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Worked Solutions

Monday 11 November 2024 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

First name(s) ______________________________

Last name ______________________________

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE

- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

1 Sasha has these two sets of number cards.

Set A: 1 2 3 4   Set B: 8 9 10

One card is taken at random from each set.
Sasha adds the numbers on the two cards to get a total.

(a) Complete the table to show all the possible totals.

1 + 8 = 9

		Set A			
	Total	1	2	3	4
	8	9	10	🟩 11	12
Set B	9	10	🟩 11	12	🟩 13
	10	🟩 11	12	🟩 13	14

[2]

(b) Find the probability that the total is a prime number.
Give your answer as a fraction.

The totals shaded in green are prime as they are only divisible by themselves and 1. This is 5 out of the 12 totals.

The calculator can be used to check if a number is prime by formatting it as a product of prime factors. If it does not change it must be prime

(b) $\frac{5}{12}$ **[2]**

2 The price of a holiday increases from £320 to £340.

Work out the percentage increase in the price of the holiday.

$340 - 320$ ← This works out that the increase is £20

$\frac{20}{320} \times 100$ ← Putting the £20 increase over the original £320 expresses the increase as a fraction. Multiplying this by 100 converts it to a percentage

6.25 % **[3]**

3 A bag contains only blue, green and red counters in the ratio 7 : 3 : 2.
There are 76 more blue counters than green counters in the bag.

Work out the **total** number of counters in the bag.

76 ÷ 4 ← 7 - 3 = 4 more parts for blue than for green. So dividing the 76 by 4 works out that 1 part of the ratio is worth 19 counters

19 × 12 ← 7 + 3 + 2 = 12 parts in total in the ratio. So multiplying the value of 1 part of the ratio by 12 works out that the total number of counters is 228

........................228........................ **[4]**

4 A farmer has 60 pear trees.
The table shows the heights, h metres, of the pear trees.

Height (h metres)	**Frequency**		
$1 < h \leqslant 2$	5	1.5	7.5
$2 < h \leqslant 3$	8	2.5	20
$3 < h \leqslant 4$	32	3.5	112
$4 < h \leqslant 5$	15	4.5	67.5

$\boxed{A}$ $\boxed{B}$

$\underset{\boxed{C}}{207} \div \underset{\boxed{D}}{60}$

(a) Calculate an estimate of the mean height of the 60 pear trees.

A: writing the midpoint of each interval. Halfway between 1 and 2 is 1.5. Halfway between 2 and 3 is 2.5. Halfway between 3 and 4 is 3.5. Halfway between 4 and 5 is 4.5.

B: multiplying the midpoints by the frequencies works out an estimate for the total height of all the pear trees for each interval.

C: Adding all of these totals gives an overall height of 207 m for all of the pear trees.

D: Dividing the total height by the 60 pear trees works out an estimate for the mean

(a) ....3.45........ m **[4]**

(b) Explain why it is not possible to use the information from this table to calculate the **exact** value of the mean height.

Exact heights are not known

The heights are given in intervals and the midpoints were used for the estimate. However without knowing the exact heights it is impossible to calculate the exact mean as the heights are probably not the midpoints

[1]

5 Rearrange this formula to make *f* the subject.

$$e = \frac{k}{f}$$

$ef = k$ ← Multiplying both sides by f to eliminate it as a denominator

Dividing both sides by e to get f on its own

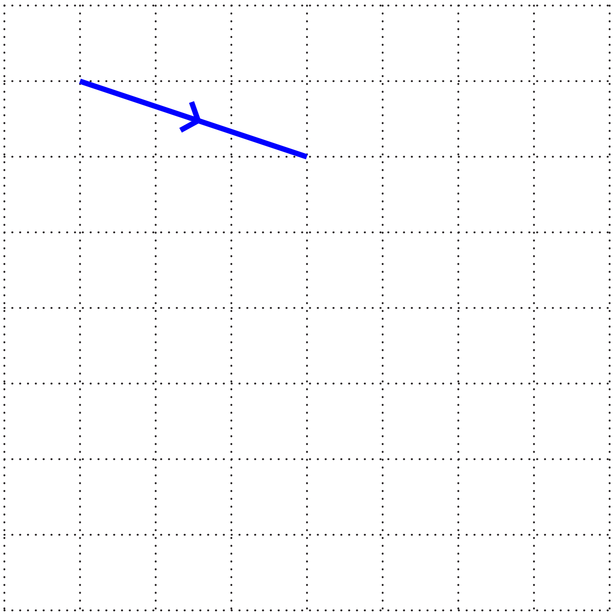
$f = \frac{k}{e}$ .... **[2]**

x-component

6 $\overrightarrow{AB} = \begin{pmatrix}3\\-1\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}2\\6\end{pmatrix}$.

y-component

(a) On the grid below, draw $\overrightarrow{AB}$. 3 to the right and 1 down



[2]

(b) Work out $\overrightarrow{AC}$. ← $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$3 + 2$ ← Adding the x-components works out that the x-component of $\overrightarrow{AC}$ is 5

$-1 + 6$ ← Adding the y-components works out that the y-component of $\overrightarrow{AC}$ is 5

$\begin{pmatrix}5\\5\end{pmatrix}$ **[2]**

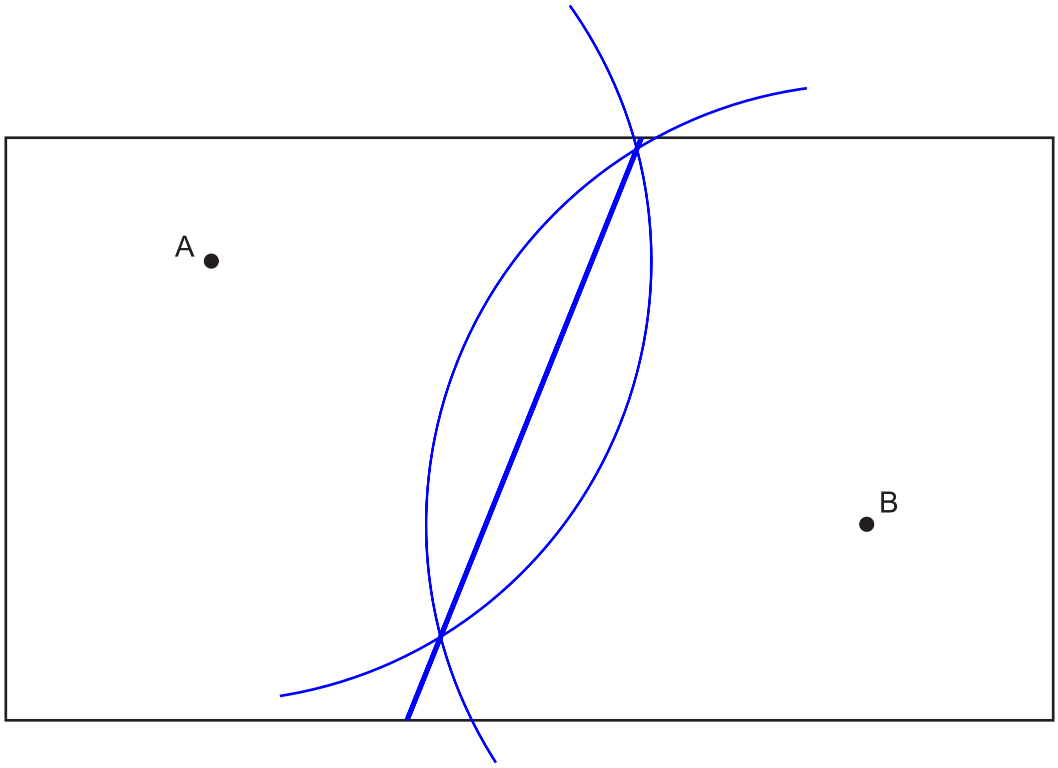
(c) Write down $\overrightarrow{BA}$.

$\overrightarrow{BA}$ is in the opposite direction to $\overrightarrow{AB}$. So it goes 3 to the left and 1 up → $\begin{pmatrix}-3\\1\end{pmatrix}$ **[1]**

7 The diagram represents a rectangular field.
A and B are two trees.

A straight path goes across the field.
The path is always the same distance from A and B.

Construct the route followed by the path.
Show all your construction lines.



[2]

Constructing a perpendicular bisector by scribing arcs from A and B with the same radius using a compass. Then drawing a straight line through the two crosses where the arcs meet

8 **(a)** 198 and 495 are written below as the product of their prime factors.

$198 = 2 \times 3^2 \times 11 \qquad 495 = 3^2 \times 5 \times 11$

Work out the highest common factor (HCF) of 198 and 495.

$3^2 \times 11$ ← Multiplying the lowest power of each prime in both product of prime factors works out the HCF

Newer Casio calculators can work out the HCF of two numbers without having to do this method

(a) .... 99 .... **[2]**

(b) A five-digit passcode is created using the lowest common multiple (LCM) followed by the highest common factor (HCF) of two numbers.
The two numbers chosen are 198 and 495.

(i) To try and find the passcode, a computer hacker multiplies the highest common factor (HCF) of 198 and 495 by 5 and uses this as the lowest common multiple (LCM) in the passcode.
The computer hacker's passcode is incorrect.

Write down the omission in the computer hacker's method.

Should also multiply by 2

The LCM is the highest power of each prime in both product of prime factors. 2 is the highest power of the 2s and this was missed **[1]**

(ii) Work out the correct five-digit passcode.

$2 \times 3^2 \times 5 \times 11$ ← The LCM is the highest power of each prime in both product of prime factors

Newer Casio calculators can work out the LCM of two numbers without having to do this method

The LCM is 990 and the HCF is 99. The LCM followed by the HCF is 99099

(ii) .... 99099 .... **[2]**

9 The next term in a Fibonacci sequence is found by adding together the previous two terms.

In a particular Fibonacci sequence:

- the first term is 3
- the second term is x.

(a) Show that the fifth term in the sequence is $6+3x$. **[2]**

$3 + x \leftarrow$ Adding the 1st and 2nd term gives the 3rd term

$x + 3 + x \leftarrow$ Adding the 2nd and 3rd term gives the 4th term

$3 + x + x + 3 + x \leftarrow$ Adding the 3rd and 4th term gives the 5th term

$6 + 3x \leftarrow$ Collecting like terms

(b) The sixth term in the sequence is 74.

Find the value of x.

$x + 3 + x + 6 + 3x \leftarrow$ Adding the 4th and 5th term gives the 6th term

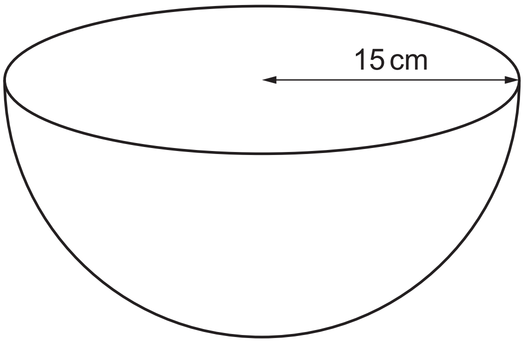
$5x + 9 = 74 \leftarrow$ Collecting like terms and setting equal to the 74

$5x = 65 \leftarrow$ Subtracting 9 from both sides gets the x term on its own

Dividing both sides by 5 gets the x on its own

(b) $x =$ ....13........ **[4]**

10 A bowl in the shape of a hemisphere with radius 15 cm is used to collect raindrops.



Assume each raindrop has the volume of a sphere of radius 3×10^{-4} cm.

Calculate how many raindrops it takes to completely fill the bowl.
Give your answer in standard form.
You must show your working.

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$\frac{1}{2} \times \frac{4}{3}\pi \times 15^3 = 2250\pi$ ←

This works out that the volume of the bowl is 2250π cm^3. Doing half of the volume of the whole sphere and substituting the radius of 15 cm into the formula

$\frac{4}{3}\pi \times (3 \times 10^{-4})^3$ ←

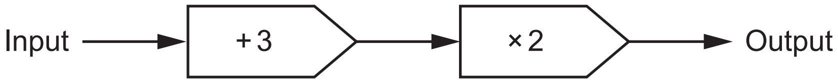
This works out that the volume of each raindrop is $1.1\ldots \times 10^{-10}$ cm^3. Substituting the radius of 3×10^{-4} cm into the formula

$2250\pi \div 1.1\ldots \times 10^{-10}$ ←

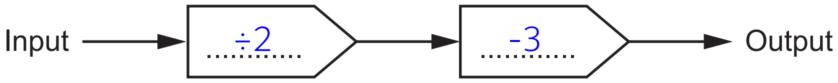
Dividing the volume of the bowl by the exact value of the volume of each raindrop works out how many raindrops it takes to completely fill the bowl

6.25×10^{13} **[6]**

11 **(a)** Here is a function.



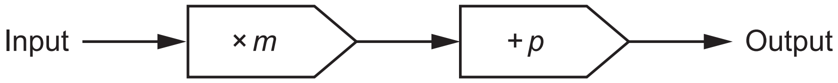
Complete the diagram below to show the **inverse** of the function.



Doing the opposite operations in the opposite order

[2]

(b) Here is another function.



When the input is 5, the output is 8.5.
When the input is 10, the output is 11.

Find the value of *m* and the value of *p*.

$5m + p = 8.5$ ← Multiplying the input of 5 by m then adding p gives the output of 8.5. This forms the 1st equation

$10m + p = 11$ ← Multiplying the input of 10 by m then adding p gives the output of 11. This forms the 2nd equation

$5m = 2.5$ ← Doing simultaneous equations by subtracting the 1st equation from the 2nd equation. This cancels out the p. Then dividing both sides by 5 finds that m = 0.5

$5(0.5) + p = 8.5$ ← Substituting 0.5 for m in the 1st equation

(b) $m =$ 0.5

Subtracting 5(0.5) from both sides gets p on its own → $p =$ 6 **[5]**

12 **(a)** Find all the possible integer values of *x* that satisfy the inequality $10 < 3x - 2 \leqslant 21$.

$12 < 3x \leq 23$ ← Adding 2 to all sides to eliminate the -2 in the centre and get the x term on its own

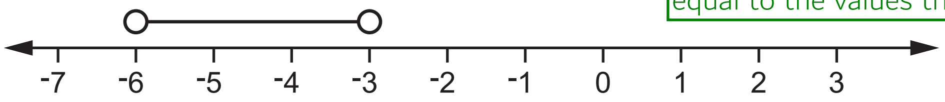
$4 < x \leq 7.\dot{6}$ ← Dividing all sides by 3 to eliminate the 3 and get x on its won

These are the integers which are greater than 4 and less than or equal to $7.\dot{6}$

(a) ......5, 6, 7...... **[3]**

(b) An inequality is shown on the number line below.

Open circles mean that x cannot be equal to the values the circles are over



Taylor says,

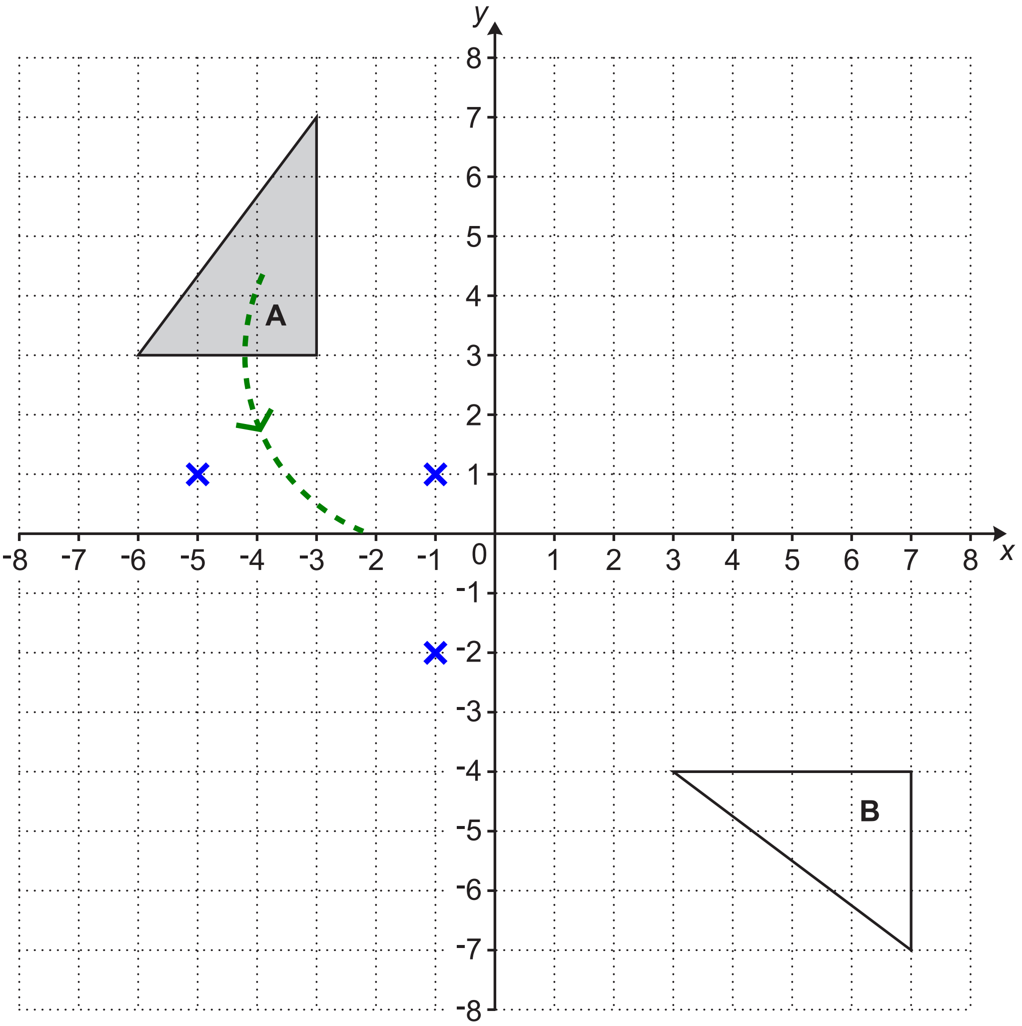
You can write this inequality as {x: -3 < x < -6}. ← This is meaning that x is greater than -3 and less than -6. This is not possible

Explain why Taylor is **not** correct.

It should be {x: -6 < x < -3} ← x is greater than -6 and less than -3

.... **[1]**

13 Triangle **A** and triangle **B** are shown on the coordinate grid.



Triangle **A** is mapped onto triangle **B** using a combination of two transformations:

- a transformation T, followed by
- a translation of $\begin{pmatrix} 8 \\ -5 \end{pmatrix}$. ← This means 8 to the right and 5 down

Describe fully transformation T.

First moving B 8 to the left and 5 up to do the opposite of the translation

Rotation 90° anticlockwise centre (-1, 3) .... **[4]**

The centre can be found using tracing paper and considering the circular motion of the rotation. (-1, 3) is at the centre of the circles A rotates around to get the corners of the triangle drawn. When using tracing paper, sketching around A, putting something sharp in at (-1, 3) and rotating the paper around gets the drawn triangle

14 $N = 4a^6$.

Write the following in the form ka^m.

Raising both the 4 and the a^6 to the power of -1 and 3/2. When raising a power to a power, the indices are multiplied

(a) $N^{-1} = \frac{1}{4}\, a^{-6}$ **[2]**

(b) $N^{\frac{3}{2}} = 8\, a^{9}$ **[2]**

15 Prove that $1.8\dot{6}$ converts to the fraction $\frac{28}{15}$.

You must show your working. **[3]**

$x = 1.8\dot{6}$ ← Let x be the recurring decimal

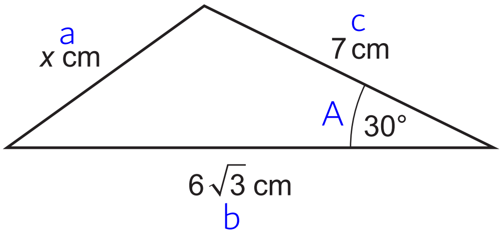
$10x = 18.6\dot{6}$ ← There is 1 recurring digit. So multiplying by ten 1 time writes another decimal with the recurring digit in the same decimal place

$9x = 16.8$ ← Subtracting x form 10x cancels out the recurring digit

$x = \frac{16.8}{9}$ ← Dividing both sides by 9 expresses x as a fraction

$= \frac{28}{15}$ ← Using the calculator to simplify the fraction

16 Work out the exact value of *x* in this triangle.



Not to scale

Labelling the triangle. Side a is opposite angle A

$x^2 = (6\sqrt{3})^2 + 7^2 - 2 \times 6\sqrt{3} \times 7 \times \cos 30$

There are not opposite pairs of sides and angles so the sine rule cannot be used. So the cosine rule should be used. $a^2 = b^2 + c^2 - 2bc\cos A$

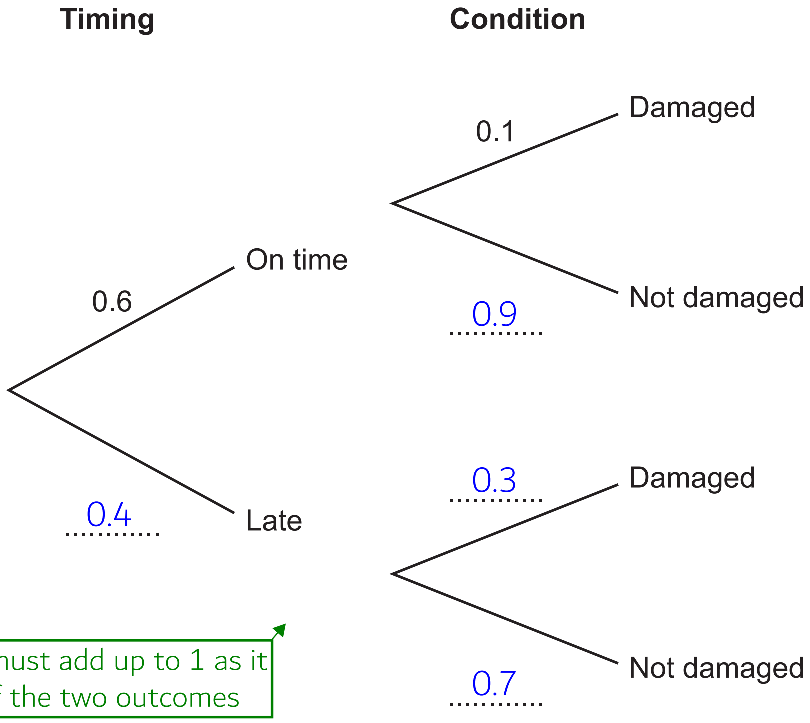
Square rooting both sides finds x

$x =$ .... $\sqrt{31}$ .... **[4]**

17 An online company is tracking the timing and condition of its deliveries.

The probability that a parcel arrives on time is 0.6.
When the parcel arrives on time, the probability that it is damaged is 0.1.
When the parcel arrives late, the probability that it is damaged is 0.3.

(a) Use the information to complete the tree diagram.



Each set of branches must add up to 1 as it is certain to get one of the two outcomes

[3]

(b) Given that a parcel arrives damaged, find the probability that it also arrived on time.

$0.6 \times 0.1 = 0.06$

On time AND damaged. AND means to multiply the probabilities

$0.6 \times 0.1 + 0.4 \times 0.3 = 0.18$

On time AND damaged OR late AND damaged. AND means to multiply the probabilities. OR means to add the probabilities. So the probability it is damaged is 0.18

$\frac{0.06}{0.18}$

Expressing the probability it is on time and damaged as a fraction of the probability that it is damaged

Using the calculator to simplify the fraction

(b) .... $\frac{1}{3}$ .... **[4]**

18 *y* is inversely proportional to x^2.

Find the percentage decrease in *y* when *x* is increased by 25%.

$\frac{100 + 25}{100}$ ← Adding the 25% to 100% expresses the percentage x increases to. Putting this over 100 converts it to the decimal multiplier 1.25

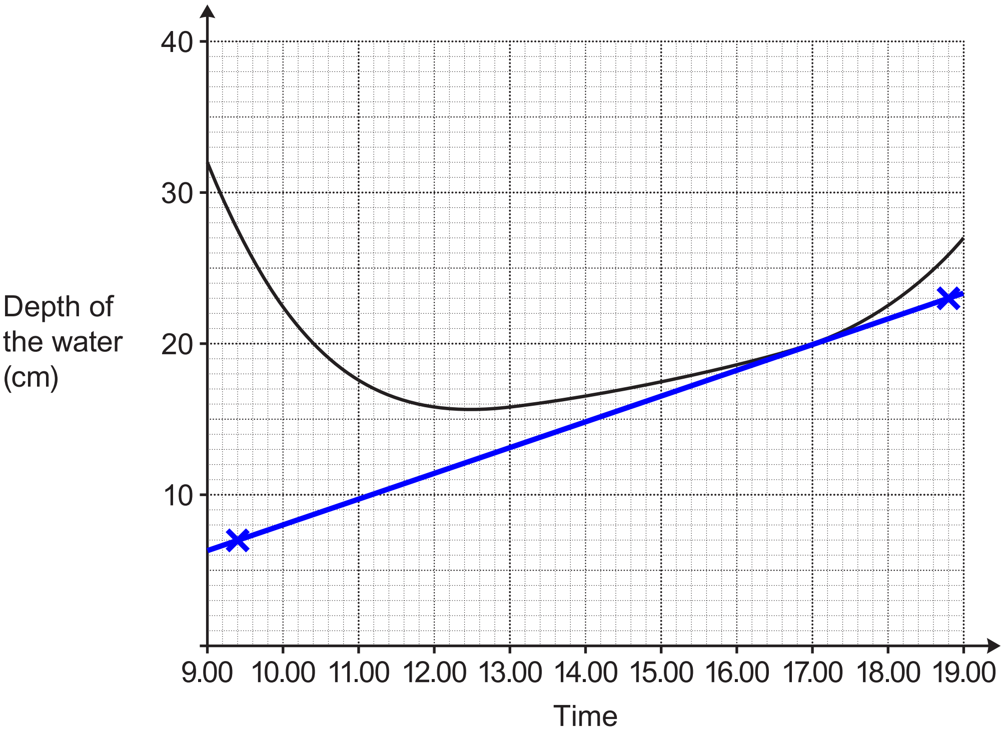
$1 \div 1.25^2$ ← This expresses the decimal multiplier for y. If x is multiplied by 1.25, y will be divided by 1.25^2

0.64×100 ← Multiplying the decimal multiplier for y my 100 converts it to a percentage

$100 - 64$ ← So y decreases to 64%. Subtracting this from 100% works out the percentage decrease

........................36........................ % **[4]**

19 This graph shows the depth of the water, in centimetres, at a particular point in a river over a period of 10 hours.



(a) Work out the average rate of change in the depth of the water over the 10 hours.

$-5 \div 10$ ← The depth went down 5 cm over the 10 hours. Dividing -5 cm by 10 hours gives the rate in cm per hour. Per means to divide

(a) ....-0.5.......... cm per hour **[2]**

(b) Use the graph to estimate the rate of change in the depth of the water at 17.00.
You must show working to support your estimate.

$$\frac{23 - 7}{18.8 - 9.4}$$

← The gradient of the curve at 17.00 is the rate. Drawing a tangent to estimate the gradient at this point. Picking two points on the tangent which are on grid lines and are far away from each other. Gradient = (change in y)/(change in x). y changed from 7 to 23 and x changed from 9.4 to 18.8. Using a decimal number of hours rather than hours and minutes

(b) ....1.7.......... cm per hour **[4]**

20 $(4x + a)(4x - a)(x^2 + 2) = 16x^4 + bx^2 - 50$

Find the **two** possible pairs of values for a and b.
You must show your working.

$16x^2 - 4ax + 4ax - a^2$ ← Expanding the first two brackets

$(16x^2 - a^2)(x^2 + 2)$ ← Collecting like terms and writing the third bracket

$16x^4 + 32x^2 - a^2x^2 - 2a^2$ ← Expanding the brackets

$16x^4 + (32 - a^2)x^2 - 2a^2$ ← Collecting like terms so that it is in the same form as the right side of the identity

$-2a^2 = -50$ ← The $-2a^2$ and -50 are both constants in the identity so must be equal

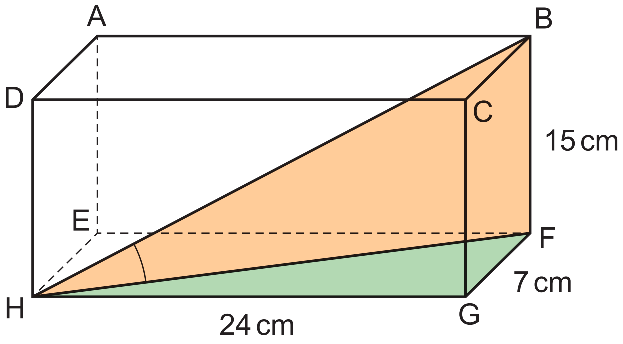
$a^2 = 25$ ← Dividing both sides by -2. Then doing the positive and negative square root of 25 finds the possible values of a

$32 - a^2 = b$ ← Equating the x^2 coefficients. Substituting in the possible values of a works out the possible values of b

Pair 1: a = ...5....... and b = ...7.......

Pair 2: a = ...-5....... and b = ...7....... **[6]**

21 The diagram shows a cuboid ABCDEFGH.



Not to scale

FB = 15 cm, GF = 7 cm and HG = 24 cm.

Calculate the angle BHF.
You must show your working.

$24^2 + 7^2 = HF^2$

Using Pythagoras' Theorem in the green right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side

$HF = 25$

Square rooting both sides finds that HF is 25 cm

$$\underset{S\quad H}{\overset{\checkmark}{O}}\quad \underset{C\quad H}{\overset{\checkmark}{A}}\quad \underset{T\quad \overset{\checkmark}{A}}{\overset{\checkmark}{O}}$$

Using right-angled trigonometry in the orange right-angled triangle. Ticking O as the 15 cm is the opposite and ticking A as the 25 cm is the adjacent. There are two ticks on the TOA formula triangle so this one can be used

$\tan x = \frac{15}{25}$

Covering T in the TOA formula triangle finds that tan of the angle = opposite/adjacent

Doing the inverse tan of both sides finds the angle

Angle BHF = ...31.0........ ° **[5]**

Turn over

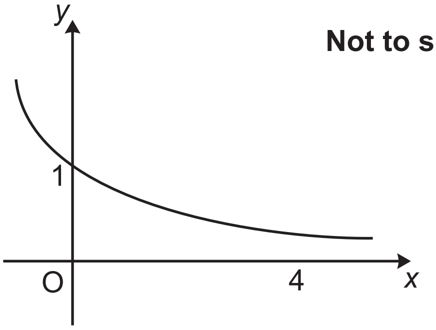
22 **(a)** For each graph below, select its possible equation from this list.

$y = \sqrt{x-4}$ $y = 4^x$ $y = x^4$ $y = \frac{4}{x}$

$y = \left(\frac{1}{4}\right)^x$ $y = -4x^2$ $y = 4\cos x$ $y = \sqrt{4^2 - x^2}$

Use table mode on the calculator to give a table of values for each equation. Start: -5. End: 5. Step: 1

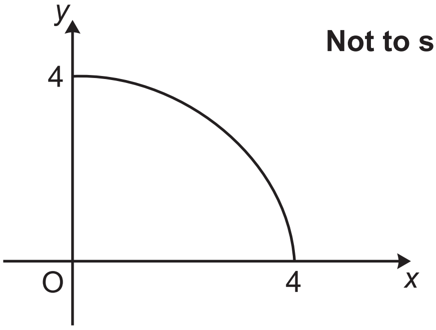
(i)



(a)(i) $y = \left(\frac{1}{4}\right)^x$ **[1]**

This is the only equation with a y-intercept of 1, and which decreases as x increases

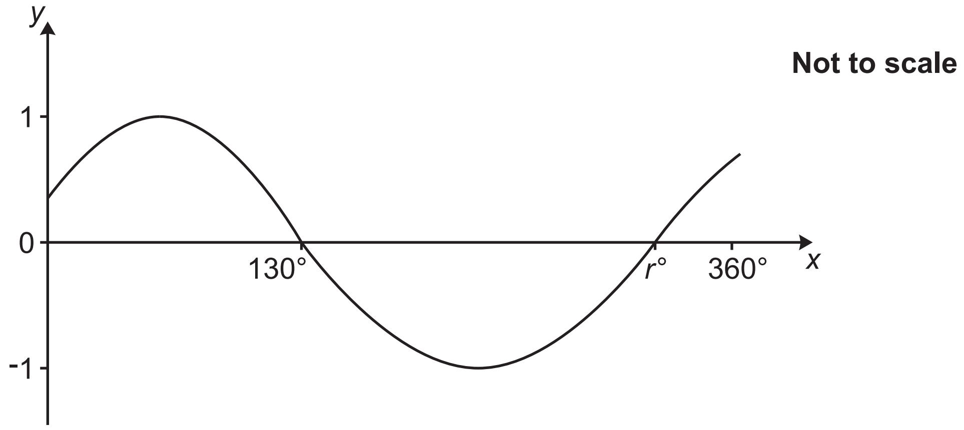
(ii)



(ii) $y = \sqrt{4^2 - x^2}$ **[1]**

This is the only equation with a y-intercept of 4, and which goes through (4, 0)

(b) A graph is drawn on the axes below.



The equation of the graph is $y = \sin(x + p)$, where $0° \leqslant x \leqslant 360°$.
The x-intercepts are 130° and r°.

Write down the value of p and the value of r.

On a y = sinx graph, it goes though (180, 0). So the graph has been translated 50 to the left. So p must be 50

p = 50

r = 310 **[2]**

On a y = sinx graph, it goes though (360, 0). The graph has been translated 50 to the left

END OF QUESTION PAPER