

Please note that these worked solutions have neither been provided nor approved by Pearson Education and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

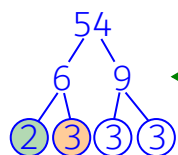
If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer ALL questions.

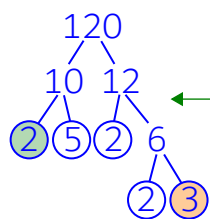
Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 Find the highest common factor (HCF) of 54 and 120



Doing a factor tree to find 54 as a product of prime factors. So $54 = 2 \times 3 \times 3 \times 3$



Doing a factor tree to find 120 as a product of prime factors. So $120 = 2 \times 2 \times 2 \times 3 \times 5$

$$2 \times 3$$

The highest common factor is what is shared by both of the product of prime factors. One 2 and one 3 are shared. Multiplying these finds the highest common factor

6

(Total for Question 1 is 2 marks)

- 2 There are only red counters, white counters, blue counters and green counters in a bag.

Chris is going to take at random a counter from the bag.

The table shows the probability that he will take a red counter and the probability that he will take a white counter.

Colour	red	white	blue	green
Probability	0.3	0.1	$2g$	g

There are twice as many blue counters as there are green counters in the bag.

- (a) Work out the probability that Chris will take a blue counter.

Let g be the probability of green.
 $2g$ must be the probability of blue

$$0.3 + 0.1$$

Adding the probability of red and the probability of white finds that there is 0.4 total probability in the table so far

$$\begin{array}{r} 1.0 \\ -0.4 \\ \hline 0.6 \end{array}$$

It is certain to get one of the colours so the probabilities must all add up to 1. So subtracting the 0.4 from 1 finds that the probabilities for blue and green must add up to 0.6

$$3 \overline{) 0.6}$$

$2g + g = 3g = 0.6$. Dividing both sides by 3 finds that $g = 0.2$

$$0.2 \times 2$$

Multiplying the value of g by 2 finds that $2g$ is 0.4, which is the probability of blue

$$0.4$$

(3)

There are 45 red counters in the bag.

- (b) Work out the total number of counters in the bag.

$$0.3T = 45$$

Let T be the total number of counters. Multiplying T by the probability of red expresses the number of red counters, which is 45

$$\frac{45}{0.3}$$

Dividing both sides by 0.3 eliminates the 0.3 on the left and gets T on its own

$$3 \overline{) 150}$$

Multiplying both the numerator and denominator by ten gives $450/3$ and eliminates the decimal on the denominator

$$150$$

(2)

(Total for Question 2 is 5 marks)

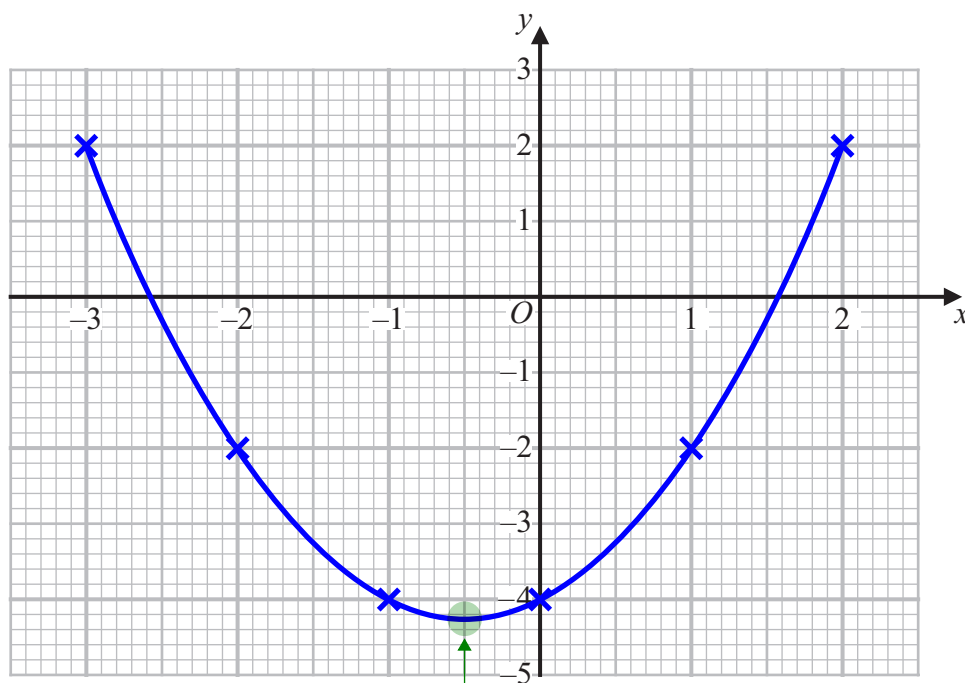
- 3 (a) Complete the table of values for $y = x^2 + x - 4$

x	-3	-2	-1	0	1	2
y	2	-2	-4	-4	-2	2

$(-2)^2 = 4$, then $4 + (-2) = 2$, then $2 - 4 = -2$.
 $0^2 = 0$, then $0 + 0 = 0$, then $0 - 4 = -4$.
 $1^2 = 1$, then $1 + 1 = 2$, then $2 - 4 = -2$.
 $2^2 = 4$, then $4 + 2 = 6$, then $6 - 4 = 2$

(2)

- (b) On the grid, draw the graph of $y = x^2 + x - 4$ for values of x from -3 to 2



This is the turning point

(2)

- (c) Write down the coordinates of the turning point of the graph of $y = x^2 + x - 4$

(.....-0.5.....,-4.25.....)

(1)

(Total for Question 3 is 5 marks)

- 4 There are 280 chocolates in a box.
There are only dark chocolates, milk chocolates and white chocolates.

$\frac{1}{7}$ of the 280 chocolates are dark chocolates.

The number of milk chocolates : the number of white chocolates = 1 : 3

The number of white chocolates : the number of dark chocolates = n : 1

- (a) Work out the value of n .

You must show all your working.

$$\begin{array}{r} 0\ 4\ 0 \\ 7 \overline{) 280} \end{array}$$

Dividing the 280 chocolates by 7 finds that $\frac{1}{7}$ of the chocolates is 40, so there are 40 dark chocolates

$$\begin{array}{r} 280 \\ - 40 \\ \hline 240 \end{array}$$

Subtracting the 40 dark chocolates from the 280 chocolates finds that the total of the milk and white chocolates is 240

$$\begin{array}{r} 0\ 6\ 0 \\ 4 \overline{) 240} \end{array}$$

$1 + 3 = 4$ parts in total in the 1 : 3 ratio. These 4 parts represent 240 chocolates. So dividing the 240 by 4 finds that 1 part of the ratio is worth 60 chocolates

$$\begin{array}{r} 60 \\ \times 3 \\ \hline 180 \end{array}$$

Multiplying the value of 1 part of the ratio by 3 finds that the 3 parts represent 180 chocolates, so there are 180 white chocolates

$$180 : 40$$

Writing the ratio of the number of white chocolates : the number of dark chocolates

$$\begin{array}{r} 0\ 4.\ 5 \\ 4 \overline{) 180} \end{array}$$

Dividing both sides of this ratio by 40 gets 1 on the right. $180 \div 40 = 18 \div 4 = 4.5$

$$n = \frac{4.5}{(5)}$$

10 milk chocolates from the box are eaten.

- (b) Does this affect your answer to part (a)?
Give a reason for your answer.

No, there are the same numbers of white and dark chocolates

So the ratio of the number of white chocolates : the number of dark chocolates does not change and n will be the same

(1)

(Total for Question 4 is 6 marks)

- 5 Work out $5.7 \times 10^2 + 9.8 \times 10^3$
Give your answer in standard form.

$$0.57 \times 10^3$$

Dividing the 5.7 by 10 and adding 1 to the index of the 10^2 changes the 5.7×10^2 to 0.57×10^3 so that it has the same power of ten as 9.8×10^3

$$9.80$$

$$+0.57$$

$$10.37 \times 10^3$$

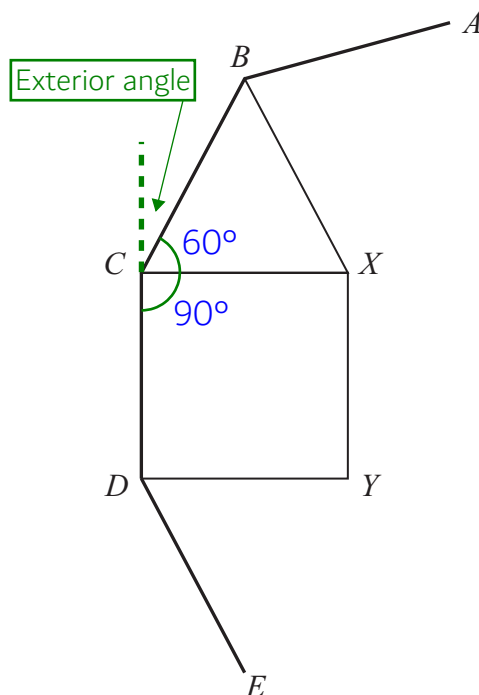
$0.57x + 9.8x = (0.57 + 9.8)x$. The numbers before the powers of ten can now be added and the power of ten stays the same

Dividing the 10.37 by 10 and adding 1 to the index of 10^3 gets the 1st number at least 1 and less than 10, so that it is now in standard form

$$1.037 \times 10^4$$

(Total for Question 5 is 3 marks)

- 6 AB , BC , CD and DE are four sides of a regular polygon with n sides.



BCX is an equilateral triangle.

$CDYX$ is a square.

Work out the value of n .

You must show all your working.

$$\begin{array}{r} 0\ 6\ 0 \\ 3 \overline{) 1\ 8\ 0} \end{array}$$

There are 180° in total in a triangle. Dividing this 180° finds that each angle in the equilateral triangle is 60° , as all three angles are equal

$$\begin{array}{r} 90 \\ +60 \\ \hline 150 \end{array}$$

The angle in the square is 90° . Adding the angle in the equilateral triangle finds that the interior angle of the polygon is 150°

$$\begin{array}{r} 1\ 8\ 0 \\ -1\ 5\ 0 \\ \hline 0\ 3\ 0 \end{array}$$

There are 180° around a point on a straight line. The interior angle and exterior angle lie around a point on a straight line. So subtracting the 150° interior angle from 180° finds that the exterior angle is 30°

$$\begin{array}{r} 1\ 2 \\ 3 \overline{) 3\ 6} \end{array}$$

The exterior angles of any polygon add up to 360° in total. So dividing 360° by each 30° exterior angle (which are all the same as the shape is regular) finds that there are 12 exterior angles. $360 \div 30 = 36 \div 3$. So there must be 12 sides of the polygon

$$n = \dots\dots\dots 12$$

(Total for Question 6 is 4 marks)

7 (a) Simplify $\frac{3(2-m)^2}{2-m}$

$3(2-m)$ ← The denominator cancels out with one of the $(2-m)$ on the numerator

Expanding the brackets

$6 - 3m$

(1)

(b) Solve $7 + x \leq \frac{5x}{2} - 8$

$14 + 2x \leq 5x - 16$ ← Multiplying all terms on both sides by 2 to eliminate the denominator

$14 \leq 3x - 16$ ← Subtracting $2x$ from both sides gets all the x on the same side

$30 \leq 3x$ ← Adding 16 to both sides to get the x term on its own

Dividing both sides by 3 to get x on its own

$x \geq 10$

(3)

(c) Solve $9 < 2y + 4 < 12$

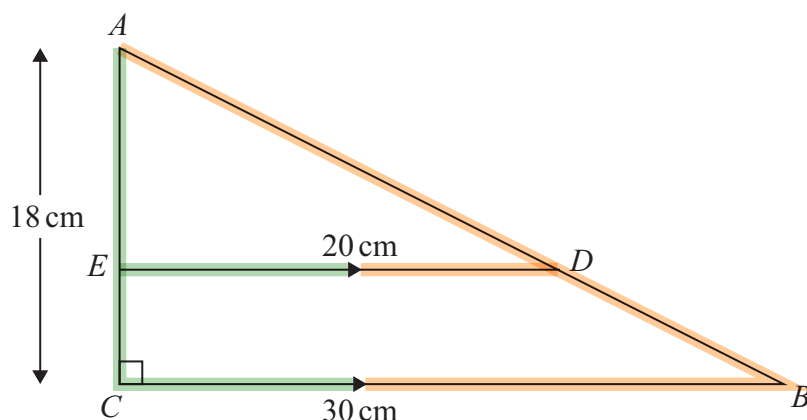
$5 < 2y < 8$ ← Subtracting 4 from all three sides to get the y term on its own in the middle

Dividing all three sides by 2 to get y on its own in the middle → $\frac{5}{2} < y < 4$

(2)

(Total for Question 7 is 6 marks)

8 ABC is a right-angled triangle.



AEC and ADB are straight lines.

ED is parallel to CB .

(a) Prove that triangle ABC is similar to triangle ADE .

Angle EAB is shared ← This angle is in both triangle ABC and triangle ADE

Angle $AED = \text{angle } ACB$ and angle $ADE = \text{angle } ABC$ as corresponding angles are equal

The insides of the green F-shape are corresponding angles.
The insides of the orange F-shape are corresponding angles

All the angles in both triangles are the same so they are similar

(2)

$$ED = 20 \text{ cm} \quad CB = 30 \text{ cm} \quad AC = 18 \text{ cm}$$

(b) Work out the length of EC .

$$\frac{20}{30} = \frac{2}{3} \quad \leftarrow \text{ED and CB are the same side in both triangles as they are opposite the same angle. Putting ED over CB expresses the scale factor from the larger triangle to the smaller triangle. This simplifies to } 2/3 \text{ by dividing both the numerator and denominator by 10}$$

$$18 \div 3 \quad \leftarrow \text{Applying the scale factor to AC. First dividing AC by 3 to find } 1/3 \text{ of AC}$$

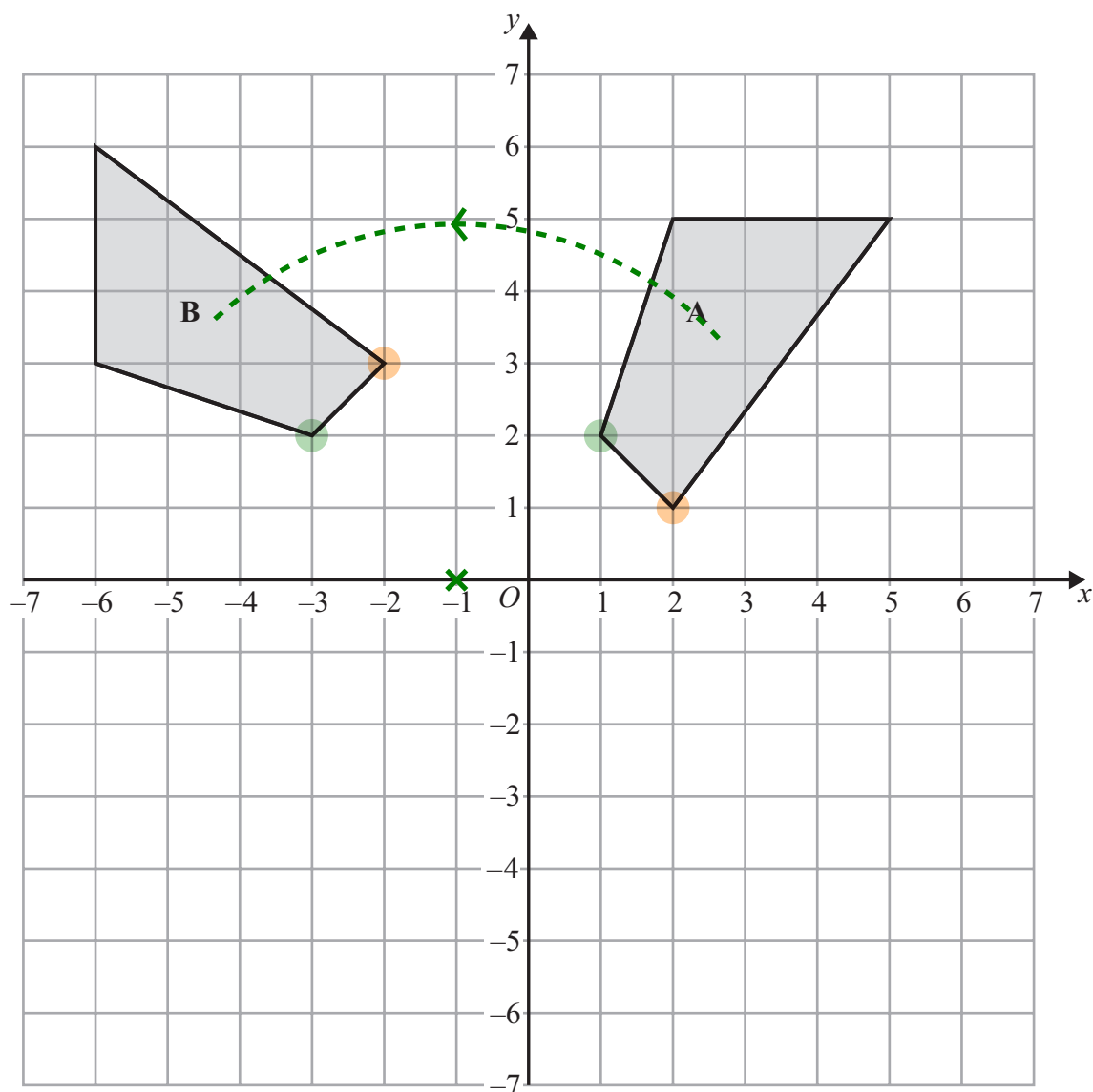
$$6 \times 2 \quad \leftarrow \text{Multiplying } 1/3 \text{ of AC by 2 finds that } 2/3 \text{ of AC is 12 cm. This is AE}$$

$$18 - 12 \quad \leftarrow \text{Subtracting AE from AC leaves EC}$$

$$\dots\dots\dots 6 \dots\dots\dots \text{ cm}$$

(3)

(Total for Question 8 is 5 marks)

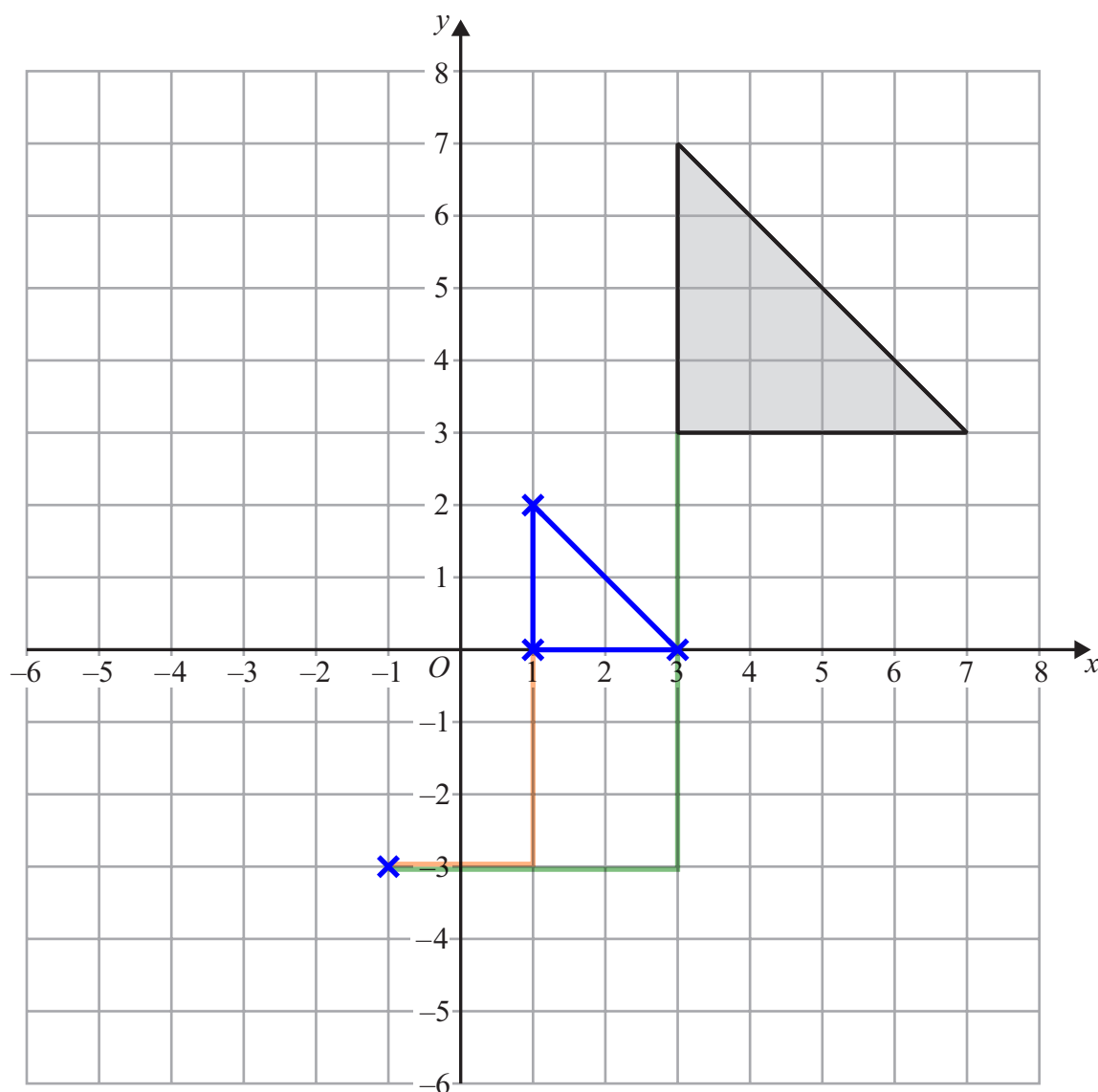


(a) Describe fully the single transformation that maps shape A onto shape B.

Rotation, 90° anticlockwise, centre $(-1, 0)$

Focussing on the green and orange corners, it must be a rotation. 90° is a quarter turn. The centre of the rotation can be found by considering the centre of the circular motion and trying various points using tracing paper

(2)



- (b) Enlarge the triangle by scale factor $\frac{1}{2}$ centre $(-1, -3)$

(2)

(Total for Question 9 is 4 marks)

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix} \times \frac{1}{2} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Writing the vector from the centre to the closest corner then doing half of this. Doing the new vector from the centre works out the new closest corner. Doing half of the other sides from this new corner finds the other new corners. Then it can be joined up

10 Prove that the difference in the squares of two consecutive even numbers is always a multiple of 4

$$2n, 2n + 2$$

Let n be an integer. Multiplying it by 2 makes it even and gives $2n$. This is the 1st even number. Adding 2 to this gives the next even number

$$(2n + 2)^2 - (2n)^2$$

Squaring the larger of the two even numbers and subtracting the square of the smaller even numbers expresses the difference in the squares of two consecutive even numbers

$$4n^2 + 8n + 4 - 4n^2$$

Expanding the brackets. The 1st square bracket can be expanded by squaring the first term, doubling the product of the two terms, squaring the last term. For the 2nd bracket, both the 2 and the n are squared

$$8n + 4$$

Collecting like terms

$$4(2n + 1)$$

Bringing 4 out as a factor shows that it is a multiple of 4. n was an integer so $2n + 1$ is also an integer. Multiplying it by 4 expresses a multiple of 4

(Total for Question 10 is 3 marks)

11 T is inversely proportional to w . $\leftarrow T = \frac{1}{w}$

w is directly proportional to the cube root of d . $\leftarrow w = \sqrt[3]{d}$

When $w = 6$, $T = 20$

When $w = 1$, $d = 8$

Find the value of d when $T = 48$

$$T = \frac{k}{w}$$

Writing the 1st proportion as an equation by multiplying the right side by k , which represents a constant

$$20 \times 6 = k$$

Multiplying both sides by w to get k on its own. Substituting 20 for T and 6 for w . So $k = 120$

$$T = \frac{120}{w}$$

Putting the value of k back into the equation

$$w = c \sqrt[3]{d}$$

Writing the 2nd proportion as an equation by multiplying the right side by c , which represents a constant

$$1 = c \sqrt[3]{8}$$

Substituting 1 for w and 8 for d

$$c = \frac{1}{2}$$

$\sqrt[3]{8} = 2$ as $2^3 = 8$. Dividing both sides by 2 gets c on its own

$$w = \frac{1}{2} \sqrt[3]{d}$$

Putting the value of c back into the equation

$$48 = \frac{120}{\frac{1}{2} \sqrt[3]{d}}$$

Substituting the right side of the previous equation for w in the other equation. Substituting 48 for T

$$24 \sqrt[3]{d} = 120$$

Multiplying both sides by the denominator to eliminate it

$$\sqrt[3]{d} = \frac{120}{24} = \frac{10}{2} = 5$$

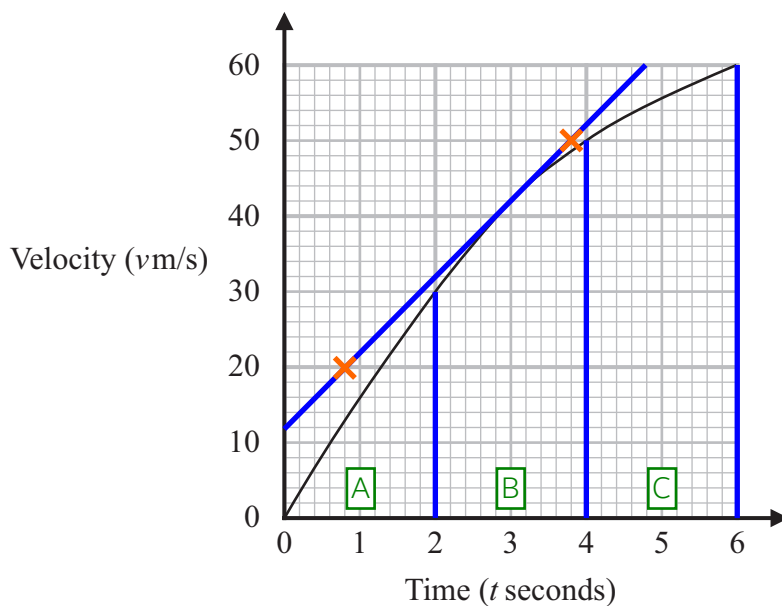
Dividing both sides by 24 then simplifying

Cubing both sides gets d on its own

$$d = \dots\dots\dots 125$$

(Total for Question 11 is 5 marks)

12 The graph shows the velocity, v m/s, of a particle t seconds after it starts to move.



- (a) (i) Work out an estimate of the gradient of the graph at $t = 3$
You must show how you get your answer.

Drawing a tangent to the curve at $t = 3$ then estimating its gradient. Picking two convenient points which are quite far apart and are on grid lines

$$\frac{50 - 20}{3.8 - 0.8}$$

← Gradient = rise/run. It rises from 20 to 50 and runs from 0.8 to 3.8

$$\frac{30}{3}$$

10

(3)

- (ii) What does this gradient represent?

Acceleration

Rate of change in velocity is acceleration

(1)

- (b) Work out an estimate for the distance the particle travelled in the first 6 seconds.
Use 3 strips of equal width.

The width for the first 6 seconds is 6. Dividing this by the 3 equal strips finds that each strip needs width of 2. Drawing on the strips by drawing lines up from 2, 4, 6

$$\frac{1}{2} \times 2 \times 30 = 30$$

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$. So the area of triangle A is 30

$$\frac{1}{2} \times (30 + 50) \times 2 = 80$$

Area of trapezium = $\frac{1}{2} \times (a + b) \times h$, where a and b are parallel sides and h is the distance between them. So the area of trapezium B is 80

$$\frac{1}{2} \times (50 + 60) \times 2 = 110$$

Area of trapezium = $\frac{1}{2} \times (a + b) \times h$, where a and b are parallel sides and h is the distance between them. So the area of trapezium C is 110

$$30 + 80 + 110$$

Adding the areas of all three shapes estimates that the area under the curve is 220

Distance is the area under the line on a velocity-time graph $\rightarrow 220$ m
(3)

(Total for Question 12 is 7 marks)

13 $x = 0.\dot{2}$ $y = 0.6\dot{8}1$

Work out the value of xy .

Give your answer as a fraction in its simplest form.

$$10x = 2.\dot{2}$$

Multiplying x by ten once as there is one recurring digit gets a different decimal with the recurring digit in the same decimal place

$$9x = 2$$

Subtracting x from $10x$ cancels out the recurring digit

$$x = \frac{2}{9}$$

Dividing both sides by 9 gets x on its own and expresses it as a fraction

$$100y = 68.1\dot{8}1$$

Multiplying y by ten twice as there are two recurring digits gets a different decimal with the recurring digits in the same decimal places

$$99y = 67.5$$

Subtracting y from $100y$ cancels out the recurring digits

$$y = \frac{67.5}{99}$$

Dividing both sides by 99 gets y on its own and expresses it as a fraction

$$\frac{2}{9} \times \frac{67.5}{99}$$

Expressing xy as fractions

$$\begin{array}{r} 67.5 \\ \times 2 \\ \hline 135.0 \\ 1 \end{array}$$

Multiplying the fractions by multiplying the numerators and multiplying the denominators. So the answer is $135/891$, which needs to be simplified. $9 \times 99 = 9 \times 100 - 9 = 900 - 9 = 891$

$$\begin{array}{r} 0 \ 4 \ 5 \\ 3 \overline{) 1 \ 13 \ 15} \\ \underline{2 \ 9 \ 7} \\ 3 \overline{) 8 \ 29 \ 21} \end{array}$$

Dividing both the numerator and denominator of $135/891$ by 3 gives $45/297$

$$\begin{array}{r} 1 \ 5 \\ 3 \overline{) 4 \ 15} \\ \underline{0 \ 9 \ 9} \\ 3 \overline{) 2 \ 29 \ 27} \end{array}$$

Dividing both the numerator and denominator of $45/297$ by 3 gives $15/99$

$$\begin{array}{r} 3 \ 3 \\ 3 \overline{) 9 \ 29} \end{array}$$

Dividing both the numerator and denominator of $15/99$ by 3 gives $5/33$, which cannot go simpler

$$\frac{5}{33}$$

(Total for Question 13 is 5 marks)

14 There are nine balls labelled 1 to 9 in a box.

Lee will take at random two balls from the box.

Lee says,

“The probability that the sum of the numbers on the two balls will be an even number is greater than the probability that the product of the numbers will be an even number.”

Is Lee correct?

You must show how you get your answer.

1, 2, 3, 4, 5, 6, 7, 8, 9 ← Listing the numbers 1 to 9 to help visualise which are even or odd

EE, EO, OE, OO

E O O E
E E E O

Systematically listing the possible outcomes of the two balls, E standing for even and O standing for odd. Under these, writing the result of the sum of the two balls. Under these, writing the result of the product of the two balls

$$\frac{4}{9} \times \frac{3}{8} + \frac{5}{9} \times \frac{4}{8} = \frac{12}{72} + \frac{20}{72} = \frac{32}{72}$$

Working out the probability of the sum being even. Even AND even OR odd AND odd. AND means to multiply the probabilities, OR means to add the probabilities.

To begin with, 4 out of the 9 balls are even. Once one is taken, there is one fewer even ball and one fewer ball in total. Applying similar logic to the other probabilities.

To multiply fractions, multiply the numerators and multiply the denominators. To add fractions, the numerators can be added when the denominators are the same

$$\frac{4}{9} \times \frac{3}{8} + \frac{4}{9} \times \frac{5}{8} + \frac{5}{9} \times \frac{4}{8} = \frac{12}{72} + \frac{20}{72} + \frac{20}{72} = \frac{52}{72}$$

Working out the probability of the product being even. Even AND even OR even AND odd OR odd AND even. AND means to multiply the probabilities, OR means to add the probabilities.

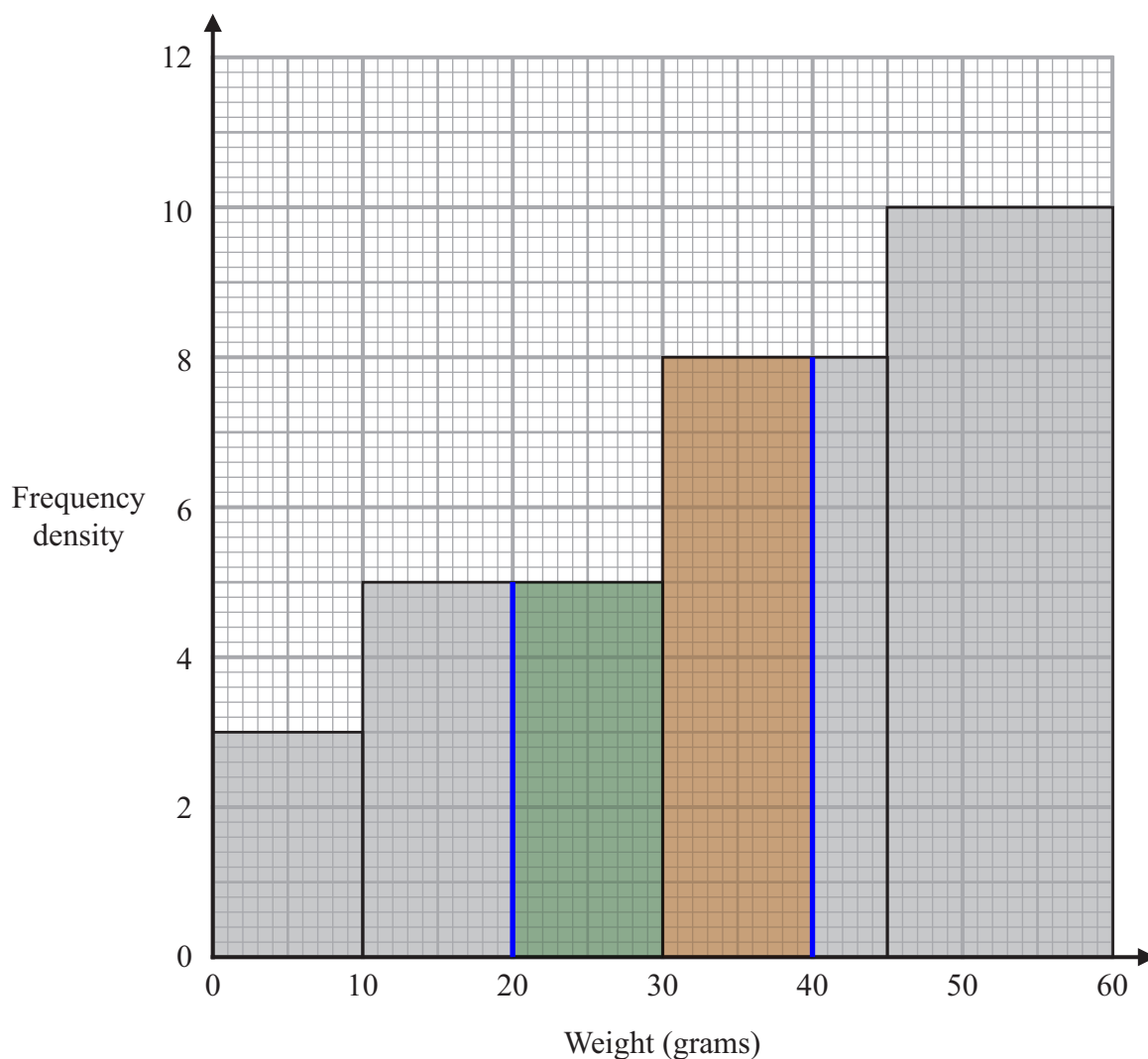
To begin with, 4 out of the 9 balls are even. Once one is taken, there is one fewer even ball and one fewer ball in total. Applying similar logic to the other probabilities.

To multiply fractions, multiply the numerators and multiply the denominators. To add fractions, the numerators can be added when the denominators are the same

No ← $\frac{32}{72}$ is not greater than $\frac{52}{72}$

(Total for Question 14 is 5 marks)

15 The histogram gives information about the weights, in grams, of some biscuits.



One of these biscuits is taken at random.

Work out an estimate for the probability that the biscuit will have a weight between 20 grams and 40 grams.

$10 \times 3 = 30$
 $20 \times 5 = 100$
 $15 \times 8 = 120$
 $15 \times 10 = 150$
 400

Frequency on a histogram is the area of each box. Area of rectangle = base $\times$ height

Adding all the frequencies finds that there are 400 biscuits in total

$10 \times 5 = 50$
 $10 \times 8 = 80$
 130

Splitting the bars to create the green and orange bars. Frequency on a histogram is the area of each box. Area of rectangle = base $\times$ height

Adding these frequencies estimates that there are 130 biscuits between 20 grams and 40 grams

130 out of the 400 biscuits are estimated to be between 20 grams and 40 grams $\rightarrow \frac{130}{400}$

(Total for Question 15 is 4 marks)

16 (a) Rationalise the denominator of $\frac{35}{\sqrt{7}}$

Give your answer in its simplest form.

$$\frac{35\sqrt{7}}{7}$$

Multiplying both the numerator and denominator by root 7 eliminates the square root on the denominator so rationalises it

$$\boxed{35/7 = 5} \longrightarrow 5\sqrt{7} \quad (2)$$

$\frac{\sqrt{27}-1}{2-\sqrt{3}}$ can be written in the form $a + b\sqrt{3}$ where a and b are integers.

(b) Work out the value of a and the value of b .

$$\frac{(\sqrt{27}-1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

Flipping the sign of the denominator and multiplying both the numerator and denominator by the result of this will rationalise the denominator

$$\frac{2\sqrt{27} + \sqrt{81} - 2 - \sqrt{3}}{4 + 2\sqrt{3} - 2\sqrt{3} - \sqrt{9}}$$

Expanding the brackets. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$$\frac{6\sqrt{3} + 9 - 2 - \sqrt{3}}{4 - 3}$$

Simplifying. $2\sqrt{27} = 2 \times \sqrt{9} \times \sqrt{3} = 2 \times 3 \times \sqrt{3} = 6\sqrt{3}$

$$7 + 5\sqrt{3}$$

Collecting like terms to put into the desired form. The denominator of 1 can be ignored

$$a = 7$$

$$b = 5 \quad (4)$$

(Total for Question 16 is 6 marks)

Turn over for Question 17

17 $g(x) = 1 - 3x$ $h(x) = 2x^2 - 1$

Show that $3gh(x) + hg(x) = 0$ has just one solution for x .

$1 - 3(2x^2 - 1)$ ← Expressing the composite function $gh(x)$ by substituting $h(x)$ for x in $g(x)$

$1 - 6x^2 + 3$ ← Expanding the brackets

$3(4 - 6x^2)$ ← Collecting like terms and multiplying by 3 to express $3gh(x)$

$12 - 18x^2$ ← Expanding the brackets. This is $3gh(x)$ simplified

$(1 - 3x)(1 - 3x)$ ← Expressing $(g(x))^2$

$1 - 3x - 3x + 9x^2$ ← Expanding the brackets

$2(1 - 6x + 9x^2) - 1$ ← Collecting like terms and substituting $(g(x))^2$ for x^2 in $h(x)$ to express the composite function $hg(x)$

$2 - 12x + 18x^2 - 1$ ← Expanding the brackets

$12 - 18x^2 + 1 - 12x + 18x^2 = 0$ ← Substituting the simplified expressions for $3gh(x)$ and $hg(x)$ into the equation $3gh(x) + hg(x) = 0$

$13 - 12x = 0$ ← Collecting like terms

$13 = 12x$ ← Adding $12x$ to both sides to get the x term positive and on its own

$x = \frac{13}{12}$ ← Dividing both sides by 12 gets x on its own and solves the equation, showing that there is only one solution

(Total for Question 17 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS