

Please write clearly in block capitals.

Centre number       Candidate number

Surname \_\_\_\_\_

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

# GCSE MATHEMATICS

# H

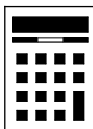
Higher Tier Paper 3 Calculator

Wednesday 8 November 2017 Morning Time allowed: 1 hour 30 minutes

### Materials

For this paper you must have:

- a calculator
- mathematical instruments.



### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

### Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
<b>TOTAL</b>	



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

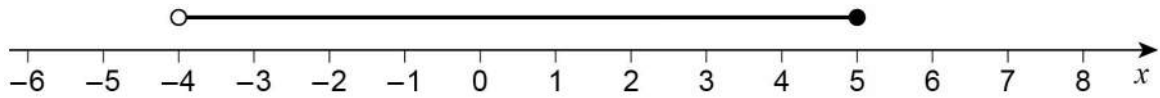
Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)

Answer **all** questions in the spaces provided

- 1 Circle the inequality shown by the diagram.



[1 mark]

$$-4 \leq x < 5$$

$$-4 \leq x \leq 5$$

$$-4 < x < 5$$

$$-4 < x \leq 5$$

The closed dot means it can be equal to the value it is above. The open dot means it cannot be equal to the value it is above.  $x$  is greater than 4 but less than or equal to 5

- 2  $y$  is 100% **more** than  $x$ .

Circle the ratio  $x : y$

[1 mark]

$$1 : 100$$

$$100 : 1$$

$$1 : 2$$

$$2 : 1$$

Increasing by 100% makes it 200%. So the ratio would be 100 : 200, which simplifies to 1 : 2

- 3 The first four terms of a sequence are  $-10$   $-8$   $-6$   $-4$

Circle the expression for the  $n$ th term of the sequence.

[1 mark]

$$-12 - 2n$$

$$-8 - 2n$$

$$n + 2$$

$$2n - 12$$

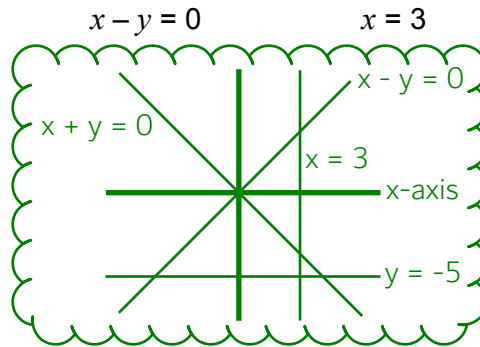
The sequence increases by 2 between each term so it must involve positive  $2n$



- 4 Circle the equation of the line that is parallel to the  $x$ -axis.

[1 mark]

$$y = -5$$



$$x + y = 0$$

- 5 Multiply out and simplify  $(x - 8)^2$

[2 marks]

$$(x - 8)(x - 8)$$

The power of 2 means that the bracket is squared, which means to multiply it by itself

$$x^2 - 8x - 8x + 64$$

Expanding the brackets

Answer  $x^2 - 16x + 64$

Simplifying by collecting like terms

Turn over for the next question

Turn over ►

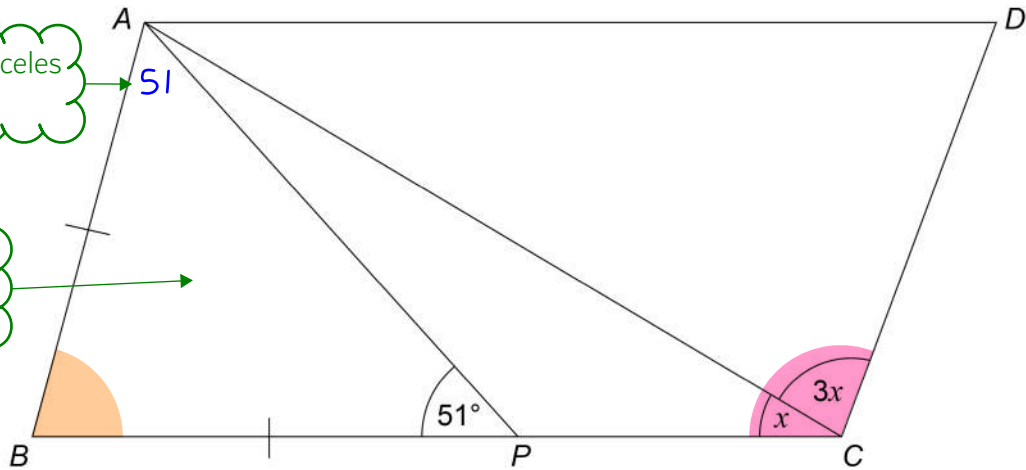




8  $ABCD$  is a parallelogram.

$$AB = BP$$

Not drawn  
accurately



Base angles of an isosceles triangle are equal

This triangle is isosceles as two of the sides are equal

Work out the size of angle  $x$ .

[4 marks]

$$180 - 51 - 51$$

There are 180 degrees in total in a triangle. Subtracting both of the 51 degree angles in triangle  $ABP$  from 180 degrees works out that the orange angle is 78 degrees

$$180 - 78$$

The orange and pink angle are co-interior angles so they add up to 180 degrees. Subtracting the orange angle from 180 degrees works out that the pink angle is 102 degrees

$$102 \div 4$$

$x + 3x = 4x$  so the pink angle is also  $4x$ . Dividing the 102 degrees by 4 works out  $x$

Answer 25.5 degrees

Turn over for the next question

Turn over ►



9 (a) Rearrange  $v = u + at$  to make  $t$  the subject of the formula.

[2 marks]

$$v - u = at$$

Subtracting  $u$  from both sides gets the term involving  $t$  on its own

Answer  $\frac{v-u}{a} = t$

Dividing both sides by  $a$  gets  $t$  on its own

9 (b) Complete this table with consistent metric units.

[2 marks]

Distance	Time	Speed	Acceleration
m	s	m/s	m/s <sup>2</sup>

Speed = distance/time. Dividing the metres by seconds give the unit of speed.

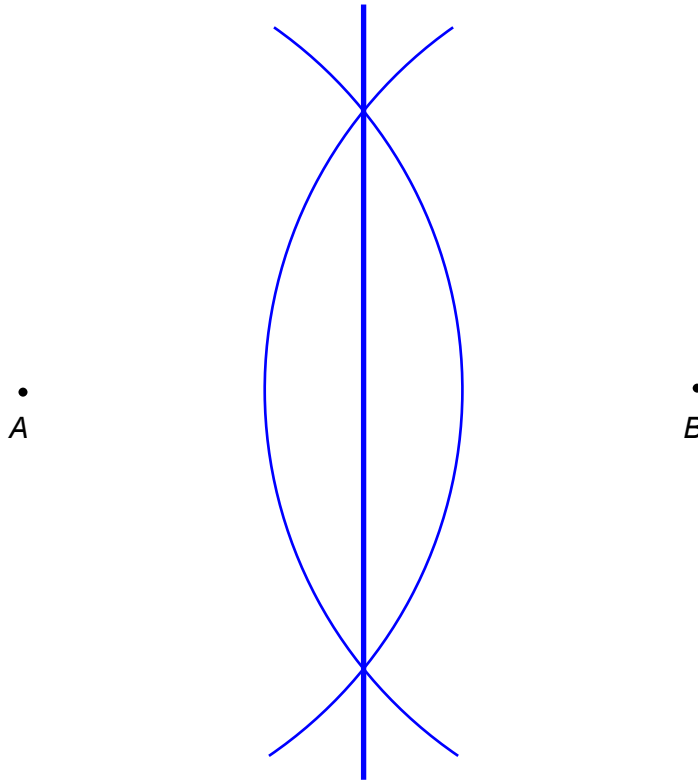
Acceleration = (change in speed)/(change in time). Dividing the unit of speed by seconds gives the unit of acceleration



10

Construct a locus of points that are the same distance from points  $A$  and  $B$ .

[2 marks]



Creating a perpendicular bisector of line  $AB$ . Scribing an arc from  $A$  which is at least halfway between  $A$  and  $B$ . Scribing an arc from  $B$  which is at least halfway between  $A$  and  $B$ . Drawing a straight line through the points where the two arcs cross

Turn over for the next question

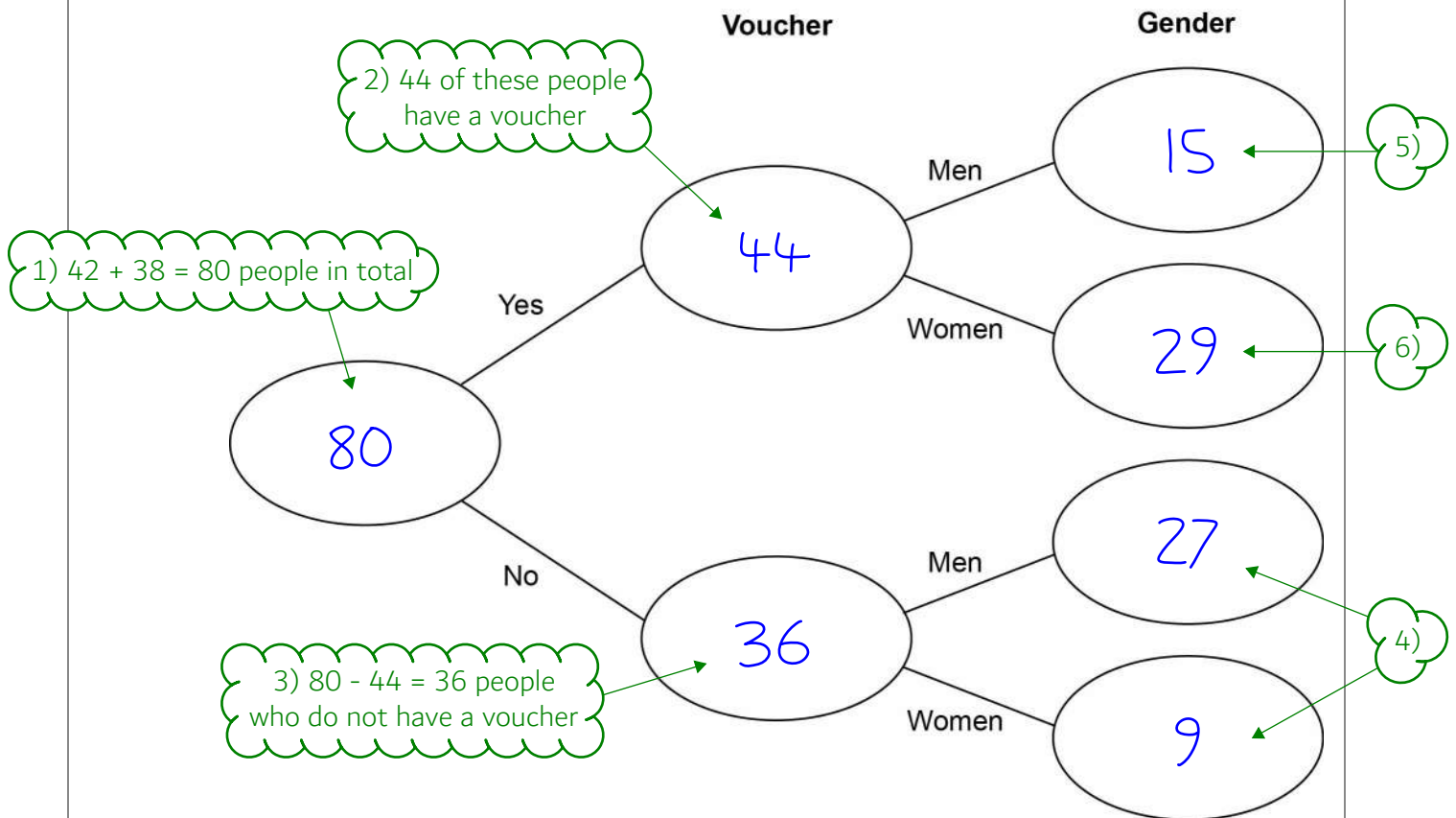
Turn over ►



- 11 42 men and 38 women visit a restaurant.  
44 of these people have a voucher.  
Three times as many men as women do **not** have a voucher.

11 (a) Complete the frequency tree.

[4 marks]



- 11 (b)** A voucher takes **15% off** the bill.  
After using the voucher, the bill for a meal is £27.20  
How much was the bill before using the voucher?

[3 marks]

$100 - 15$

Subtracting the 15% from 100% works out that the bill was reduced to 85%

$27.20 \div 85$

Dividing the £27.20 by 85 works out that 1% of the bill before using the voucher was £0.32

$0.32 \times 100$

Multiplying the £0.32 by 100 works out that 100% of the bill before using voucher is £32

Answer £ 32

Turn over for the next question

Turn over ►



12 The distance by road from Newport to London is 140 miles.

Tom travels by coach from Newport to London.  
The coach leaves Newport at 1.30 pm

12 (a) He assumes the coach will travel at an average speed of 50 mph

Use his assumption to work out the arrival time in London.

[3 marks]

$s^d t$

This is a speed, distance, time problem so writing the formula triangle

$140 \div 50$

Covering  $t$  in the formula triangle finds that time = distance  $\div$  speed. So the time taken for the journey is 2.8 hours. The speed was in miles per hour which is why the unit of time is hours

$1^{\circ}30' + 2.8^{\circ}$

Adding the 2.8 hours to 1 hour and 30 minutes as sexagesimals on the calculator gives an answer of  $4^{\circ}18'0''$ , which can be read as 4 hours and 18 minutes

Converting 4 hours and 18 minutes back into time format.  
It is still pm as it is after noon and before midnight

Answer 4.18 pm

12 (b) In fact, the coach has a lower average speed.

How does this affect the arrival time?

[1 mark]

It will be later

Time = distance  $\div$  speed so having a lower speed will mean dividing by less, which increases the time



**13** Here is some information about the length of time cars stayed in a car park.

Shortest time 30 minutes

Lower quartile 2 hours

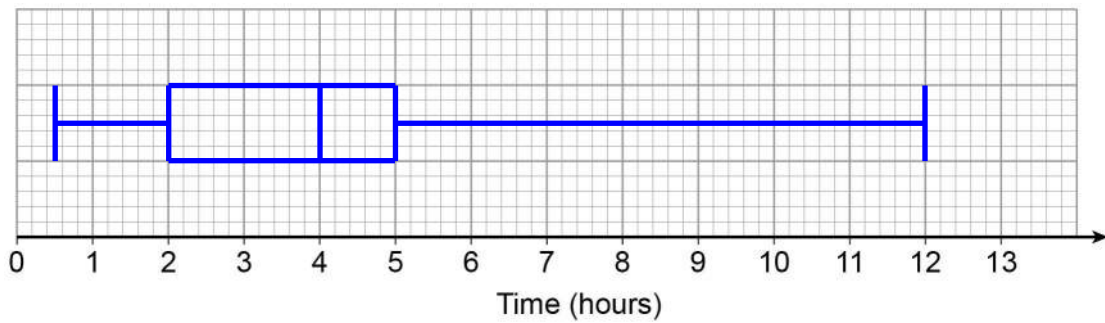
Longest time 12 hours

Interquartile range 3 hours

Median time 4 hours

Draw a box plot to show this information.

**[3 marks]**



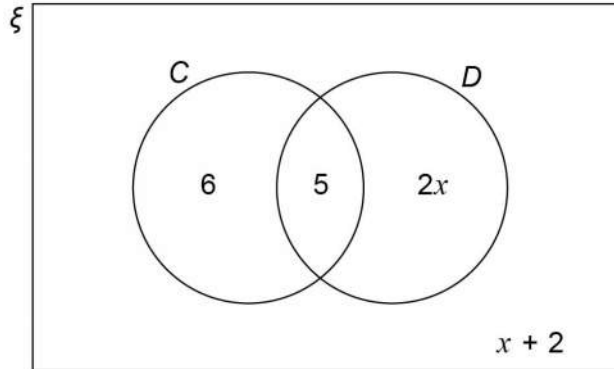
**Turn over for the next question**

Drawing vertical lines for the lowest, lower quartile, median, upper quartile and highest.  
Connecting the lowest value to the lower quartile and the upper quartile to the highest value using horizontal lines. Drawing a box around the quartiles and median. The upper quartile is found by adding the interquartile range to the lower quartile.  $2 + 3 = 5$

Turn over ►



- 14** In the Venn diagram  
 $\xi$  represents 31 students in a class  
 C is students who have a cat  
 D is students who have a dog



- 14 (a)** One student from the class is picked at random.

Work out the probability that the student has a dog.

[3 marks]

$$6 + 5 + 2x + x + 2$$

Expressing the total number of students in the class in terms of  $x$  by adding all the expressions in the Venn diagram

$$3x + 13 = 31$$

Simplifying the expression by collecting like terms. This must be equal to the 31 total students

$$3x = 18$$

Subtracting 13 from both sides to get the  $x$  term on its own

$$x = 6$$

Dividing both sides by 3 to get  $x$  on its own

$$2 \times 6 + 5$$

Substituting 6 for  $x$  in  $2x + 5$ , which is the number of students who have a dog

Answer  $\frac{17}{31}$

17 out of the 31 students have a dog

- 14 (b)** One of the students who has a cat is picked at random.

Work out the probability that this student has a dog.

[1 mark]

$$6 + 5$$

Adding the 6 who only have a cat and the 5 who have both a cat and a dog works out that 11 students have a cat

Answer  $\frac{5}{11}$

5 out of the 11 students who have a cat also have a dog



15 Circle the highest common factor (HCF) of  $6xy^2$  and  $4x^3y$

[1 mark]

$2xy^2$

$2xy$

$12x^3y^2$

$24x^4y^3$

2 is the highest common factor of 6 and 4. x is the highest common factor of x and  $x^3$ . y is the highest common factor of  $y^2$  and y. So  $2xy$  is the highest common factor

16  $f(x) = x^2 - x^3$

Circle the value of  $f(-3)$

[1 mark]

18

-18

36

-36

$(-3)^2 - (-3)^3 = 36$ . Negatives must go into brackets when raised to a power.  $f(-3)$  basically means to substitute -3 for x in  $f(x)$

**Turn over for the next question**



17

At a football game

number of men : number of women : number of children = 13 : 5 : 7

There are 4152 **more** men than women.

Work out the number of children at the game.

**[3 marks]**

$13-5$

Subtracting the number of parts representing the women from the 13 parts representing the men works out that 8 parts of the ratio represent the 4152 more men than women

$4152 \div 8$

Dividing the 4152 by the 8 parts which represent it works out that 1 part of the ratio is worth 519

$519 \times 7$

Multiplying the value of 1 part of the ratio by the 7 parts which represent the children works out that there are 3633 children

Answer 3633

18

Expand and simplify

$(3x^2 + 2)(2x + 5) - 6x(x^2 - 3)$

**[4 marks]**

$6x^3 + 15x^2 + 4x + 10 - 6x^3 + 18x$

Expanding all the brackets.

$3x^2 \times 2x = 6x^3$

$3x^2 \times 5 = 15x^2$

$2 \times 2x = 4x$

$2 \times 5 = 10$

$-6x \times x^2 = -6x^3$

$-6x \times -3 = 18x$

Answer  $15x^2 + 22x + 10$ 

Collecting like terms.

$6x^3 - 6x^3 = 0$

$4x + 18x = 22x$

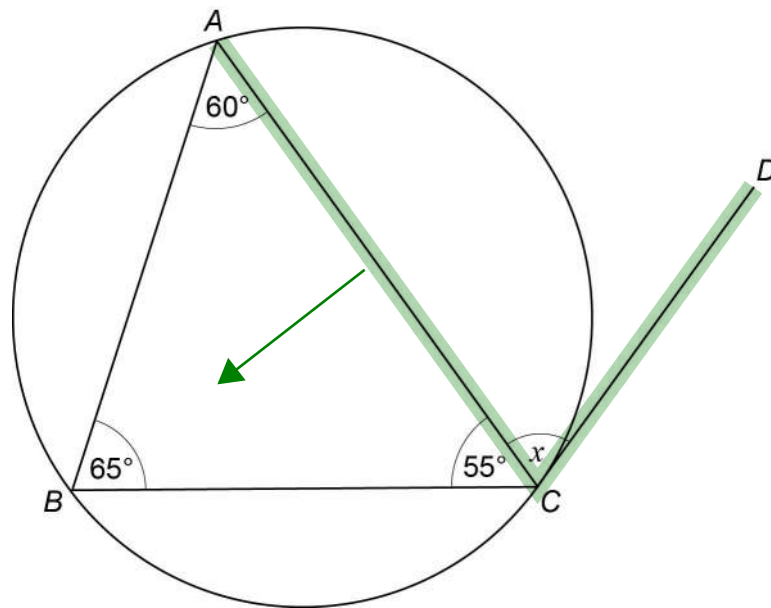


19

$A$ ,  $B$  and  $C$  are points on a circle.

$CD$  is a tangent to the circle.

Not drawn  
accurately



Write down the size of angle  $x$ .

Give a reason for your answer.

[2 marks]

Answer 65 degrees

Reason Alternate segment theorem

**Turn over for the next question**

The angle between a tangent and a chord is equal to the interior opposite angle.  
 $CD$  is the tangent.  $AC$  is the chord. So  $x$  is the angle between the tangent and chord. The 65 degree angle is the interior opposite angle as it is opposite the chord.

Turn over ►



20

 $w$  is a positive number. $x$  is 10% more than  $w$ . $y$  is 10% less than  $x$ .

Which statement is true?

Tick **one** box.

Let  $w = 100$ . 10% of 100 is 10 then  $100 + 10 = 110$ , so  $x = 110$ . 10% of 110 is 11 then  $110 - 11 = 99$ , so  $y = 99$ .  $x$  is greater than  $y$  and  $w$  is greater than  $y$

[1 mark]

 $w < x$  and  $w < y$  $w < x$  and  $w = y$  $x > y$  and  $w > y$  $x > y$  and  $w = y$ 

21

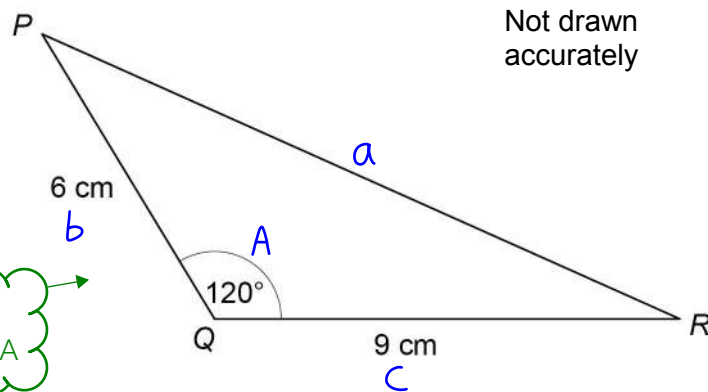
 $N$  is a number.As a product of prime factors in index form  $N = 2 \times 3^4 \times y^3$ Work out  $3N^2$  as a product of prime factors in index form.Give your answer in terms of  $y$ .

[3 marks]

$3N^2 = 3(2 \times 3^4 \times y^3)^2$ . Everything in the bracket is raised to the power of 2 first. This doubles the power of each of them as  $(a^x)^y = a^{xy}$ . Then multiplying by 3 adds 1 to the power of the  $3^8$  as  $a^x \times a^y = a^{x+y}$

Answer            $2^2 \times 3^9 \times y^6$           

22 Here is a triangle.



Labelling the triangle.  
Side a is opposite angle A

Work out the length  $PR$ .

[3 marks]

$$a^2 = b^2 + c^2 - 2bc \cos A$$

The sine rule can't be used as there are not two opposite pairs of sides and angles. So the cosine rule should be used

$$PR = \sqrt{6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 120}$$

Square rooting both sides and substituting in the values from the diagram

Answer                      $3\sqrt{19}$                      cm

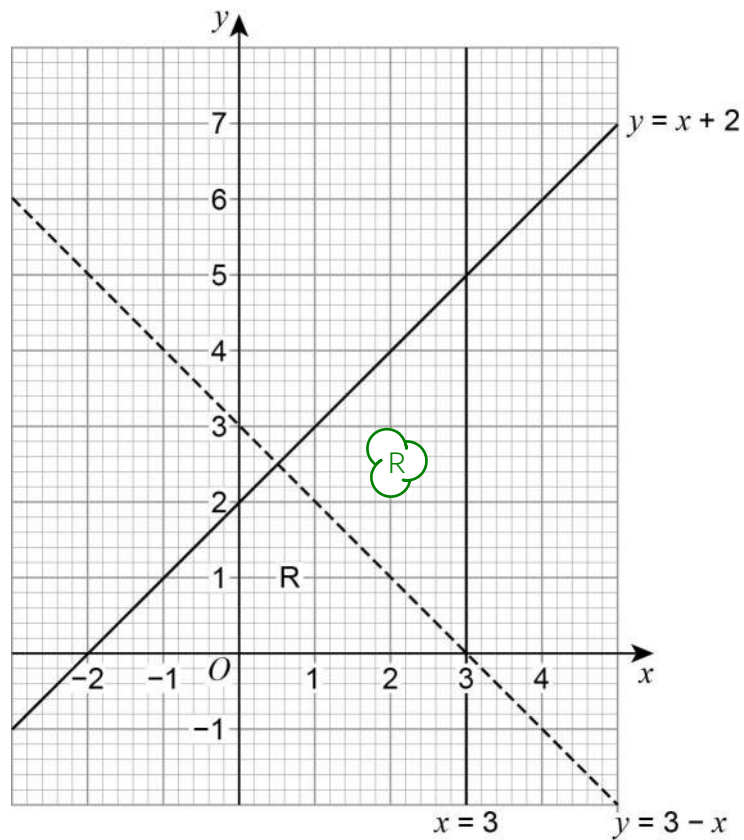
Turn over for the next question



23

Joe draws this graph to identify the region R represented by

$$y \leq x + 2 \quad \text{and} \quad y > 3 - x \quad \text{and} \quad x < 3$$

Make **two** criticisms of his graph.**[2 marks]**

Criticism 1  $x = 3$  should be dashed ← As  $x$  cannot be equal to 3

Criticism 2  $R$  is in the wrong place ← It should go in the place indicated



24  $a : b = 9 : 4$  and  $10b = 7c$

Work out  $a : c$  in its simplest form.

[3 marks]

$a$	$b$	$c$
$9$	$4$	
	$7$	$10$
$63$	$28$	$40$

In the equation  $10b = 7c$ ,  $b$  could be 7 and  $c$  could be 10

28 is a common multiple of 4 and 7. The 4 must be multiplied by 7 to get 28 so the 9 must also be multiplied by 7. The 7 must be multiplied by 4 to get 28 so the 10 must also be multiplied by 4. Now the ratios can be combined as they both have  $b$  in common and they have the same number of parts for it

Answer         63         :         40        

The fraction  $63/40$  does not simplify when put in the calculator so the ratio  $63 : 40$  will not either

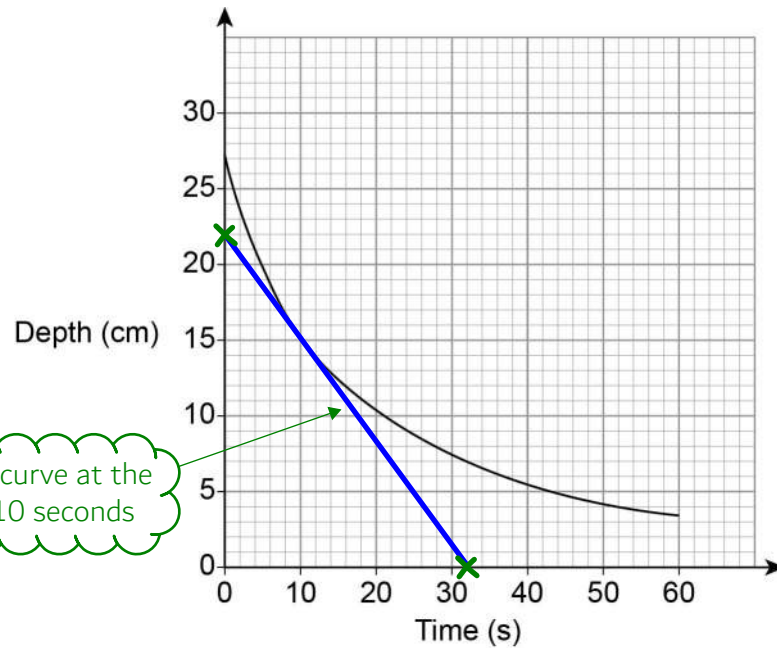
**Turn over for the next question**



25

Liquid is leaking out of a container.

The graph shows the depth of the liquid for 60 seconds.



Drawing a tangent to the curve at the point where the time is 10 seconds

Use the graph to work out an estimate of the rate of decrease of depth at 10 seconds.

You **must** show your working.**[3 marks]**

$$\frac{0-22}{32-0}$$

The rate of change of depth is the gradient, which can be found using (change in y)/(change in x). Picking the two end points of the line as they are on grid lines.  
Change in y = (y-coordinate of the second point) - (y-coordinate of the first point).  
Change in x = (x-coordinate of the second point) - (x-coordinate of the first point)

Answer 0.7 cm/s

The rate of change of depth is negative but as it asks for the rate of decrease the negative is ignored



26

$$a^2 - b^2 \equiv (a + b)(a - b)$$

$a$  and  $b$  are positive whole numbers with  $a > b$

$a^2 - b^2$  is a **prime** number.

Why are  $a$  and  $b$  consecutive numbers?

[2 marks]

$(a + b)$  or  $(a - b)$  must be 1

As  $(a + b)(a - b)$  is the factorised form and prime numbers only have two factors: themselves and 1.  $(a + b)$  and  $(a - b)$  are factors of  $a^2 - b^2$

$(a + b)$  cannot be 1 so  $(a - b)$  must be 1

The smallest  $b$  could be is 1. The smallest  $a$  could be is 2. So the smallest  $(a + b)$  could be is 3. Therefore  $(a - b)$  must be 1. Subtracting  $b$  from  $a$  works out the difference and as this difference is 1, they must be consecutive (next to each other)

Turn over for the next question

5

Turn over ►



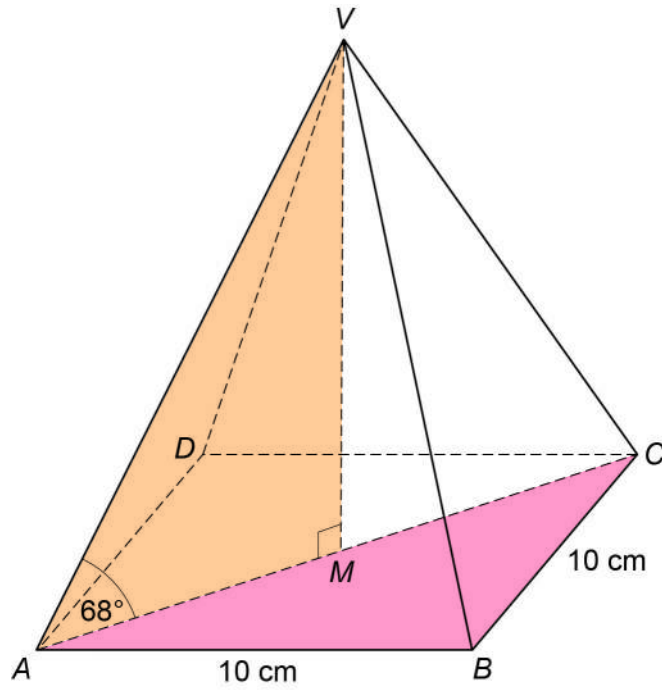
27

$VABCD$  is a square-based pyramid.

The horizontal base  $ABCD$  has side length 10 cm and centre  $M$ .

Angle  $VMA$  is  $90^\circ$

Angle  $VAM$  is  $68^\circ$



$$\text{Volume of pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$$



Work out the volume of the pyramid.

[6 marks]

$$10^2 + 10^2 = AC^2$$

Using Pythagoras' Theorem in the pink right-angled triangle.  $a^2 + b^2 = c^2$ , where a and b are the shorter sides and c is the longest side. Substituting 10 cm for a, 10 cm for b and AC for c

$$AC = \sqrt{200}$$

$10^2 + 10^2 = 200$ . Then square rooting both sides finds that  $AC = 5\sqrt{2}$

$$AM = 10\sqrt{2} \div 2$$

M is the centre of the square so AM is half of AC. So  $AM = 5\sqrt{2}$

$$\begin{array}{c} \text{O} \quad \text{A} \quad \text{O} \\ \text{S} \quad \text{H} \quad \text{C} \quad \text{H} \quad \text{T} \quad \text{A} \end{array}$$

Using right-angled trigonometry in the orange right-angled triangle. Ticking O as we have the adjacent and ticking O as we are looking for the opposite. There are two ticks on the TOA formula triangle so this one can be used

$$\tan 68 \times 5\sqrt{2}$$

Covering O in the formula triangle finds that opposite = tan of the angle x adjacent. AM is the adjacent. This works out that the perpendicular height, VM, is 17.5... cm

$$\frac{1}{3} \times 10^2 \times 17.5...$$

Using the formula for the volume of the pyramid. Area of square = length<sup>2</sup>, so  $10^2$  is the area of the base. Using the exact value of the perpendicular height

Answer 583.4 cm<sup>3</sup>

Turn over for the next question



28  $y = p \times q^{x-1}$  where  $p$  and  $q$  are numbers.

$y = 10$  when  $x = 1$

$y = 0.3125$  when  $x = 6$

Work out the value of  $y$  when  $x = 3$

[5 marks]

$$10 = p \times q^{1-1}$$

Substituting 10 for  $y$  and 1 for  $x$  in the equation.  $1 - 1 = 0$  and anything to the power of 0 is 1. So  $10 = p \times 1$  meaning  $p = 10$

$$0.3125 = 10 \times q^{6-1}$$

Substituting 0.3125 for  $y$ , 6 for  $x$  and 10 for  $p$  in the equation

$$\sqrt[5]{\frac{0.3125}{10}} = q$$

$6 - 1 = 5$ . Rearranged to make  $q$  the subject by dividing both sides by 10 then doing the 5th root of both sides. So  $q = 0.5$

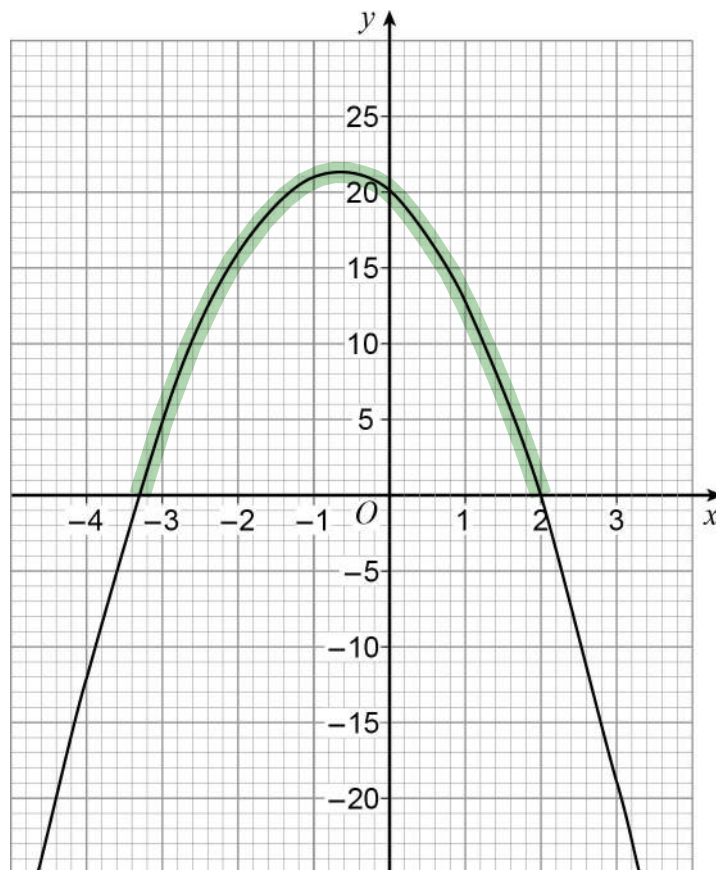
$$y = 10 \times 0.5^{3-1}$$

Substituted 10 for  $p$ , 0.5 for  $q$  and 3 for  $x$  to find the value of  $y$

Answer 2.5



29 Here is the graph of  $y = f(x)$  where  $f(x)$  is a quadratic function.



Write down all the **integer** solutions of  $f(x) \geq 0$

[2 marks]

The highlighted region of the line is where  $f(x) \geq 0$ . It is basically asking what the integer values of  $x$  are when the line is above or on the  $x$  axis

Answer -3, -2, -1, 0, 1, 2

Turn over for the next question

Turn over ►



30  $f(x) = \frac{x}{3} + 4$  for all values of  $x$ .

$g(x) = 6x^2 + 3$  for all values of  $x$ .

Work out  $fg(x)$ .

Give your answer in the form  $ax^2 + b$  where  $a$  and  $b$  are integers.

[2 marks]

$$\frac{6x^2+3}{3} + 4$$

Substituting  $g(x)$  for  $x$  in  $f(x)$

$$6x^2/3 = 2x^2. \quad 3/3 = 1. \quad 1 + 4 = 5$$

Answer \_\_\_\_\_

$$2x^2 + 5$$

END OF QUESTIONS

