

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

GCSE MATHEMATICS

H

Higher Tier

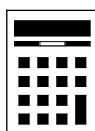
Paper 3 Calculator

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Advice

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided.

1 b is 3 more than the square root of a .

Circle the correct equation.

[1 mark]

$$b = \sqrt{a} + 3$$

$$b = \sqrt{a} - 3$$

$$b = \sqrt{a+3}$$

$$b = \sqrt{a-3}$$

Adding 3 to a value expresses 3 more than the value

2 Circle the largest number.

[1 mark]

$$0.\dot{5}$$

$$0.55$$

$$0.545$$

$$0.5\dot{4}5$$

$$0.555\dots$$

$$0.550$$

$$0.545$$

$$0.545\dots$$

Writing the numbers truncated to 3 decimal places is enough for the largest to be identified

3 A line has equation $3y = 3x - 2$

Circle the coordinates of the intercept of the line with the y -axis.

[1 mark]

$$(0, 1)$$

$$(0, -1)$$

$$\left(0, \frac{2}{3}\right)$$

$$\left(0, -\frac{2}{3}\right)$$

Dividing both sides of the equation by 3 puts it into the form $y = mx + c$, where m is the gradient and c is the y -intercept. c would be $-2/3$. The x coordinate must be 0 as it is a point on the y -axis



- 4 Factorise $x^2 - 64$
Circle your answer.

[1 mark]

$(x + 8)^2$

$(x - 8)^2$

$(x + 8)(x - 8)$

$x(x - 64)$

Difference of two squares can be used here. $a^2 - b^2 = (a + b)(a - b)$

- 5 Six positive numbers have
a mean of 10
a range of 19

Four of the numbers are 12 7 15 3

Work out the other two numbers.

[3 marks]

10×6 ← Multiplying the mean by the 6 numbers works out that the total of the numbers must be 60

$60 - 12 - 7 - 15 - 3 = 23$ ← Subtracting the other numbers from the total of 60 works out that the other two numbers must add up to 23

$3 + 19 = 22$ ← Assuming that the smallest number is 3, adding the range of 19 to this works out that the largest number would be 22. This does not work as a number smaller than 3 must be added to 22 to get the 23 and we assumed that 3 was the smallest

$2 + 19 = 21$ ← Assuming that the smallest number is 2, adding the range of 19 to this works out that the largest number would be 21. This works as $2 + 21 = 23$ so the other two numbers must be 2 and 21

Answer 2 and 21

Turn over ►



- 6 At a country park there is a house, a museum and a garden.
The table shows the prices per person to visit the park.

	Price per person
Garden only	Free
House and museum	£12.50
House only	£8
Museum only	£7

One day, 480 people visit the park.

67 visit the garden **only**.

40% visit the house **and** the museum.

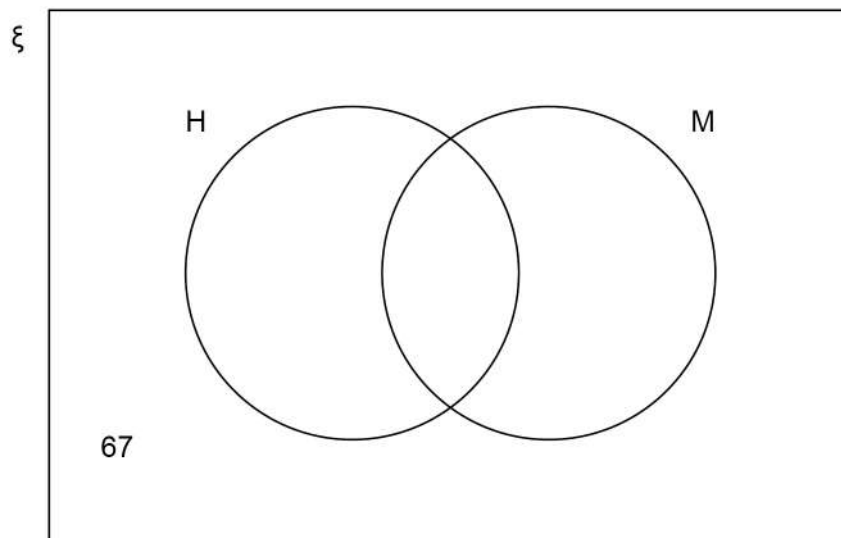
$\frac{3}{8}$ visit the house **only**.

The rest visit the museum **only**.

In total, how much do the 480 people pay to visit the park?

You may use the Venn diagram to help you.

[5 marks]



$$\frac{40}{100} \times 480 = 192$$

Putting the 40% over 100 converts it into a fraction. Multiplying this fraction by the 480 people works out that the number of people who visited the house and the museum is 192

$$\frac{3}{8} \times 480 = 180$$

This works out that the number of people who visited the house only is 180

$$480 - 192 - 180 - 67 = 41$$

Subtracting the number of people who visited the house and the museum, the house only and the garden only leaves 41 who visited the museum only

$$192 \times 12.50 + 180 \times 8 + 41 \times 7$$

Adding together the amounts paid to visit the house and the museum, the house only and the museum only gives the total amount paid. The garden is ignored as it is free

Multiplying the 41 people who visited the museum only by the £7 to visit the museum only expresses the amount paid to visit the museum only

Multiplying the 180 people who visited the house only by the £8 to visit the house only expresses the amount paid to visit the house only

Multiplying the 192 people who visited the house and museum by the £12.50 to visit the house and museum expresses the amount paid to visit the house and museum

Answer £ 4127

7 Jeff and Kaz share £270 in the ratio Jeff : Kaz = 2.6 : 1

How much **more** than Kaz does Jeff get?

[3 marks]

$$2.6 + 1$$

Adding the 2.6 and 1 parts of the ratio works out that there are 3.6 parts in total which represent the £270

$$270 \div 3.6$$

Dividing the £270 by the 3.6 parts of the ratio which represent it works out that 1 part of the ratio is worth £75. So Kaz gets £75

$$75 \times 2.6$$

Multiplying the value of 1 part of the ratio by the 2.6 parts which represent Jeff works out that Jeff gets £195

$$195 - 75$$

Subtracting the £75 Kaz gets from the £195 Jeff gets works out how much more Jeff gets than Kaz

Answer £ 120



8 The heel of a shoe exerts a pressure of 198 pounds per square inch.

Convert this pressure into kilograms per square centimetre.

Use

1 pound = 0.45 kilograms

1 square inch = 6.25 square centimetres

[3 marks]

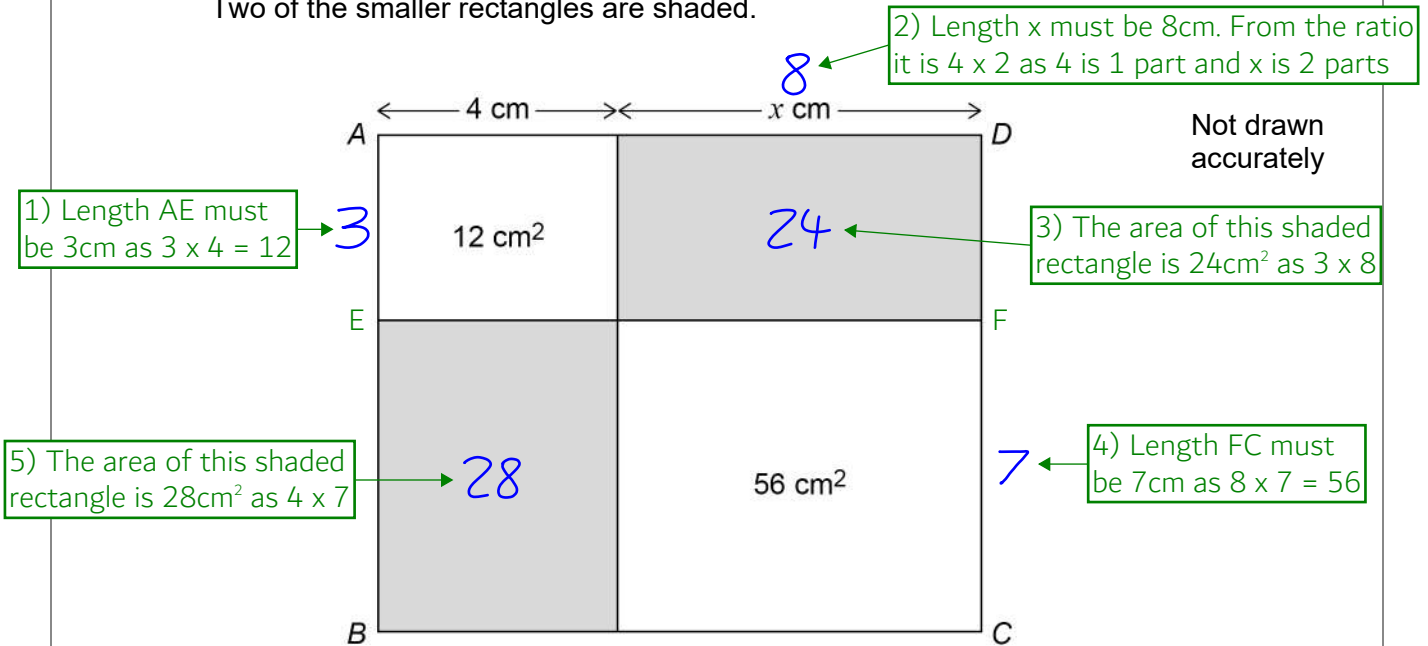
198×0.45 ← Multiplying 198 pounds by 0.45 converts it into 89.1 kg

$89.1 \div 6.25$ ← Kilograms per square centimetre means to divide the kilograms by the square centimetres

Answer _____ 14.256 _____ kg/cm²



- 9 Rectangle $ABCD$ is split into four smaller rectangles.
Two of the smaller rectangles are shaded.



$$4 : x = 1 : 2$$

Area of rectangle = length \times width

For rectangle $ABCD$, work out the ratio shaded area : unshaded area

Give your answer in its simplest form.

[4 marks]

$$24 + 28 = 52$$

Adding the areas of the two shaded rectangles works out that the shaded area is 52cm^2

$$12 + 56$$

Adding the areas of the two unshaded rectangles works out that the unshaded area is 68cm^2

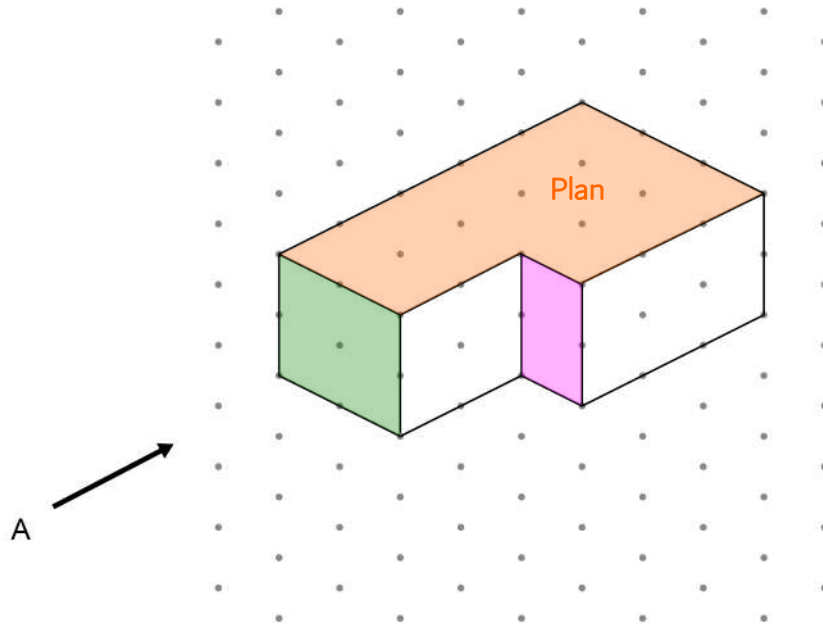
$$\frac{52}{68} = \frac{13}{17}$$

Putting the shaded area as a fraction of the unshaded area in the calculator simplifies it to $\frac{13}{17}$. Ratios simplify in a similar way to fractions so the ratio must be $13 : 17$

Answer 13 : 17

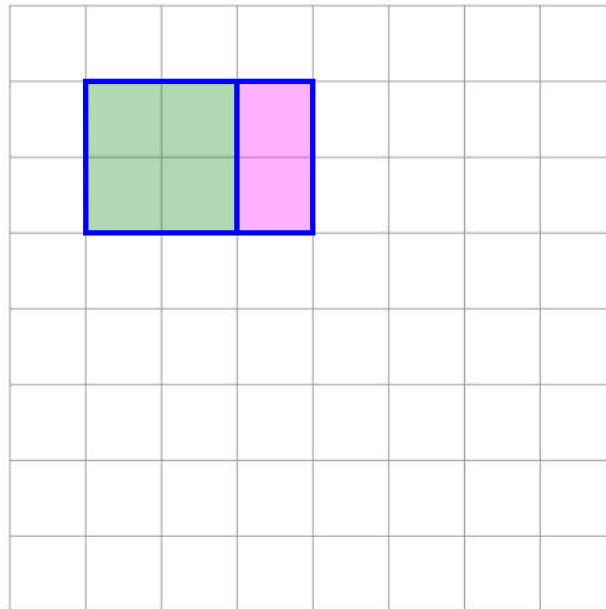


- 10 A solid shape is drawn on isometric paper.



- 10 (a) On the centimetre grid, draw the elevation of the shape from A.

[1 mark]

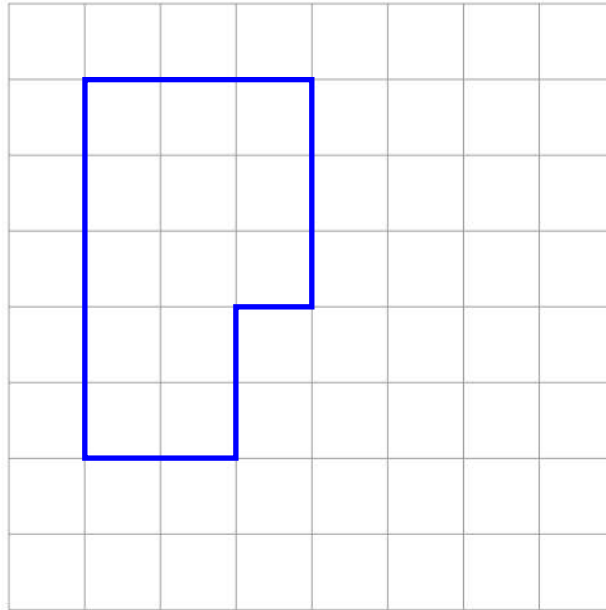


The shading is only to illustrate the faces seen and is not needed as part of the answer



- 10 (b) On the centimetre grid, draw a plan of the shape.

[1 mark]



- 11 Erik thinks of a prime number between 20 and 30

His number is $x\%$ of 125

Work out **one** possible value of x .

[3 marks]

$$\frac{23}{125} \times 100$$

23 is prime as it only has two factors, itself and 1. Writing 23 as a fraction of 125 then multiplying by 100 to convert it into a percentage

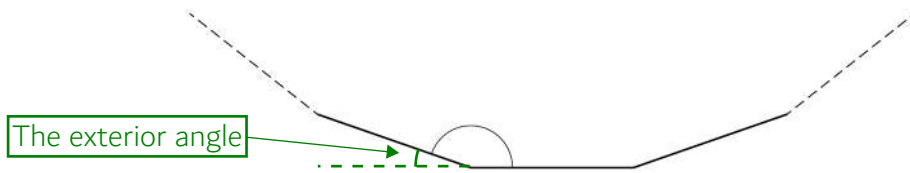
Answer 18.4

5

Turn over ►



12 Part of a regular polygon with 15 sides is shown.



Not drawn
accurately

Work out the size of an **interior** angle.

$$360 \div 15$$

The exterior angles of any polygon add up to 360° . So dividing 360° by the 15 exterior angles works out that one of the exterior angles is 24°

[2 marks]

$$180 - 24$$

The exterior and interior angle form a straight line and there are 180° around a point on a straight line. So subtracting the exterior angle from 180° work out that the interior angle is 156°

Answer 156 degrees



14

Phil sells ties.

He increases the original price of each tie by 10% to £13.20

A month later he announces a sale.



Phil says,

"The ties will be back to their original price, because each change was by 10%"

Is he correct?

Tick a box.

Yes

No

Show working to support your answer.

[3 marks]

$$x \times \frac{100+10}{100} \times \frac{100-10}{100}$$

Let x be the original price. To increase by 10%: $100 + 10$ expresses the percentage it increases to then dividing by 100 converts it into a fraction which when multiplied by will increase by 10%. To decrease by 10%: $100 - 10$ expresses the percentage it decreases to then dividing by 100 converts it into a fraction which when multiplied by will decrease by 10%

$$0.99x \leftarrow x \text{ was the original price and this is not } x \text{ so the ties will not be back to their original price}$$



15

A biased spinner can land on A, B or C.

The table shows the probabilities, in terms of k , of A, B and C.

	A	B	C
Probability	$0.5k$	$7k - 0.15$	$2.5k$

Work out the probability of B.

[3 marks]

$$0.5k + 7k - 0.15 + 2.5k = 1$$

It is certain to be either A, B or C so the probabilities of them all added together must equal 1

$$10k = 1.15$$

Collecting the k terms and adding 0.15 to both sides to get the k term on its own

$$k = 0.115$$

Dividing both sides by 10 finds k

$$7(0.115) - 0.15$$

Substituting 0.115 for k in the expression for the probability of B

Answer 0.655

Turn over for the next question

Turn over ►



- 16 P is the point (2, 14)
 Q is the point (6, 8)
 R is the point (2, 5)

Use gradients to show that angle PQR is **not** a right angle.

[3 marks]

$$\frac{8-14}{6-2} = -\frac{3}{2}$$

This works out the gradient of PQ . Gradient = (change in y)/(change in x). Change in y is found by subtracting the first y -coordinate from the second y -coordinate. Change in x is found by subtracting the first x -coordinate from the second x -coordinate

$$\frac{5-8}{2-6} = \frac{3}{4}$$

This works out the gradient of QR . Gradient = (change in y)/(change in x). Change in y is found by subtracting the first y -coordinate from the second y -coordinate. Change in x is found by subtracting the first x -coordinate from the second x -coordinate

$$-\frac{3}{2} \times \frac{3}{4} \neq -1$$

Perpendicular gradients should multiply to -1 . As they don't the angle PQR cannot be a right angle



17 $m^2 > 9$

Circle the possible value of m .

[1 mark]

$-2\frac{7}{8}$

2.8

3

$\frac{7}{2}$

$(-2\frac{7}{8})^2 = 8.2\dots$ which is not greater than 9.
 $2.8^2 = 7.84$ which is not greater than 9.
 $3^2 = 9$ which is not greater than 9.
 $(\frac{7}{2})^2 = 12.25$ which is greater than 9

18 Simplify $w^1 \times w^0$

Circle your answer.

[1 mark]

1

0

w

w^2

$a^x \times a^y = a^{x+y}$. So $w^1 \times w^0 = w^{1+0} = w^1 = w$

19 The equation of a circle is $x^2 + y^2 = 11$

Work out the length of the **diameter**.

Circle your answer.

[1 mark]

$\sqrt{11}$

$2\sqrt{11}$

$\sqrt{22}$

22

The general equation of a circle with its centre at $(0, 0)$ is $x^2 + y^2 = r^2$, where r is the radius. $r^2 = 11$ so the radius is $\sqrt{11}$. The diameter is double the radius

Turn over for the next question



20

$$\frac{a}{b} = 3c$$

$$\frac{b}{c} = 2$$

Work out the value of a when $c = 8$

[3 marks]

$$a = 3bc$$

Rearranging to make a the subject in the first equation by multiplying both sides by b

$$b = 2c = 2 \times 8 = 16$$

Rearranging to make b the subject in the second equation by multiplying both sides by c . Substituting 8 for c finds that $b = 16$

$$a = 3 \times 16 \times 8$$

Substituting 16 for b and 8 for c

Answer _____ 384 _____

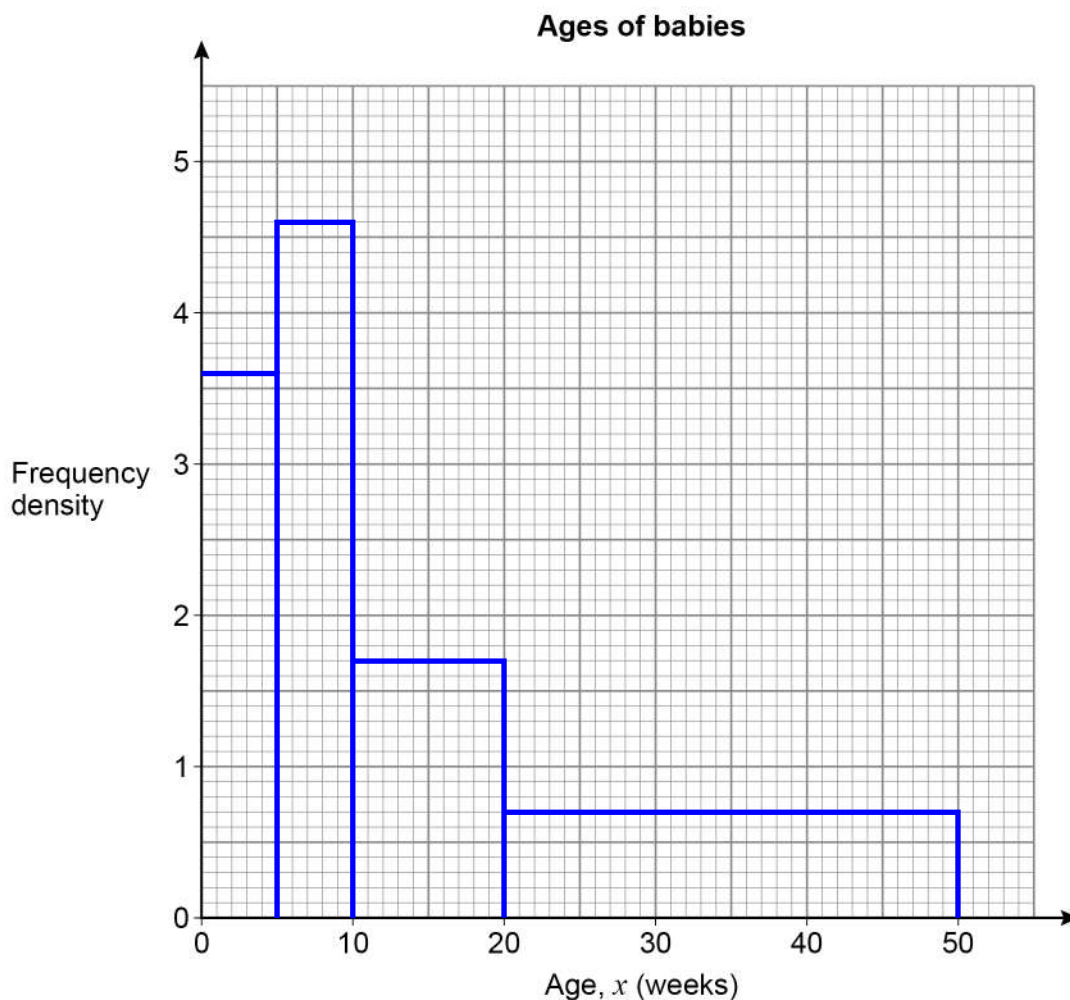


21 Here is some information about the ages of babies at a clinic.

Age, x (weeks)	Frequency	Class width	Frequency density
$0 \leq x < 5$	18	5	3.6
$5 \leq x < 10$	23	5	4.6
$10 \leq x < 20$	17	10	1.7
$20 \leq x < 50$	21	30	0.7

Draw a histogram to represent the information.

[4 marks]



The class width is how wide each interval is. For example, the $0 \leq x < 5$ interval goes from 0 to 5 so has a class width of 5. Frequency on a histogram is the area of each box. Area of rectangle = length \times width. So dividing the frequencies by the class widths works out the heights of each box, which is the frequency density



22

A sequence of patterns is made using horizontal sticks and vertical sticks.

Pattern 1



Pattern 2



Pattern 3



The table shows the number of horizontal sticks and vertical sticks in each pattern.

Pattern	Number of horizontal sticks	Number of vertical sticks
1	2	2
2	4	3
3	6	4

What fraction of the total number of sticks in Pattern n are horizontal?

Give your answer in terms of n .

[3 marks]

Expressing the n th term for the horizontal sticks as a fraction of the n th term for the total number of sticks gives the desired fraction. The number of horizontal sticks increases by 2 between each term so must involve $2n$. The 0th term, the term before the first term, in the sequence 2, 4, 6 would be 0 so the n th term of the horizontal sticks is $2n + 0$, or just $2n$. The number of vertical sticks increases by 1 between each term so must involve n . The 0th term, the term before the first term, in the sequence 2, 3, 4 would be 1 so the n th term of the vertical sticks is $n + 1$. Adding the n th term of the horizontal and vertical sticks gives the n th term of the total number of sticks

Answer $\frac{2n}{2n+n+1}$



23 The equation of a curve is $y = 16^x$

23 (a) Circle the point that lies on the curve.

[1 mark]

(2, 32)

(32, 2)

(2, 256)

(256, 2)

The y-coordinate must be equal to 16 to the power of the x-coordinate. $16^2 = 256$

23 (b) A different point on the curve has y-coordinate $\frac{1}{16}$

Work out the x-coordinate.

[1 mark]

Answer _____ -1 _____

The power of -1 does the reciprocal, which means 1 over the number. So $16^{-1} = 1/16$

24 $a^b = 3$ where a is an integer and b is a proper fraction.

Work out **one** possible pair of values of a and b .

[1 mark]

$a =$ _____ 9 _____ $b =$ _____ $\frac{1}{2}$ _____

The power of $1/2$ means to do the square root. The square root of 9 is 3



25 Expand and simplify fully $(x-3)(x+2)(x+5)$

[3 marks]

$$x^2 + 2x - 3x - 6 \leftarrow \text{Expanding the first two brackets}$$

$$(x^2 - x - 6)(x + 5) \leftarrow \text{Simplifying by collecting like terms then writing multiplied by the third bracket}$$

$$x^3 + 5x^2 - x^2 - 5x - 6x - 30 \leftarrow \text{Expanding these two brackets}$$

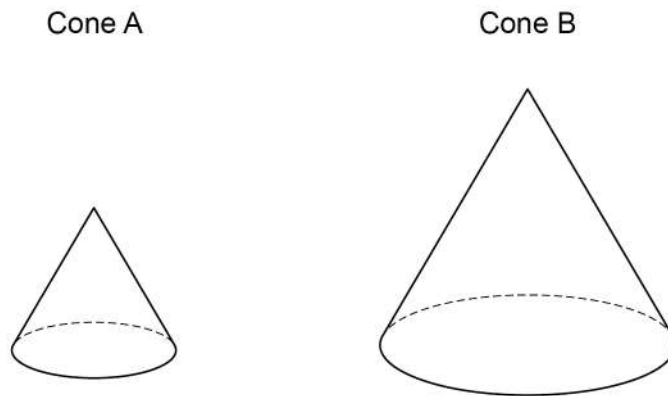
Answer $x^3 + 4x^2 - 11x - 30$

Simplifying by collecting like terms



26

Here are two similar cones.

The surface area of cone A is 2 m^2 The surface area of cone B is 4.5 m^2

Work out the ratio radius of cone A : radius of cone B

Give your answer in the form $1 : n$

$$\sqrt{2} : \sqrt{4.5}$$

The ratio of the areas is $2 : 4.5$. Square rooting both sides gives the ratio of the lengths. As the radius is a length this is the ratio we are looking for

[3 marks]

Answer 1 : 1.5

Dividing both sides of the ratio by $\sqrt{2}$ to get 1 part on the left



27

In the diagram

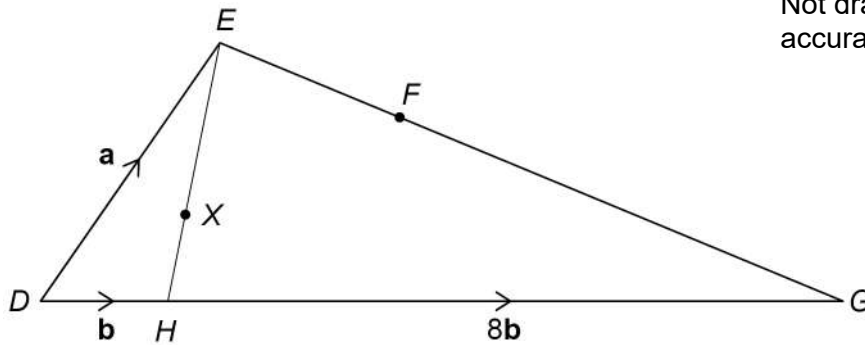
$$\overrightarrow{DE} = \mathbf{a}$$

$$\overrightarrow{DH} = \mathbf{b}$$

$$\overrightarrow{HG} = 8\mathbf{b}$$

$$EX : XH = 3 : 1$$

$$EF : FG = 1 : 3$$

Not drawn
accurately

27 (a) Show that $\overrightarrow{DX} = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

[2 marks]

$$\overrightarrow{DX} = \overrightarrow{DH} + \overrightarrow{HX}$$

$$= \mathbf{b} + \frac{1}{4}\overrightarrow{HE}$$

$\overrightarrow{HX} = \frac{1}{4}\overrightarrow{HE}$ as 1 out of the 4 parts going from E to H is XH in the ratio EX : XH

$$= \mathbf{b} + \frac{1}{4}(-\mathbf{b} + \mathbf{a})$$

$\overrightarrow{HE} = \overrightarrow{HD} + \overrightarrow{DE}$. $\overrightarrow{HD} = -\mathbf{b}$ as it is going the opposite direction to \overrightarrow{DH} . $\overrightarrow{DE} = \mathbf{a}$

$$= \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$$

Expanding the brackets and collecting like terms



27 (b) Is DXF a straight line?

Show working to support your answer.

[4 marks]

$$\vec{DF} = \vec{DE} + \vec{EF}$$

$$= a + \frac{1}{4}\vec{EG}$$

$\vec{EF} = 1/4 \vec{EG}$ as 1 out of the 4 parts going from E to G is EF in the ratio EF : FG

$$= a + \frac{1}{4}(-a + 9b)$$

$\vec{EG} = \vec{ED} + \vec{DG}$. $\vec{ED} = -a$ as it is going the opposite direction to \vec{DE} . $\vec{DG} = \vec{DH} + \vec{HG} = b + 8b$

$$= \frac{3}{4}a + \frac{9}{4}b$$

Expanding the brackets and collecting like terms

$$= 3\left(\frac{1}{4}a + \frac{3}{4}b\right)$$

Bringing out 3 as a factor shows that \vec{DF} is a multiple of \vec{DX}

Yes

So DXF must be a straight line as \vec{DF} is a multiple of \vec{DX}

Turn over for the next question

Turn over ►



28 $a = 4.72$ to 3 significant figures.

$b = 158$ to 3 significant figures.

Work out the upper bound of $\frac{a}{b}$

You **must** show your working.

[3 marks]

$$\frac{4.72 + \frac{0.01}{2}}{158 - \frac{1}{2}}$$

The third significant figure in 4.72 has a resolution of 0.01 as this is what it goes up in for that place. Adding half of this to the 4.72 gives the upper bound of a. The third significant figure in 158 has a resolution of 1 as this is what it goes up in for that place. Subtracting half of this from the 158 gives the lower bound of b, which is needed as b is the denominator so should be as small as possible in order for a/b to be as great as possible

Answer 0.03

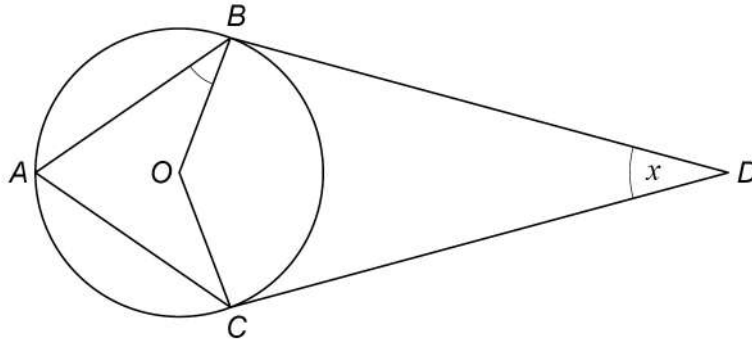


29 A , B and C are three points on the circumference of a circle, centre O .

BD and CD are tangents to the circle.

$ABDC$ is a kite.

Angle BDC is x



Not drawn
accurately

Prove that angle ABO is $45^\circ - \frac{x}{4}$

[4 marks]

Angles OBD and OCD are 90° as the angle between a tangent and radius is a right angle.

$360 - 90 - 90 - x = 180 - x = \text{BOC obtuse}$ as angles in a quadrilateral add up to 360° .

Subtracting angles OBD , OCD and BDC from 360° in quadrilateral $OBDC$

$BAC = 90 - \frac{x}{2}$ as angles at the circumference are half the angle at the centre.

$ABD + ACD = 360 - x - (90 - \frac{x}{2}) = 270 - \frac{x}{2}$ as angles in a quadrilateral add up to 360° .

Subtracting angles BDC and BAC from 360° in quadrilateral $ABDC$

$ABD = 135 - \frac{x}{4}$ as two of the angles in a kite are equal so ABD is $\frac{1}{2}$ of $ABD + ACD$.

$ABD = ACD$

$ABO = 135 - \frac{x}{4} - 90 = 45 - \frac{x}{4}$ Subtracting OBD from ABD



30 A sphere has radius r cm

An approximate value of r can be found using the iterative formula

$$r_{n+1} = \sqrt{\frac{239}{r_n}}$$

The starting value is $r_1 = 7$

30 (a) Work out the values of r_2 and r_3

[2 marks]

$$r_2 = \underline{\hspace{10em} 5.84 \hspace{10em}}$$

$$r_3 = \underline{\hspace{10em} 6.40 \hspace{10em}}$$

Enter 7 then press = or exe. Enter $\sqrt{239}/\text{ANS}$ then press = or exe to get r_2 . Press = or exe again to get r_3

30 (b) Continue the iteration to work out the radius to 1 decimal place.

[1 mark]

$$\text{Answer } \underline{\hspace{10em} 6.2 \hspace{10em}} \text{ cm}$$

Keep pressing = or exe until the 2nd decimal place stops changing

END OF QUESTIONS

