

Wednesday 8 November 2023 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:

- the Formulae Sheet for Higher Tier (inside this document)

You can use:

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



H



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

Last name

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **20** pages.

ADVICE

- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

- 1 The price of a phone increases from £240 to £262.80.

Calculate the percentage increase in the price of the phone.

$$\frac{262.80-240}{240} \times 100$$

Subtracting the original price from the new price expresses the change in price. Putting this over the original price expresses the increase as a fraction. Multiplying this fraction by 100 converts it into a percentage

..... 9.5 % [3]

- 2 A prime number is a whole number that has exactly two factors.

- (a) Explain why 1 is not a prime number.

It has only one factor

1 is the only factor of 1

..... [1]

- (b) a and b are prime numbers.

Write down the 6 factors of a^2b .

Writing the factors in pairs.

$1 \times a^2b = a^2b$ so both 1 and a^2b are factors.

$a \times ab = a^2b$ so both a and ab are factors.

$a^2 \times b = a^2b$ so both a^2 and b are factors

(b) 1 a^2b a ab a^2 b [2]

3 (a) The table shows the results for a sports club's 'A' team.

Result	Frequency
Win	18
Draw	10
Lose	12
Total	40

$360 \div 40$

162

90

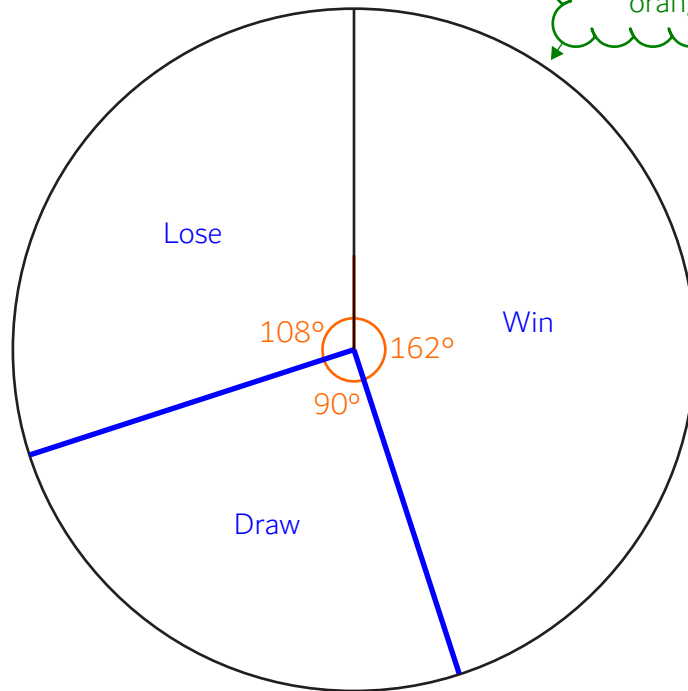
108

The total frequency of 40 is represented by 360°. So dividing 360 by 40 works out that 1 game is represented by 9°

Multiplying the frequencies by 9 works out the angle which needs to be drawn for each sector

Drawing the angles on with a protractor and labelling each sector. The angles in orange do not need to be shown

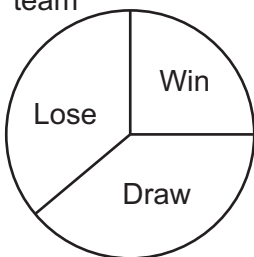
Complete a labelled pie chart to show these results.



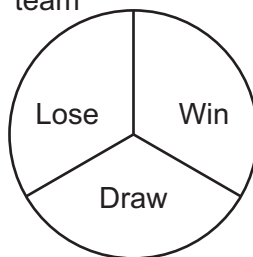
[4]

(b) Here are the results for the sports club's 'B' team and 'C' team.

'B' team



'C' team



The 'C' team manager says

The pie charts show that the 'C' team won more games than the 'B' team.

Referring to the pie charts, explain why the 'C' team manager may not be correct.

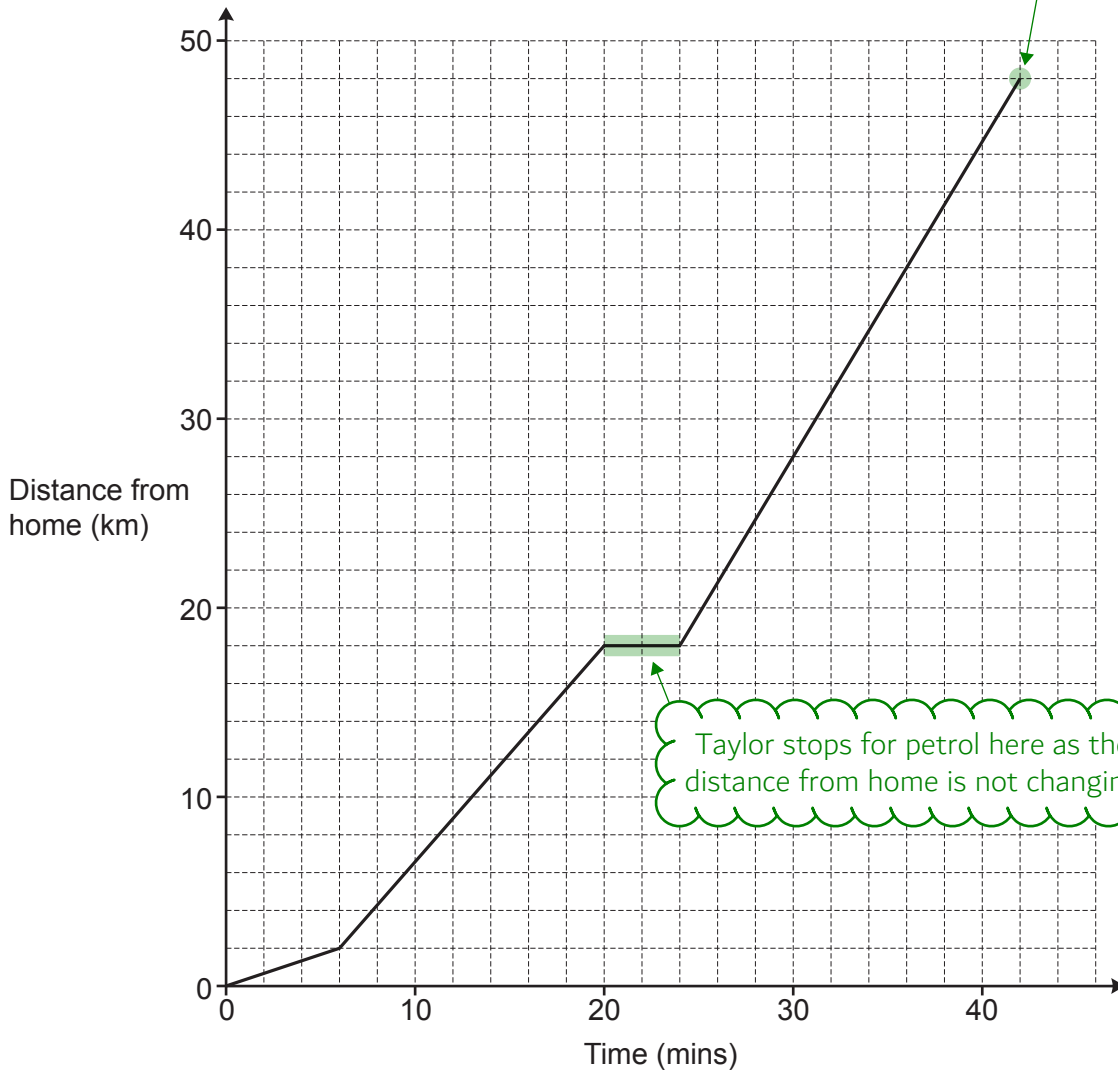
They only show proportion

The 'C' team have won a greater proportion as the angle of the sector for win is larger but they may have played less games in total meaning that they actually won less games than the 'B' team

[1]

Turn over

- 4 The graph shows Taylor's journey from home to an airport. During the journey Taylor stops for petrol.



The total distance travelled is 48 km

Taylor stops for petrol here as the distance from home is not changing

- (a) For how long did Taylor stop for petrol?

2 small boxes is worth $2 \times 2 = 4$ minutes

The scales increase by 10 over 5 small boxes. $10 \div 5 = 2$ so each small box is worth 2

(a) 4 mins [1]

- (b) Taylor drives the same route back home from the airport at an average speed of 45 km/h. Taylor leaves the airport at 22:00.

Work out the time when Taylor arrives home.

s^d_t ← Writing the formula triangle for speed, distance, time

$48 \div 45$ ← From the formula triangle, time = distance \div speed

$22:00 + \frac{16}{15}$ ← Writing the time as a sexagesimal and adding the time taken in hours

The calculator gives the answer of $23^{\circ}4'0''$ when converted into a sexagesimal, which can be read as 23:04

(b) 23:04 [4]

- 5 (a) Write an expression for the weight, in grams, of an object weighing x kilograms.

There are 1000 g in 1 kg so multiplying the x kg by 1000 converts it into grams

(a) $1000x$ g [1]

- (b) Write an expression for the area, in m^2 , of a garden of area ycm^2 .

There are 100 cm in 1 m. As the units are squared, dividing $y cm^2$ by 100^2 converts it into m^2

(b) $\frac{y}{100^2}$ m^2 [1]

- 6 2 kg of carrots and 5 kg of potatoes cost £6.36.
3 kg of carrots and 2 kg of potatoes cost £5.25.

Find the cost of 1 kg of carrots and the cost of 1 kg of potatoes.
You must show your working.

$$2c + 5p = 6.36$$

Let c be the cost of 1 kg of carrot and p be the cost of 1 kg of potatoes. Writing that 2 kg of carrots and 5 kg of potatoes cost £6.36 to form the first equation

$$3c + 2p = 5.25$$

Writing that 3 kg of carrots and 2 kg of potatoes cost £5.25 to form the second equation

$$6c + 15p = 19.08$$

$$6c + 4p = 10.50$$

Multiplying the first equation by 3 to form the third equation and multiplying the second equation by 2 to form the fourth equation. This makes the number of c the same in the third and fourth equations

$$11p = 8.58$$

Subtracting the fourth equation from the third equation cancels out the c terms leaving an equation just in terms of p

$$p = 0.78$$

Dividing both sides by 11 eliminates the 11 on the left and gets p on its own. So 1 kg of potatoes costs £0.78

$$2c + 5 \times 0.78 = 6.36$$

Substituting p for 0.78 in the first equation

$$2c = 2.46$$

Subtracting 5×0.78 from both sides to get the c term on its own

$$c = 1.23$$

Dividing both sides by 2 gets c on its own. So 1 kg of carrots costs £1.23

1 kg of carrots cost £ 1.23

1 kg of potatoes cost £ 0.78

[5]

7 Find all the possible integer values that satisfy the inequality $-10 < 3x + 2 \leq 8$.

$$-12 < 3x \leq 6$$

Subtracting 2 from all sides eliminates the +2 in the middle and gets the x term on its own

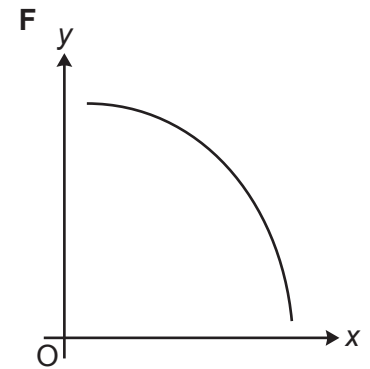
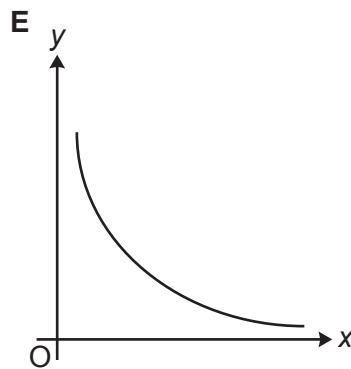
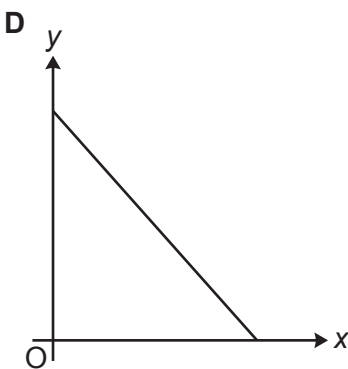
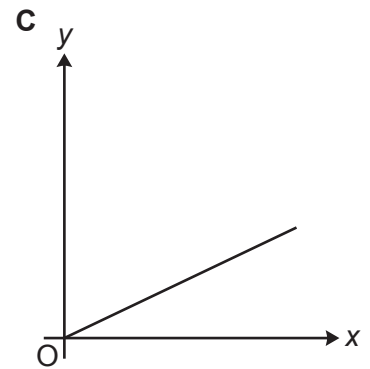
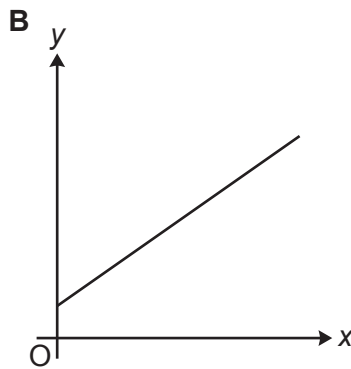
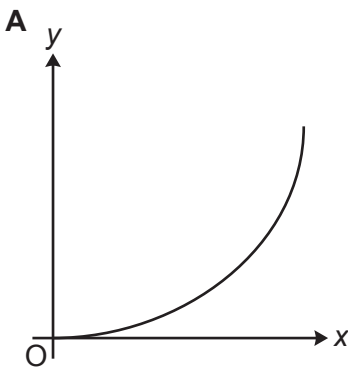
$$-4 < x \leq 2$$

Dividing all sides by 3 eliminates the 3 in the middle and gets x on its own

x must be greater than -4 and less than or equal to 2.
Integer means that it cannot be a decimal or fraction

$$x = \dots\dots\dots -3, -2, -1, 0, 1, 2 \dots\dots\dots [3]$$

8 Here are sketches of six graphs, labelled A to F.



Write the letter of the graph that represents the following relationships.

(a) y is directly proportional to x .

The equation could be $y = 2x$

(a) C [1]

(b) y is inversely proportional to x .

The equation could be $y = 1/2x$

(b) E [1]

- 9 Here are two pieces of work.
Each shows a question and an incorrect solution.

For each part, describe the error made and write out a correct solution.

(a)

Question:
Factorise. $x^2 + x - 20$

Solution:
 $(x + 4)(x - 5)$

The error is the brackets expand to give $-x$

$4x - 5x = -x$, this is not x

A correct solution is $(x - 4)(x + 5)$

[2]

(b)

Question:
Solve. $4x + 5 = x + 2$

Solution:
 $4x + 5 = x + 2$
 $3x + 5 = 2$
 $3x = 5 - 2$
 $3x = 3$
 $x = 1$

The error is it should be $2 - 5$ on the third line

5 should have been subtracted from both sides on the second line to give the third line

A correct solution is

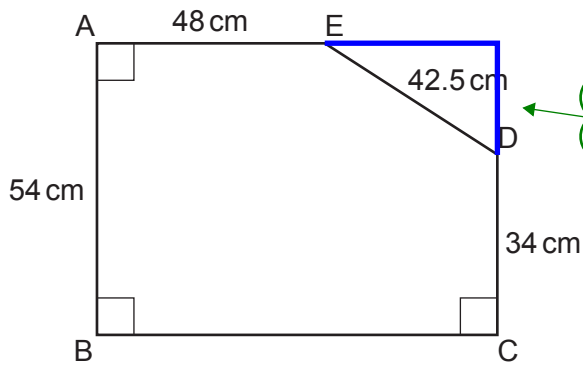
$$3x = 2 - 5$$

$$3x = -3$$

$$x = -1$$

[2]

10 The diagram shows a pentagon ABCDE.



Not to scale

Drawing lines here forms a right-angled triangle

Find the area of the pentagon.
You must show your working.

$$54 - 34 = 20$$

Subtracting the 34 cm from the 54 cm works out that the height of the right-angled triangle is 20 cm

$$a^2 + b^2 = c^2$$

Pythagoras' Theorem can be used to work out the missing length on the right-angled triangle

$$20^2 + b^2 = 42.5^2$$

Substituting 20 for a and 42.5 for c as this is the longest side

$$b = \sqrt{42.5^2 - 20^2}$$

Rearranging to find b by subtracting 20^2 from both sides then square rooting both sides. This finds that the top side of the right-angled triangle is 37.5 cm

$$37.5 + 48$$

Adding the 48 cm to the 37.5 cm work out that the length of the whole rectangle is 85.5 cm

$$85.5 \times 54$$

Area of rectangle = length x width. The length is 85.5 cm and the width is 54 cm. So the area of the whole rectangle is 4617 cm^2

$$4617 - \frac{1}{2} \times 37.5 \times 20$$

Subtracting the area of the right-angled triangle from the area of the whole rectangle works out the area of the pentagon. Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$. The base is 37.5 cm and the height is 20 cm

4242

..... cm^2 [6]

- 11 Riley and Sam are conducting surveys.
They are both given the same list of 12 463 people from which to select their sample.

Riley selects every 56th person.

Sam selects every 64th person.

They both start counting from the first name in the list.

Work out how many people will be selected to be in both surveys.

You must show your working.

$$56 = 2^3 \times 7$$

Expressing 56 as a product of prime factors using the calculator

$$64 = 2^6$$

Expressing 64 as a product of prime factors using the calculator

$$2^6 \times 7$$

Working out the lowest common multiple of 56 and 64 by multiplying the highest power of each prime in both of the products of prime factors. This means that every 448th person will be selected in both surveys. Note that the Casio fx-83/85GT CW can work out the lowest common multiple of two numbers without having to do this method

$$12463 \div 448$$

Dividing the 12463 people by the 448 works out how many lots of 448 go into the 12463 and therefore how many people will be selected to be in both surveys

The answer of 27.8... is rounded down as there are not enough people in total to have 29 people in both surveys and it needs to be a whole number of people

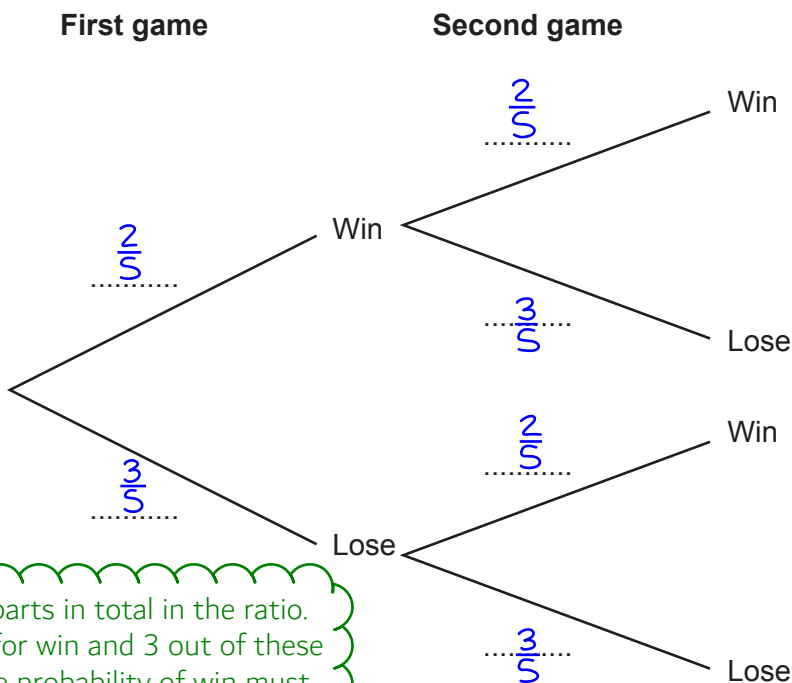
27

[5]

- 12 In a computer game the player can either win or lose.
A student thinks the ratio of the probability of winning to the probability of losing is 2 : 3.

The student plays two games.

- (a) Use the information to complete the tree diagram.



2 + 3 = 5, so there are 5 parts in total in the ratio.
2 out of these 5 parts are for win and 3 out of these 5 parts are for lose. So the probability of win must be 2/5 and the probability of lose must be 3/5

[3]

- (b) Find the probability that the student wins at least one of the two games.

$$\frac{2}{5} + \frac{3}{5} \times \frac{2}{5}$$

Win the first game (it doesn't matter what happens on the second game) OR lose the first game AND win the second game. OR means to add. AND means to multiply

(b) 0.64 [3]

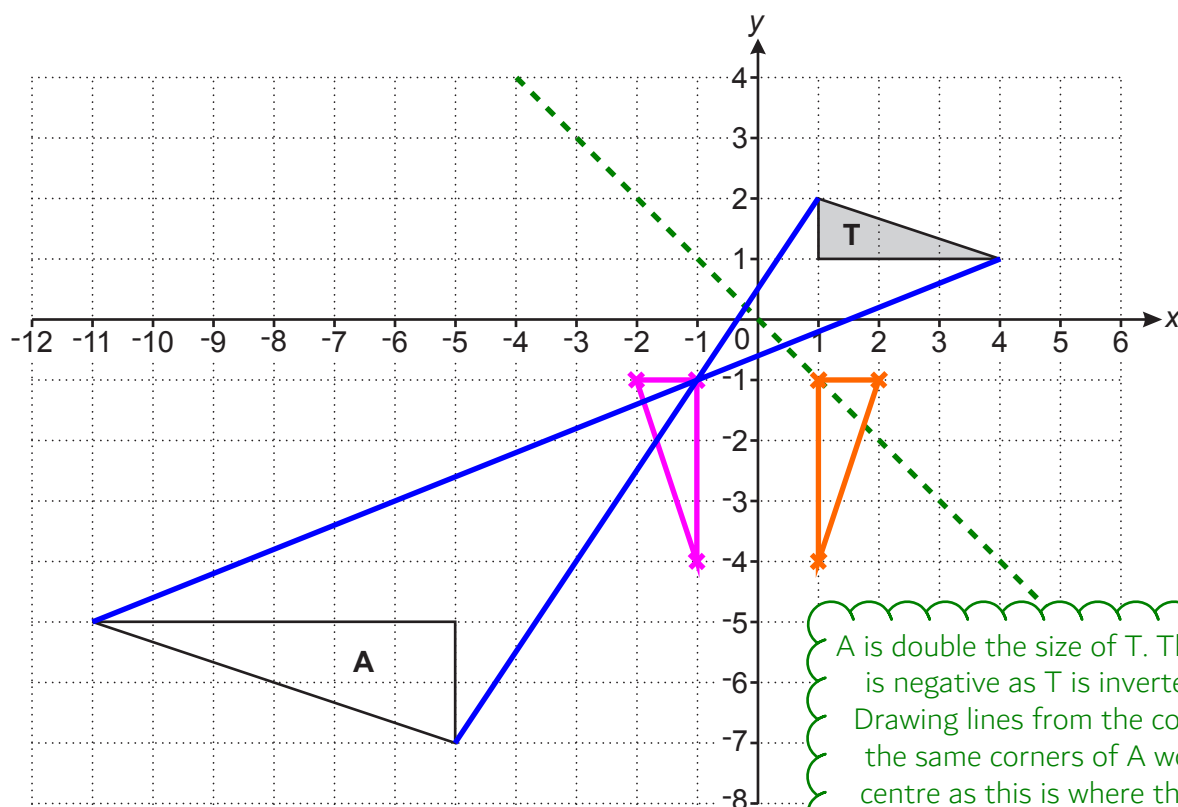
- (c) The student now thinks the ratio of the probability of winning to the probability of losing has changed to 2 : 5.

Explain the effect this change will have on your answer to part (b).

It will be less

.....
As there is now less probability of winning as there are now more parts in the ratio for losing [1]

13 Triangle **T** and triangle **A** are drawn on the coordinate grid.



A is double the size of T. The scale factor is negative as T is inverted to give A. Drawing lines from the corners of T to the same corners of A works out the centre as this is where the lines cross

(a) Describe fully the **single** transformation that maps triangle **T** onto triangle **A**.

Enlargement, scale factor -2 , centre $(-1, -1)$

[3]

(b) Describe fully the **single** transformation that is equivalent to:

- a rotation of 90° clockwise about centre $(0, 0)$, followed by
- a reflection in the y -axis.

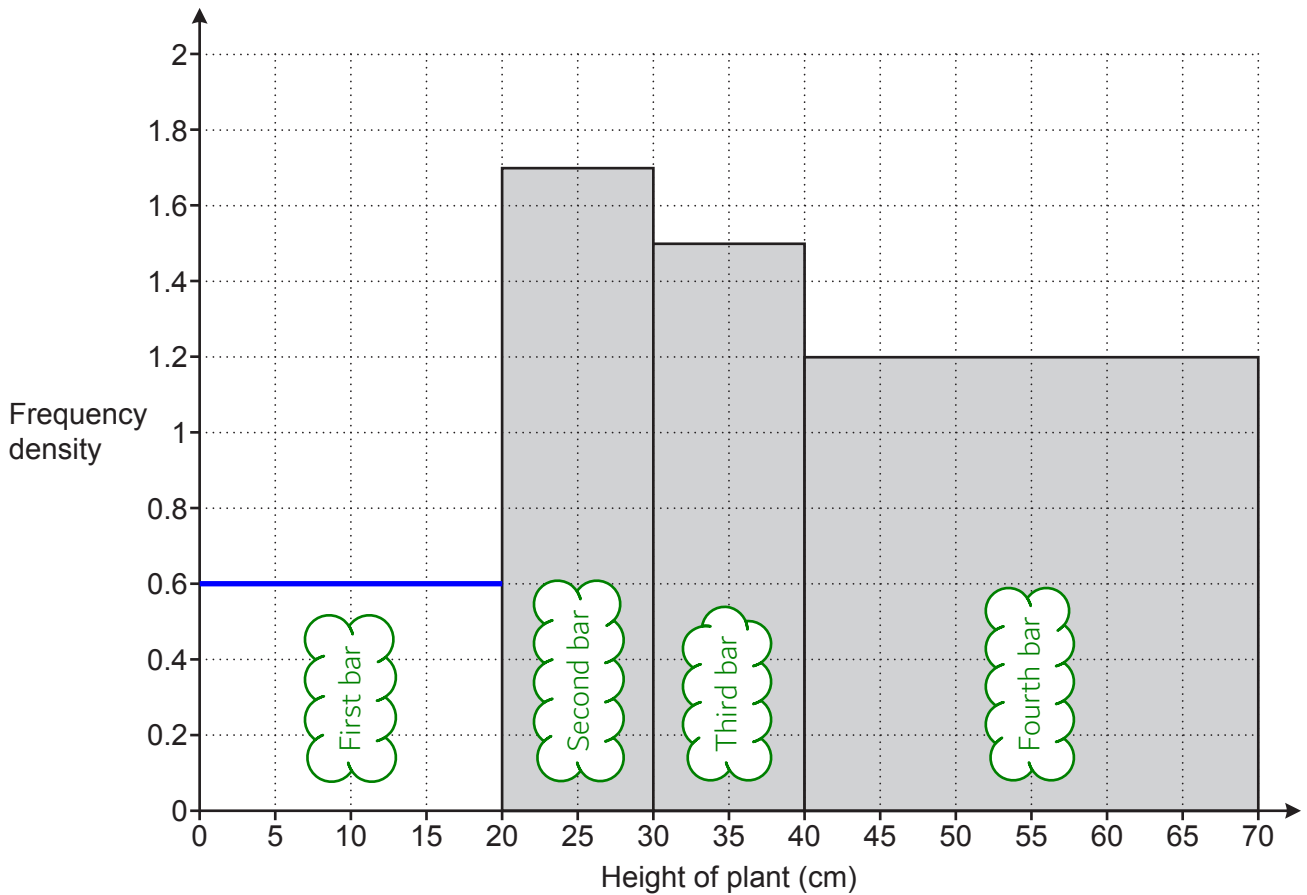
You may use the grid above to help you.

Triangle T is rotated 90° clockwise about centre $(0, 0)$ using tracing paper to give the orange triangle. This is then reflected in the y -axis to give the pink triangle. The dashed green line shows the line that T can be reflected on to give the pink triangle. The equation of this line is $y = -x$ as the y -coordinate is always the same as the x -coordinate but has the opposite sign

Reflection on the line $y = -x$

[3]

14 The histogram shows the heights of some plants out of a total of 80 plants.



Complete the histogram to show the plants with heights between 0 cm and 20 cm. [5]

cFd

Writing a formula triangle for frequency (f), class width (c), frequency density (d)

$10 \times 1.7 + 10 \times 1.5 + 30 \times 1.2$

From the formula triangle, frequency = class width \times frequency density. For the second bar, the class width is 10 as it goes from 20 to 30 and its frequency density is 1.7. For the third bar, the class width is 10 as it goes from 30 to 40 and its frequency density is 1.5. For the fourth bar, its class width is 30 as it goes from 40 to 70 and its frequency density is 1.2. Adding the frequencies of the second, third and fourth bar works out that 68 plants are shown on the histogram so far

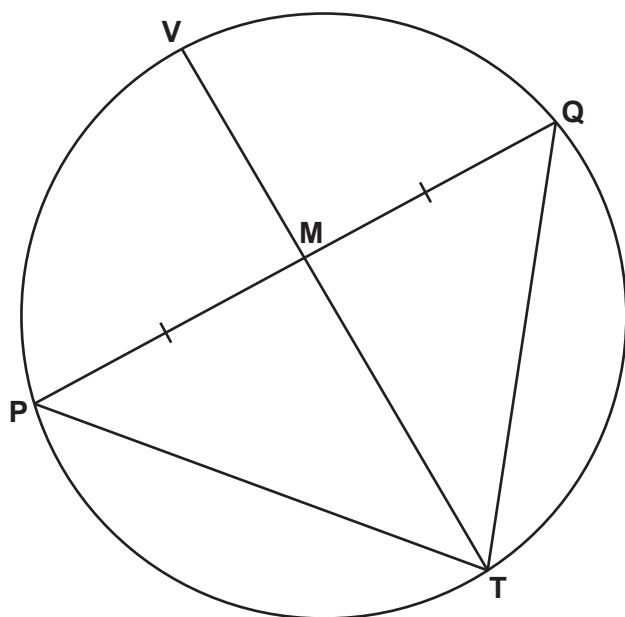
$80 - 68$

Subtracting the 68 plants from the total of 80 plants works out that the frequency of the first bar must be 12

$12 \div 20$

From the formula triangle, frequency density = frequency \div class width. The class width of the first bar is 20 as it goes from 0 to 20. So the frequency density of the first bar is 0.6

- 15 P, Q, T and V are points on the circumference of a circle.
 TV is a diameter of the circle.
 M is a point on PQ such that $PM = MQ$.



Not to scale

Complete these sentences to show that triangle TMP is congruent to triangle TMQ.

Side PM = side MQ because it is given to you.

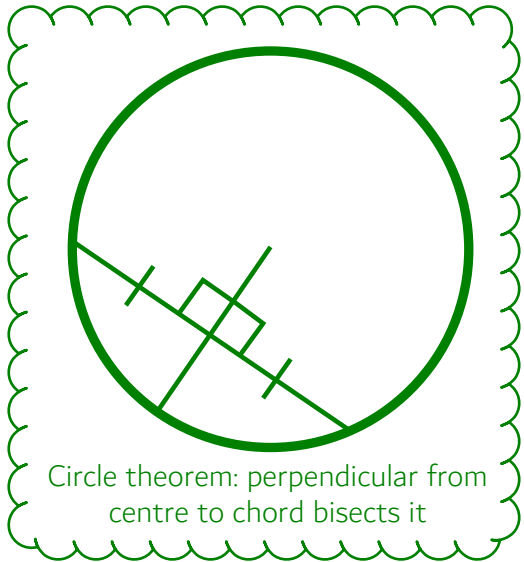
Angle PMT = angle QMT because VT bisects chord PQ and goes through the centre so VT must be perpendicular to PQ

See the circle theorem at the bottom of the page

Side MT is common As side MT is in both triangle TMP and triangle TMQ

Triangle TMP is congruent to triangle TMQ because SAS

[3]



Two sides and the angle between them are the same in both triangles. This is one of the conditions which means that triangles are congruent

16 A biologist assumes the population, P , of birds on an island can be predicted using the formula

$$P = 3800 \times 1.042^n$$

where n is the number of years after the start of 2020.

(a) Write down the percentage increase per year that is used in the formula.

1.042×100 ← Multiplying the decimal multiplier by 100 converts it into 104.2%

$104.2 - 100$ ← Subtracting 100% works out that it is increasing by 4.2%

(a) 4.2 % [1]

(b) Calculate the predicted population at the start of 2024.

$2024 - 2020$ ← Subtracting the year 2020 from the year 2024 works out that 2024 is 4 years after 2020

3800×1.042^4 ← Substituting 4 for n in the right side of the formula

Rounding 2279.7... to the nearest whole number → 4480 [2]

(c) (i) Show that the number of birds is predicted to exceed 7000 during 2034. [3]

$2034 - 2020$ ← Subtracting the year 2020 from the year 2034 works out that 2034 is 14 years after 2020

$3800 \times 1.042^{14} = 6759.7...$ ← Substituting 14 for n in the right side of the formula shows that the predicted population at the start of 2034 is less than 7000

$3800 \times 1.042^{15} = 7043.6...$ ← Substituting 15 for n in the right side of the formula shows that the predicted population at the start of 2035 is more than 7000

So the predicted population must exceed 7000 after the start of 2034 and before the start of 2035

(ii) A researcher says that between 2022 and 2030 the percentage increase per year in the population will be 2.8%.

If the researcher is correct, explain how this new information will affect the answer in part (c)(i).

It will be later than 2034

..... As 2.8% increase is significantly less than the increase of 4.8% meaning that it will take more time to exceed 7000 [1]

17 (a) A sequence is defined by

$$u_{n+1} = 3u_n + 7 \text{ and } u_1 = -2.$$

Work out the value of u_2 and the value of u_3 .

Entering -2 into the calculator and pressing =/exe to set this as the answer. Then entering 3Ans + 7. Pressing =/exe once gives u_2 then pressing =/exe again gives u_3 . This substitutes u_1 for u_n to give u_2 then substitutes u_2 for u_n to give u_3

(a) $u_2 = \dots\dots\dots 1 \dots\dots\dots$

$u_3 = \dots\dots\dots 10 \dots\dots\dots$

[2]

(b) Here are the first four terms of a quadratic sequence.

-2 7 22 43

The sequence has the formula $x_n = an^2 + b$.

Find the value of a and the value of b .

9, 15 ←

Listing the first differences. $7 - -2 = 9$ so the difference between the first two terms is 9 and $22 - 7 = 15$ so the difference between the next two terms is 15

6 ←

The second difference is 6 as $15 - 9 = 6$

$3n^2: 3, 12$ ←

Halving the second difference finds that a is 3. Writing out the first two terms of the sequence $3n^2$. $3(1)^2 = 3$ then $3(2)^2 = 12$

-5, -5 ←

Listing what must be added to each term of the $3n^2$ sequence to give the original sequence. -5 must be added to 3 to get -2 and -5 must be added to 12 to get 7

So b must be -5 as this is what is added to all of the terms of $3n^2$ to give the original sequence

(b) $a = \dots\dots\dots 3 \dots\dots\dots$

$b = \dots\dots\dots -5 \dots\dots\dots$

[3]

18 Solve this quadratic equation by factorisation.

$$2x^2 - 6x - 24 = 5x - 3$$

$$2x^2 - 11x - 21 = 0$$

Subtracting $5x$ from both sides and adding 3 to both sides to put it into the quadratic form $ax^2 + bx + c = 0$

$$2x^2 - 14x + 3x - 21 = 0$$

Multiplying a by c gives $2 \times -21 = -42$. Splitting the middle x term into $-14x$ and $+3x$ as -14 and 3 multiply to -42 and add to -11

$$2x(x-7) + 3(x-7) = 0$$

Factorising the left two terms and the right two terms

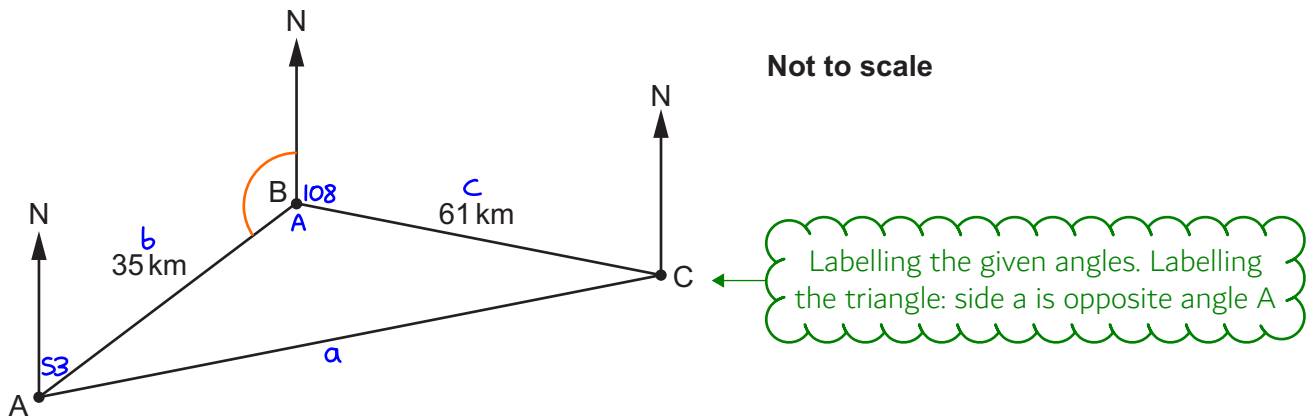
$$(2x+3)(x-7) = 0$$

Bringing it into the factorised form

Either $2x + 3 = 0$ or $x - 7 = 0$ as one of the two brackets must be 0 so that they multiply to 0 . Solving these equations finds x

$$x = \dots\dots\dots \frac{-3}{2} \dots\dots\dots \text{ or } x = \dots\dots\dots 7 \dots\dots\dots [4]$$

19 The diagram shows the positions of three towns A, B and C.



The bearing of town B from town A is 053° .
 The bearing of town C from town B is 108° .
 $AB = 35$ km and $BC = 61$ km.

Calculate AC.

You must show your working.

$$180 - 53$$

The 53° and the orange angle are co-interior so add up to 180° .
 Subtracting the 53° from 180° works out that the orange angle is 127°

$$360 - 127 - 108 = 125$$

There are 360° around a point. So subtracting the 127° and 108° from 360° works out that the angle A is 125°

$$a^2 = b^2 + c^2 - 2bc \cos A$$

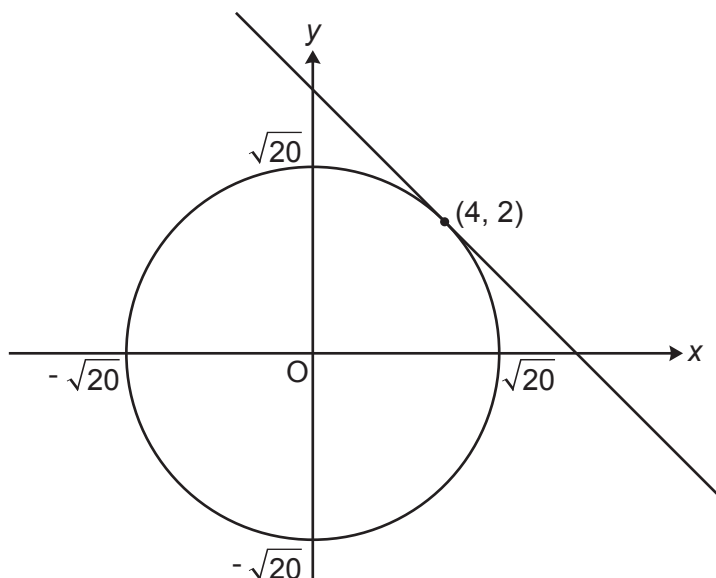
The triangle does not have a known right-angle. There are not two opposite pairs of sides and angles so the sine rule cannot be used. The cosine rule should be used

$$AC = \sqrt{35^2 + 61^2 - 2 \times 35 \times 61 \times \cos 125}$$

Square rooting both sides to make side a the subject and substituting in the values

$$AC = \dots\dots\dots 86.0 \dots\dots\dots \text{ km [5]}$$

- 20 The diagram shows a circle, centre the origin, with the tangent to the circle at the point (4, 2).



- (a) Write down the equation of the circle.

The general equation of a circle with its centre at the origin is $x^2 + y^2 = r^2$, where r is the radius. The radius is $\sqrt{20}$ and squaring this gives 20

(a) $x^2 + y^2 = 20$ [2]

- (b) (i) Show that the tangent to the circle at the point (4, 2) has gradient -2. [2]

$$\frac{2-0}{4-0}$$

Working out that the gradient of the radius from centre (0, 0) to point (4, 2) has gradient $1/2$. Gradient = (change in y)/(change in x). $2 - 0$ expresses the change in y and $4 - 0$ expresses the change in x

$$-1 \div \frac{1}{2} = -2$$

The tangent is perpendicular to the radius so its gradient is the negative reciprocal of $1/2$

- (ii) Find the equation of the tangent to the circle at the point (4, 2).

$$y = -2x + c$$

The general equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept. Substituting -2 for m as this is the gradient

$$c = 2 + 2 \times 4$$

Adding $2x$ to both sides makes c the subject and gives $c = y + 2x$. Substituting 2 for y and 4 for x from the point (4, 2) finds that $c = 10$

(b)(ii) $y = -2x + 10$ [2]

21 Solve.

$$x^{-\frac{1}{6}} = \frac{5x^{\frac{1}{3}}}{x^{\frac{3}{4}}}, \text{ where } x \neq 0$$

$$-\frac{1}{6} + \frac{3}{4}$$

Multiplying both sides by $x^{3/4}$ to get rid of x as the denominator. This gives
 $x^{-1/6} \times x^{3/4} = 5x^{1/3}$. $y^a \times y^b = y^{a+b}$, so adding the powers of $-1/6$ and $3/4$

$$\frac{7}{12} - \frac{1}{3}$$

Dividing both sides by $x^{1/3}$ to get all the x on the same sides and the 5 on the other.
 This gives $x^{7/12} \div x^{1/3} = 5$. $y^a \div y^b = y^{a-b}$, so subtracting the powers of $7/12$ and $1/3$

$$x^{\frac{1}{4}} = 5$$

Writing the simplified equation with all the x on one side and all the values on the other

$$x = \sqrt[4]{5}$$

$1/4$ rooting both sides to get rid of the power on the left and find x

$$x = \dots\dots\dots 625 \dots\dots\dots [3]$$

Turn over for question 22

22 You are given this identity.

$$\frac{2 - 3\sqrt{18}}{\sqrt{18} + 4} = a\sqrt{2} + b$$

Find the value of a and the value of b .

You must show each step in your working.

$$\frac{2 - 3\sqrt{18}}{\sqrt{18} + 4} \times \frac{\sqrt{18} - 4}{\sqrt{18} - 4}$$

Rationalising the denominator on the left of the identity by multiplying both the numerator and denominator by the denominator with the sign in the middle flipped to a negative

$$\frac{2\sqrt{18} - 8 - 54 + 12\sqrt{18}}{18 - 4\sqrt{18} + 4\sqrt{18} - 16}$$

Expanding the numerator and denominator

$$\frac{14\sqrt{18} - 62}{2}$$

Collecting like terms

$$2\sqrt{2} - 31$$

Putting it into the same form as the right side of the identity by simplifying the surd and dividing the numerator by 2 to eliminate the denominator

$$a = \dots\dots\dots 21 \dots\dots\dots$$

$$b = \dots\dots\dots -31 \dots\dots\dots$$

[6]

END OF QUESTION PAPER

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