Angle ABC = 60°, angle ACB = 20° and angle ACD = 55°.
AB = 8 cm and DC = 6 cm.

Calculate length AD.
You must show your working.

$\frac{AC}{\sin 60} = \frac{8}{\sin 20}$ ← $\frac{a}{\sin A} = \frac{b}{\sin B}$ ← Using the sine rule in triangle ABC. Side a is opposite angle A. Substituting AC for a, 60° for A, 8 cm for b, 20° for B

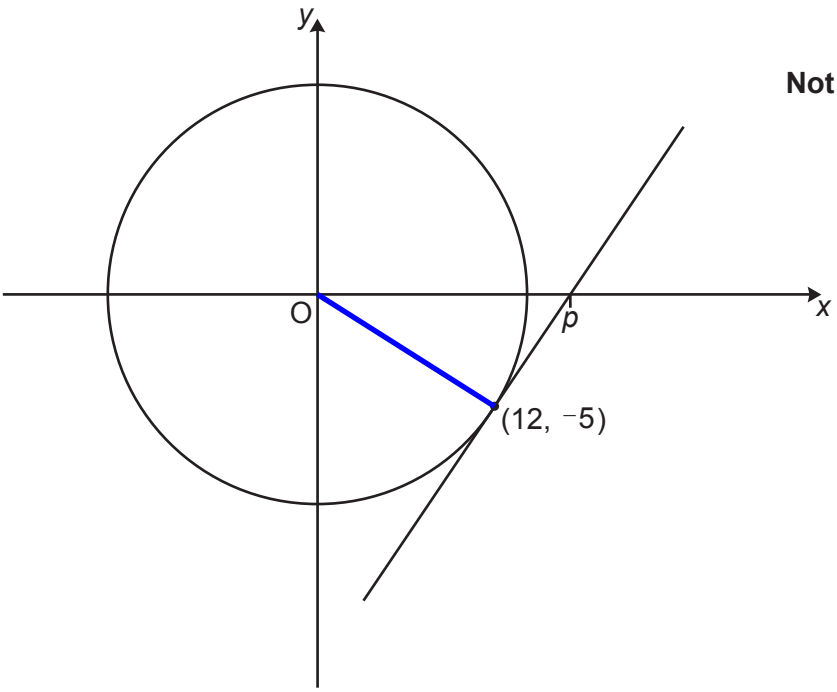
$AC = 20.2...$ ← Multiplying both sides by sin60 eliminates sin60 as the denominator on the left and gets AC on its own

$AD^2 = 20.2...^2 + 6^2 - 2 \times 20.2... \times 6 \times \cos 55$ ← Using the cosine rule in triangle ADC. $a^2 = b^2 + c^2 - 2bc\cos A$. Substituting AD for a (as it is opposite the given angle), the exact value of 20.2... for b, 6 cm for c, 55° for A

$AD = \sqrt{306.9...}$ ← Square rooting both sides eliminates the power of 2 on the left and gets AD on its own

AD = ... 17.5 ..... cm **[6]**

19 The diagram shows a circle with centre (0, 0) and a tangent at (12, −5).
The tangent at (12, −5) crosses the x-axis at (p, 0).



Not to scale

Find the exact value of p.
You must show your working.

$\frac{-5}{12}$ ← Expressing the gradient of the radius. Gradient = (change in y)/(change in x). From (0, 0) to (12, -5), y has changed -5 and x has changed 12

$5 \div \frac{12}{5}$ ← The tangent is perpendicular to the radius so its gradient is 12/5 as this is the negative reciprocal of -5/12. The gradient basically means how much it goes up for every 1 it goes across. It needs to go up 5 from (12, -5) to the x-axis. So dividing 5 by the gradient of the tangent works out that it needs to go up 25/12 lots of 12/5 and therefore has to go 25/12 across to the right

$12 + \frac{25}{12}$ ← Adding the 25/12 it needs to go to the right from the x-coordinate of 12 works out p

$p =$ $\frac{169}{12}$ **[5]**

Turn over for Question 20

20 Write as a single fraction in its simplest form.

$$4 + \frac{x^2 - 49}{(x+7)(x-3)}$$

$\frac{(x + 7)(x - 7)}{(x + 7)(x - 3)}$ ← Factorising the numerator of the fraction by using difference of two squares. $A^2 - B^2 = (A + B)(A - B)$. The $(x + 7)$ can be cancelled out from the numerator and denominator so it becomes $(x - 7)/(x - 3)$

$\frac{4(x - 3)}{(x - 3)}$ ← 4 could be considered as 4/1. Multiplying the numerator and denominator by $(x - 3)$ expresses it as a fraction with the same denominator as $(x - 7)/(x - 3)$

$\frac{4x - 12}{x - 3} + \frac{x - 7}{x - 3}$ ← Expanding the brackets on the numerator. Now writing the full expression as simplified fractions

Adding the numerators. The denominator stays the same ↓

$\frac{5x - 19}{x - 3}$

..... **[5]**

END OF QUESTION PAPER