Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

GCSE
MATHEMATICS

H

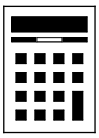
Higher Tier Paper 3 Calculator

Monday 10 June 2024 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

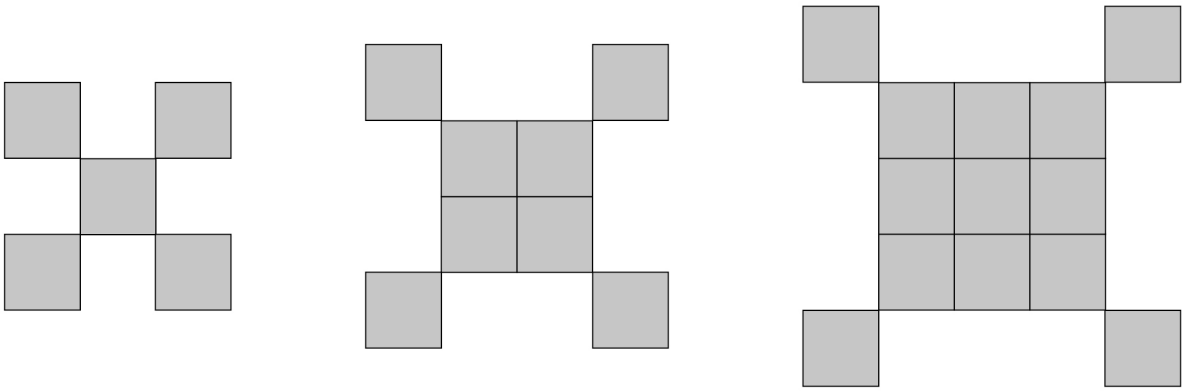
Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Do not write outside the box

Answer **all** questions in the spaces provided.

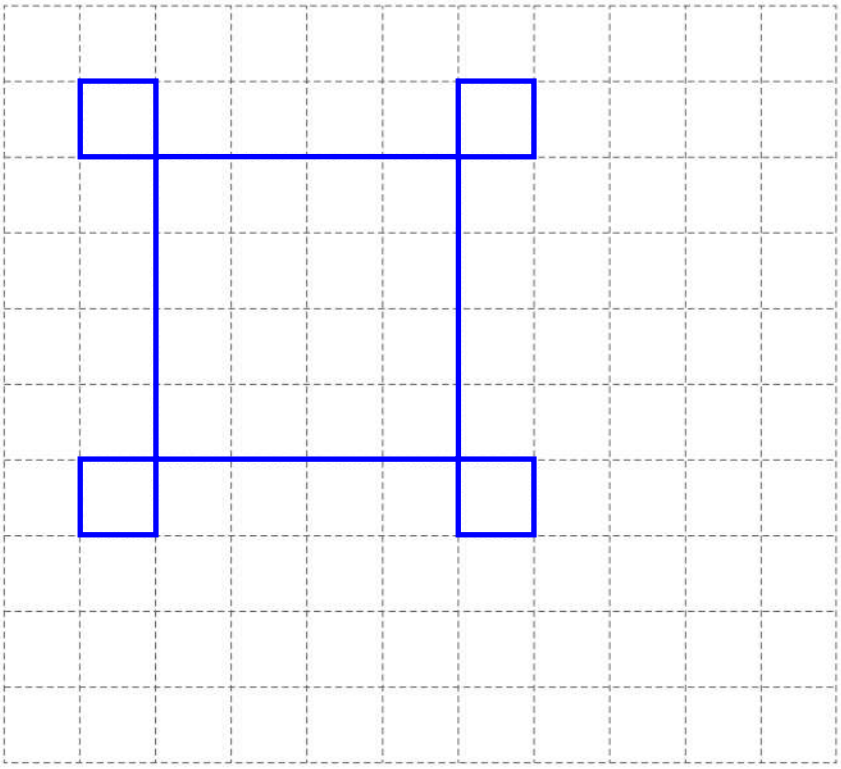
1 Here are the first three Patterns in a sequence made up of small squares.



Pattern 1 Pattern 2 Pattern 3

1 (a) On the grid, draw Pattern 4

[1 mark]



The length of the square in the middle increases by 1 between each pattern. The previous one had a length of 3 so this one needs a length of 4. There are always four small squares on the corners of the middle square

Do not write outside the box

1 (b) The expression for the number of small squares in Pattern n is $n^2 + 4$

Work out the least value of n for which the number of small squares is greater than 500

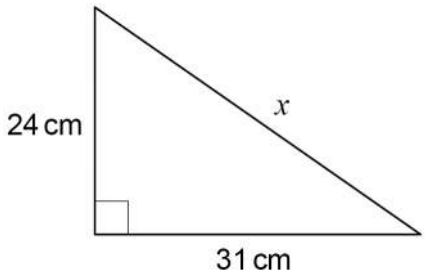
[1 mark]

Using table mode on the calculator, set f(x) = x^2 + 4. Start: 1. End: 30. Step: 1. This lists out the number of small squares in the first 30 patterns where x is n (the pattern number) and f(x) is the number of small squares

$n =$ 23

Pattern 22 has 488 small squares then pattern 23 has 533 squares. 23 is the first value of n which has more than 500 squares in its pattern

2



Not drawn accurately

Use Pythagoras' theorem to work out the value of x.

Give your answer as a decimal.

[3 marks]

$24^2 + 31^2 = x^2$ ← Pythagoras' Theorem is $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting 24 cm for a, 31 cm for b and x for c

$x = \sqrt{1537}$ ← Square rooting both sides gets rid of the power of 2 and finds x

Answer 39.2 cm

5

Turn over ►

Do not write outside the box

3 Rick claims most of the flats in his 8-floor building are energy efficient.

He samples 45 flats from floors 1 to 5

Give a reason why this sample may **not** be useful in testing Rick's claim.

[1 mark]

Not all floors were sampled

The sample is not representative of all of the flats as not all of the floors were sampled

4 $3(x-1) \equiv 3x-3$ is an identity.

Tick **one** box.

[1 mark]

☑ It is true for **all** values of x

☐ It is true for **some** values of x

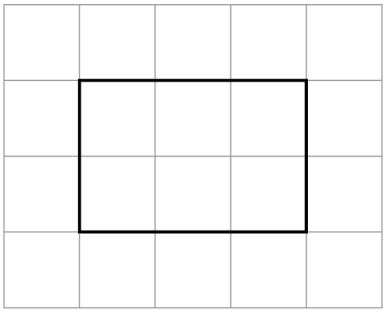
☐ It is true for **no** values of x

Both sides of an identity are equivalent, so it does not matter what value of x is put in. The right side is just a different way of writing the left side

Do not write outside the box

5 The front elevation of a cuboid is shown on this centimetre grid.

Front elevation



The volume of the cuboid is 42 cm^3

Draw the **side elevation** on this centimetre grid.

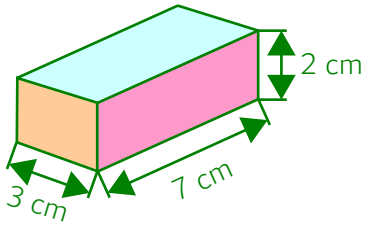
[2 marks]

Side elevation

L × 3 × 2 ← Volume of cuboid = length × width × height. Let L be the length. From the front elevation, the width is 3 cm and the height is 2 cm

6L = 42 ← Simplifying the expression of the volume in terms of L and setting it equal the actual volume of 42 cm^3

L = 7 ← Dividing both sides by 6 finds that the length of the cuboid must be 7 cm



The front elevation is highlighted in orange.
The side elevation is highlighted in pink.
The plan is highlighted in blue

4

Do not write outside the box

6 (a) On Monday, Larrs swims 50 metres in 40 **seconds** at a constant speed.

On Tuesday, Larrs swims 1.5 kilometres.

Assume he swims at the same constant speed as on Monday.

How many **minutes** does he swim for on Tuesday?

[5 marks]

s d t ← Writing the speed, distance, time formula triangle

50 ÷ 40 = 1.25 ← Covering s in the formula triangle finds that speed = distance ÷ time. So dividing the distance of 50 metres by the time of 40 seconds works out that his speed is 1.25 m/s

1.5 × 1000 ← There are 1000 m in 1 km so multiplying the 1.5 km by 1000 converts it to 1500 m. The distance needs to be in metres as the unit of speed involves metres

1500 ÷ 1.25 ← Covering t in the formula triangle finds that time = distance ÷ speed. So dividing the distance of 1500 metres by the speed of 1.25 m/s works out that his time on Tuesday is 1200 seconds. The unit of speed involved seconds so the unit of time will be seconds

1200 ÷ 60 ← There are 60 seconds in 1 minutes so dividing the 1200 seconds by 60 converts it to 20 minutes

Answer 20 minutes

6 (b) In fact, on Tuesday Larrs swims at a slower constant speed than on Monday.

What does this mean about the number of minutes he swims for on Tuesday?

Tick the correct box.

[1 mark]

☐ It is less than the answer to part (a)

☐ It is the same as the answer to part (a)

☑ It is greater than the answer to part (a)

☐ It is not possible to say

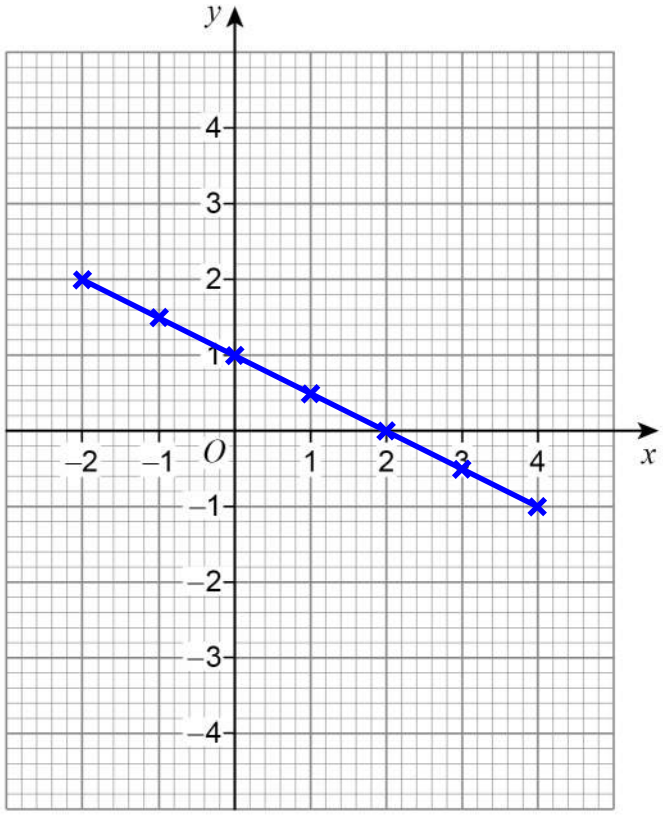
Time = distance ÷ speed. Dividing by less speed will result in a greater time

Do not write outside the box

7 Draw the graph of $y = 1 - \frac{1}{2}x$ for values of x from –2 to 4

[3 marks]

Using table mode on the calculator, set f(x) = 1 - 1/2 x. Start: -2. End: 4. Step: 1.
This creates a table of values for the values of x from -2 to 4. y is the f(x) values.
Then plotting the points and connecting them with a straight line



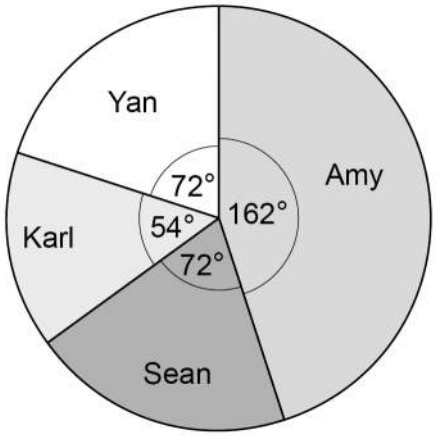
9

8 Four people are taking part in a television talent show.

Here are Amy's marks from the 6 judges.

8	9	9	6	9	10

The pie chart represents the phone vote.



Amy's total score is found by

4 × the **mean** of her marks

+

her **percentage** of the phone vote

Do not write outside the box

Work out Amy's total score.

[4 marks]

$8 + 9 + 9 + 6 + 9 + 10$ ← Adding Amy's marks from the judges works out that the total of the marks from the judges is 51

$51 \div 6$ ← Mean = total ÷ number so dividing the total 51 by the 6 judges works out that the mean of her marks is 8.5

$4 \times 8.5 = 34$ ← This works out that 4 × the mean of her marks is 34

$\frac{162}{360} \times 100$ ← There are 360° in total in a pie chart. So putting the 162° Amy got over the 360° expresses her fraction of the phone vote. Multiplying this by 100 converts it into 45%

$34 + 45$ ← Adding her percentage of the phone vote to 4 × the mean of her marks works out that her total score is 79

Answer 79

Turn over for the next question

4

Do not write outside the box

9 Town A has

a population of 84 000

an area of 7 **square miles**.

Town B has a population density of 4695 people per **square kilometre**.

$$\text{Population density} = \frac{\text{population}}{\text{area}}$$

Which town has the greater population density?

Use 1 square mile = 2.6 square kilometres

Tick a box.

Town A ☐ Town B ✓

Show working to support your answer.

[3 marks]

7×2.6 ← This converts the 7 square miles to 18.2 square kilometres

$\frac{84000}{18.2} = 4615.4$ ← This works out that the population density of town A is 4615.4 people per square kilometre

4695 is greater than 4615.4

Do not write outside the box

10 On a biased dice,

P(lands on 6) = 0.38

This dice is rolled 150 times.

How many times would you expect the dice **not** to land on 6 ?

[3 marks]

1 - 0.38 ← It is certain to land on either 6 or not 6. The probability of certain is 1. So subtracting the probability of landing on 6 from 1 works out that the probability of not landing on 6 is 0.62

150 × 0.62 ← Multiplying the 150 rolls by the probability of not landing on 6 works out an estimated number of times it will land on 6

Answer 93

Turn over for the next question

6

Do not write outside the box

11 Write a number in each box to make the calculations correct.

[2 marks]

10	÷	–2	×	-1	=	5	

On the left, the division must be done first. 10 ÷ -2 = -5. Then -5 must be multiplied by -1 to get 5

$\frac{1}{3}$	×	4π	×	6	=	8π	

The multiplication on the left can be done in any order.
So 1/3 × 6 = 2 then 2 must be multiplied by 4π to get 8π

Do not write outside the box

12 Cards are either gold or not gold.

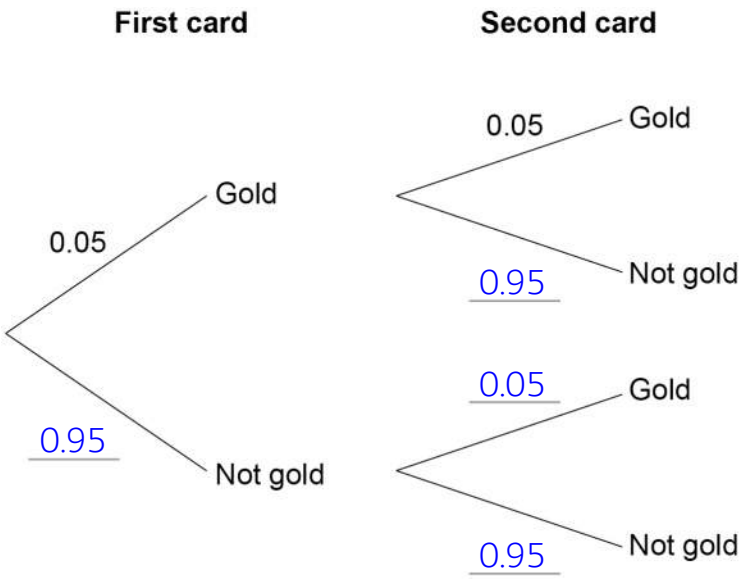
P(gold) = 0.05

Harim chooses a card at random and replaces it.

He then chooses a second card.

12 (a) Complete the tree diagram.

[2 marks]



1 - 0.05 ← It is certain to be either gold or not gold. The probability of certain is 1. So subtracting the probability of gold from 1 works out that the probability of not gold is 0.95

12 (b) What is the probability that **at least one** of Harim's cards is gold?

[3 marks]

0.95 × 0.95 ← Not gold AND not gold. AND means to multiply the probabilities. The probability of both cards not being gold is 0.9025

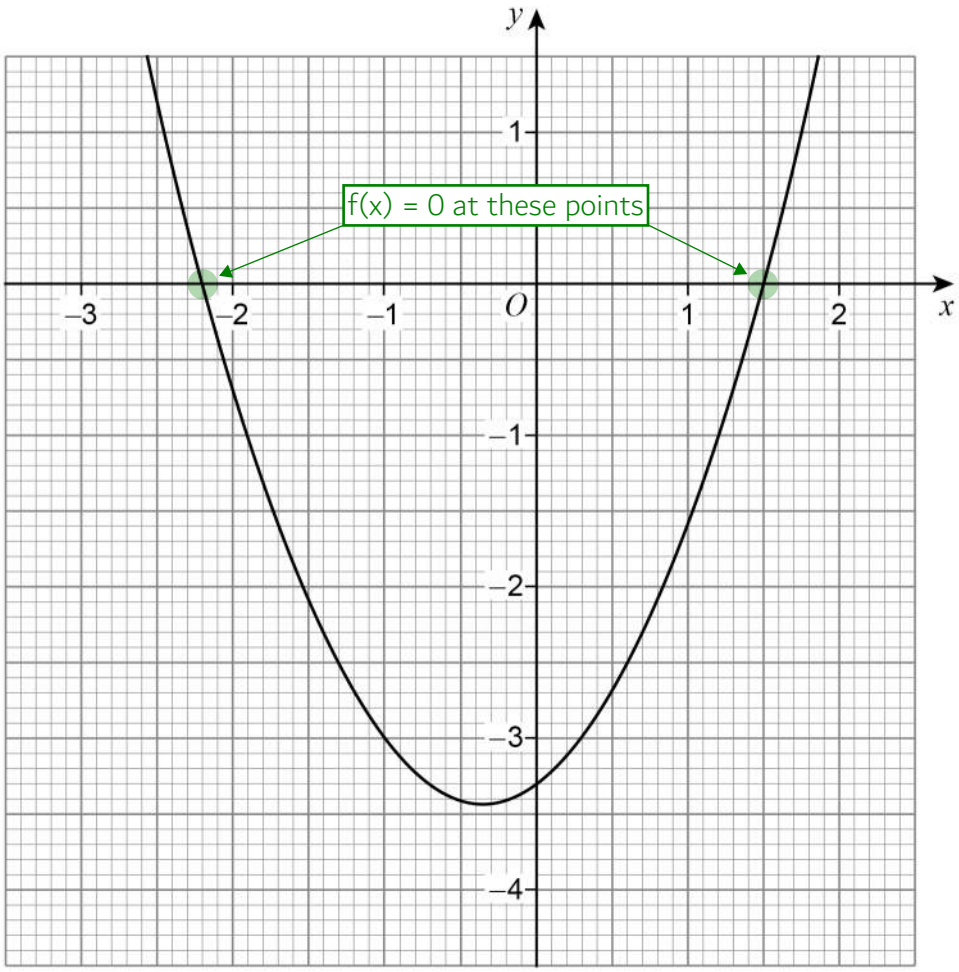
1 - 0.9025 ← It is certain that both will be not gold or that at least one will be gold. The probability of certain is 1. So subtracting the probability of both not being gold from 1 leaves the probability of at least one being gold

Answer 0.0975

7

Do not write outside the box

13 Here is a quadratic graph with equation $y = f(x)$



Write down the roots of the equation $f(x) = 0$

[2 marks]

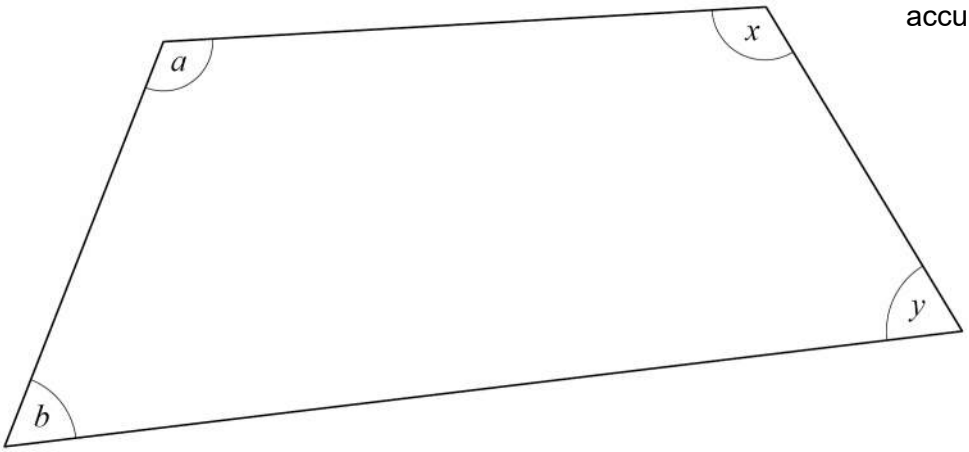
Answer -2.2 and 1.5

It is basically asking for the values of x when y is 0, so where it crosses the x-axis

Do not write outside the box

14

Not drawn accurately



$b = 45°$ and $a : b = 7 : 3$ and $x : y = 4 : 1$

Show that $a : y = 5 : 2$

[3 marks]

45 ÷ 3 ← b is represented by 3 parts of the ratio $a : b$. So dividing 45° by 3 works out that 1 part of this ratio is worth 15°

15 × 7 ← Multiplying the value of 1 part of the $a : b$ ratio by 7 works out that the 7 parts which represent a is 105°. So angle a is 105°

105 + 45 ← Adding angles a and b gives a total of 150°

360 - 150 ← There are 360° in total in a four-sided shape. Subtracting the total of angles a and b from 360° works out that the total of angles x and y is 210°

210 ÷ 5 ← 4 + 1 = 5 parts in total in the $x : y$ ratio. These 5 parts represent the 210° so dividing 210° by 5 works out that 1 part of the $x : y$ ratio is worth 42°. Angle y must be 42° as it is 1 part of this ratio

105 : 42 ← Angle a is 105° and angle y is 42°. Writing this as a ratio

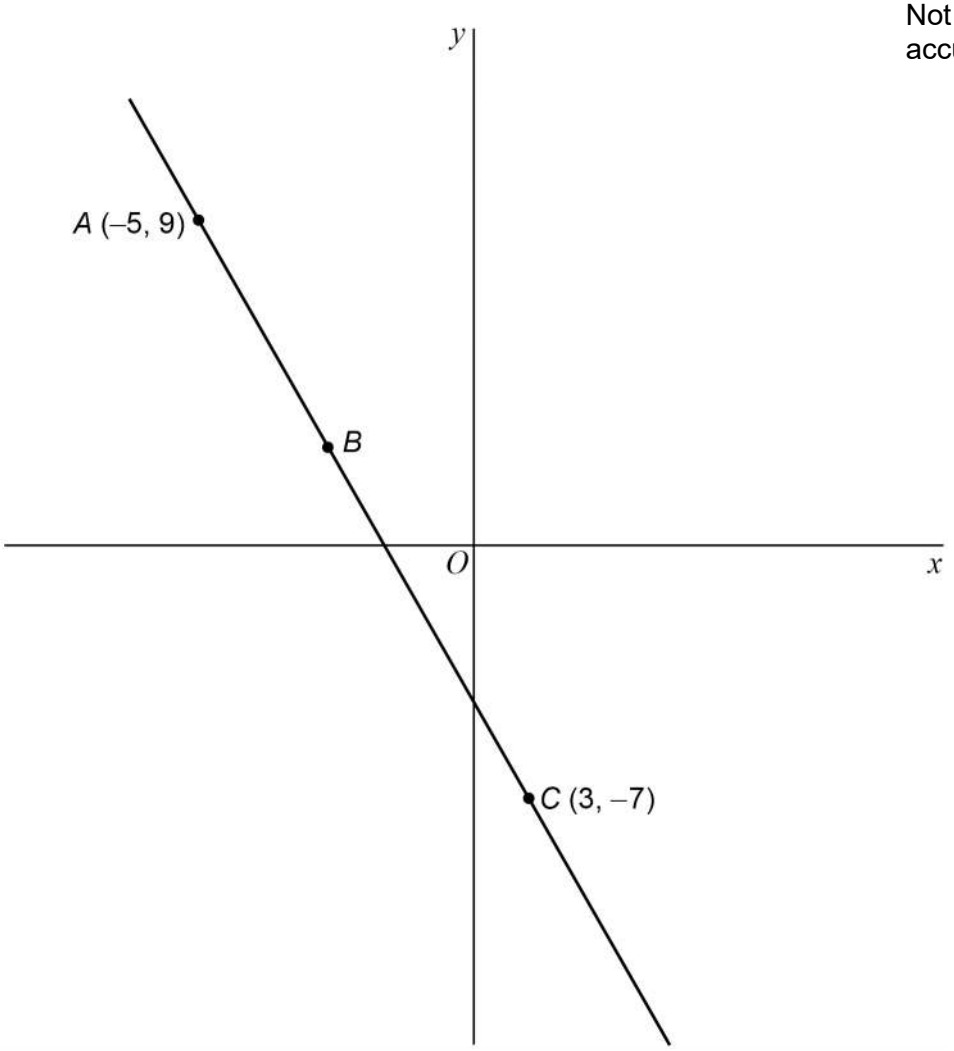
5 : 2 ← The ratio of $a : y$ simplifies to 5 : 2. Ratios simplify in a similar way to fractions and the calculator simplifies 105/42 to 5/2

5

Do not write outside the box

15 A straight line passes through points A (–5, 9), B and C (3, –7).

Not drawn accurately



15 (a) $AB : BC = 1 : 3$

Work out the coordinates of point B.

[3 marks]

See next page for method

Answer (-3 , 5)

3 - -5 ← Subtracting the x-coordinate of A from the x-coordinate of C works out that the distance between A and C in the x-direction is 8

8 ÷ 4 ← 1 + 3 = 4 parts which represent the total distance from A to C. 4 parts of the ratio represent the distance of 8 in the x-direction from A to C so dividing 8 by 4 works out that 1 part of the ratio is worth 2 in the x-direction

-5 + 2 ← The distance from A to B is represented by 1 part of the ratio. So adding the value of 1 part of the ratio in the x-direction to the x-coordinate of A works out that the x-coordinate of B is -3

-7 - 9 ← Subtracting the y-coordinate of A from the y-coordinate of C works out that the distance between A and C in the y-direction is -16

-16 ÷ 4 ← 1 + 3 = 4 parts which represent the total distance from A to C. 4 parts of the ratio represent the distance of -16 in the y-direction from A to C so dividing -16 by 4 works out that 1 part of the ratio is worth -4 in the y-direction

9 - 4 ← The distance from A to B is represented by 1 part of the ratio. So adding the value of 1 part of the ratio in the y-direction to the y-coordinate of A works out that the y-coordinate of B is 5

Do not write outside the box

15 (b) Work out the equation of the line perpendicular to AC that passes through C.

[4 marks]

$\frac{-16}{8}$ ← Gradient = (change in y)/(change in x). The change in y from A to C is -16 and the change in x from A to C is 8. So the gradient of AC is -2

$-1 \div -2$ ← Doing the negative reciprocal (-1 divided by) the gradient of AC works out that the gradient of the line perpendicular to AC is 1/2

$-7 = \frac{1}{2} \times 3 + c$ ← The general equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept. Substituting in the y-coordinate of C for y, 1/2 for m and the x-coordinate of C for x

$c = -8.5$ ← Subtracting 1/2 × 3 from both sides gets c on its own

Answer $y = \frac{1}{2}x - 8.5$

↑ The general equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept. Substituting 1/2 for m -8.5 for c

Turn over for the next question

7

Do not write outside the box

16 Jing rolls a fair six-sided dice 72 times.

	1	2	3	4	5	6
Frequency	16	11	10	8	14	13

Is the relative frequency of rolling a 5 greater than the theoretical probability?

Tick a box.

Yes [✓] No [ ]

Give a reason for your answer.

[3 marks]

$\frac{14}{72} \times 100 = 19.\dot{4}\%$ ← The relative frequency is 14/72 as 14 out of the 72 rolls were 5. Multiplying this by 100 converts it to a percentage

$\frac{1}{6} \times 100 = 16.\dot{6}\%$ ← The theoretical probability is 1/6 as 1 out of the 6 faces of the dice are a 5 and it is a fair dice (meaning that all of the outcomes are equally as likely). Multiplying this by 100 converts it to a percentage

The percentage is greater for the relative frequency

Do not write outside the box

17 (a) a and b are different prime numbers.

$a^3 \times b^2 = 200$

Work out the value of $a^4 \times b$

[3 marks]

$200 = 2^3 \times 5^2$ ← Expressing 200 as a product of its prime factors using the calculator works out that a must be 2 and b must be 5

$2^4 \times 5$ ← Substituting 2 for a and 5 for b in $a^4 \times b$

Answer 80

17 (b) c and d are different prime numbers.

Circle the equation for which $c^4 \times d^2 \times e$ is a cube number.

[1 mark]

$e = cd$ $\quad$ (**$e = c^2d$**) $\quad$ $e = c^2d^2$ $\quad$ $e = c^3d^3$

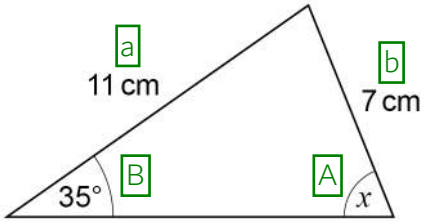
Substituting e for c^2d gives $c^4 \times d^2 \times c^2d$, which simplifies to c^6d^3. This is a cube number as cube rooting it (by dividing the powers by 3) gives c^2d, which must be a whole number. Substituting in the other options would give fractional powers of c and d when they are divided by 3, which may not give a whole number

Turn over for the next question

7

Do not write outside the box

18 Here is triangle A.



Not drawn accurately

Side a is opposite angle A and side b is opposite angle B

18 (a) Use the sine rule to show that $x = 64°$ to the nearest degree.

[3 marks]

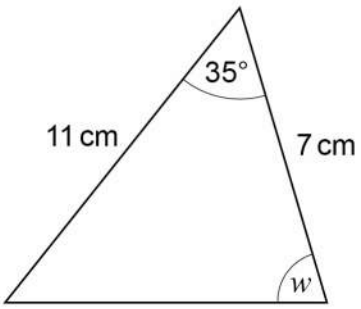
$\frac{\sin x}{11} = \frac{\sin 35}{7}$ ← $\frac{\sin A}{a} = \frac{\sin B}{b}$ ← Using the sine rule with the angles as numerators and substituting in the values as labelled on the diagram

$\sin x = 0.9...$ ← Multiplying both sides by 11 to eliminate it as the denominator on the left

$x = 64.3...$ ← Doing the inverse sin of both sides to eliminate the sin on the left

Do not write outside the box

18 (b) Here is triangle B.



Not drawn accurately

Anna thinks that w must be 64° to the nearest degree.

She says,

"This is because triangle B has two sides and one angle the same as triangle A."

Without further calculation, is she correct?

Tick a box.

Yes ☐ No ☑

Give a reason for your answer.

[1 mark]

7 cm is not opposite 35° ← Meaning that the calculation done before no longer works. Side b is opposite angle B

Turn over for the next question

4

Do not write outside the box

19 $f(x) = x - 3$ $g(x) = 4x - 7$

19 (a) Work out the value of fg(6)

[2 marks]

$4 \times 6 - 7$ ← Substituting 6 for x in g(x) works out that g(6) = 17

$17 - 3$ ← Substituting the value of g(6) for x in f(x) works out that fg(6) = 14

Answer 14

19 (b) Solve $(f(x))^2 = g(x)$

[4 marks]

$(x - 3)^2$ ← Expressing $(f(x))^2$

$x^2 - 6x + 9 = 4x - 7$ ← Expanding the square bracket by squaring the first term, doubling the product of the two terms, squaring the last term. Setting this equal to g(x)

$x^2 - 10x + 16 = 0$ ← Putting into the quadratic form $ax^2 + bx + c = 0$ by subtracting $4x$ from both sides and adding 7 to both sides

$$\frac{--10 \pm \sqrt{(-10)^2 - 4 \times 1 \times 16}}{2 \times 1}$$ ← $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← Solving using the quadratic formula. a = 1, b = -10, c = 16

Answer $x = 8$ and $x = 2$

Do not write outside the box

20 P, Q, and R have positive values.

P is directly proportional to Q

When $P = 8$, $Q = 2$

R is inversely proportional to Q^2

When $R = 10$, $Q = 3$

Work out the value of R when $P = 0.5$

[5 marks]

$P = kQ$ ← Writing P is directly proportional to Q as an equation where k is a constant which needs to be found

$k = 8 \div 2$ ← Dividing both sides by Q to make k the subject and substituting 8 for P and 2 for Q. So k = 4

$P = 4Q$ ← Substituting 4 for k in the equation

$R = \frac{c}{Q^2}$ ← Writing R is inversely proportional to Q^2 as an equation where c is a constant which needs to be found

$c = 10 \times 3^2$ ← Multiplying both sides by Q^2 to make c the subject and substituting 10 for R and 3 for Q. So c = 90

$R = \frac{90}{Q^2}$ ← Substituting 90 for c in the equation

$Q = 0.5 \div 4$ ← Dividing both sides of the P = 4Q equation by 4 and substituting 0.5 for P finds that Q = 0.125 when P = 0.5

$R = \frac{90}{0.125^2}$ ← Substituting 0.125 for Q in the $R = 90/Q^2$ equation to find R when P = 0.5

$R =$ 5760

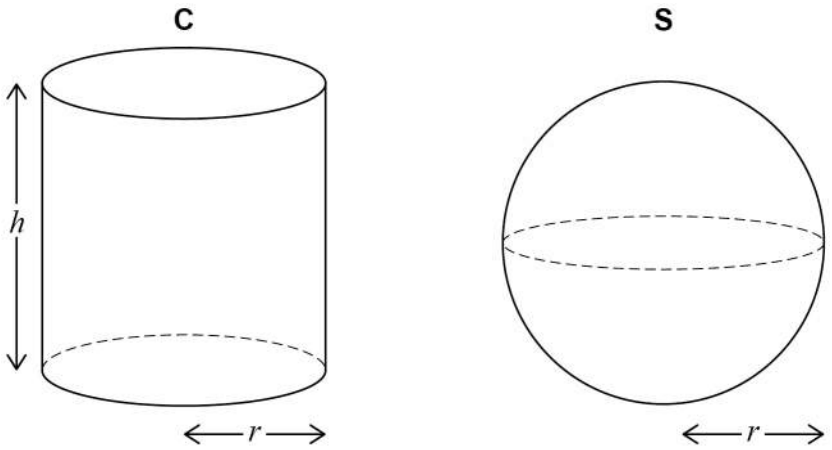
Turn over for the next question

11

Do not write outside the box

21 A cylinder, C, and a sphere, S, each have radius r

C has height h



Volume of a sphere $= \frac{4}{3}\pi r^3$

where r is the radius

21 (a) volume of C = volume of S

Work out the ratio $r : h$

You **must** show your working.

[3 marks]

$\pi r^2 h = \frac{4}{3}\pi r^3$ ← The cylinder can be treated like a prism so volume = area of cross section × length. The cross section is the circle and area of circle = πr^2. The length is h. So $\pi r^2 h$ is the volume of C. Setting this equal to the volume of S, which is expressed using the formula given

$h = \frac{4}{3}r$ ← Dividing both sides by πr^2

Answer 1 : $\frac{4}{3}$

↑ r would be 1 if h is 4/3

Do not write outside the box

21 (b) A **different cylinder** has radius $3r$ and height $2h$.

How many times bigger is the volume of this cylinder than the volume of C?

[2 marks]

$\pi(3r)^2(2h)$ ← The cylinder can be treated like a prism so volume = area of cross section × length. The cross section is the circle and area of circle = π × radius^2. The length is the height, 2h

$\pi \times 9r^2 \times 2h$ ← Simplifying

$18\pi r^2 h$ ← Simplifying

Answer 18

↑ The volume of C was $\pi r^2 h$ and the volume of the different is $18\pi r^2 h$

22 Fatima is choosing a 4-digit code.

Each digit is a whole number from 0 to 9

She decides

all her digits will be odd numbers

no digits will be repeated.

How many different codes can she make?

[2 marks]

1, 3, 5, 7, 9 ← Listing the possible digits

$5 \times 4 \times 3 \times 2$ ← Using the product rule for counting. There are 5 possibilities for the first digit. Given that 1 is already chosen, there are 4 possibilities for the second digit. Given that 2 are already chosen, there are 3 possibilities for the third digit. Given that 3 are already chosen, there are 2 possibilities for the fourth digit

Answer 120

7

Do not write outside the box

23 Quadrilateral *ABCD* is reflected in edge *BC*.

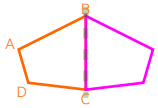
How many of the vertices are invariant?

Circle your answer.

[1 mark]

1 (2) 0 4

Both vertices B and C will be invariant as they will not change



24 Write $2x^2 - 12x + 7$ in the form $d(x + e)^2 + f$

where d, e and f are integers.

[3 marks]

$2(x^2 - 6x) + 7$ ← Bringing 2 out as a factor on the first two terms

$2(x - 3)^2 + 7 - 2(-3)^2$ ← Completing the square by halving the coefficient of x, putting this in a bracket with x and squaring the bracket. Subtracting $2(-3)^2$ from the end

Answer $2(x - 3)^2 - 11$

END OF QUESTIONS

4