Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

GCSE MATHEMATICS

H

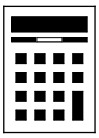
Higher Tier Paper 2 Calculator

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided.

Do not write outside the box

1 Circle the factor of $x^2 - 5x$

[1 mark]

$x - 1$ $\quad -5x$ $\quad (x - 5)$ $\quad 5x$

When factorised it is x(x - 5) so both x and (x - 5) are factors

2 A is half of B.

Work out the ratio $A : B$

Circle your answer.

[1 mark]

(1 : 2) $\quad$ 2 : 1 $\quad$ 1 : 3 $\quad$ 3 : 1

There is 1 part for A and 2 parts for B and 1 is half of 2

3 The first three terms of a geometric progression are $\frac{2}{3} \quad \frac{4}{9} \quad \frac{8}{27}$

Circle the fourth term.

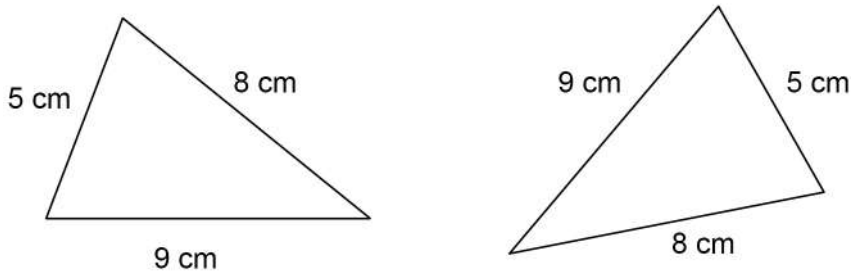
[1 mark]

$\frac{10}{81}$ $\quad \frac{14}{81}$ $\quad \left(\frac{16}{81}\right)$ $\quad \frac{32}{81}$

It multiplies by 2/3 between each term

Do not write outside the box

4



Not drawn accurately

Circle the reason why these triangles are congruent.

[1 mark]

ASA RHS SAS (SSS)

All three sides are the same in both triangles

5 Solve $10x = 62.4 - 3x$

[2 marks]

$13x = 62.4$ ← Adding 3x to both sides to get all the x on the same side

$x =$ 4.8

Dividing both sides by 13 to get x on its own

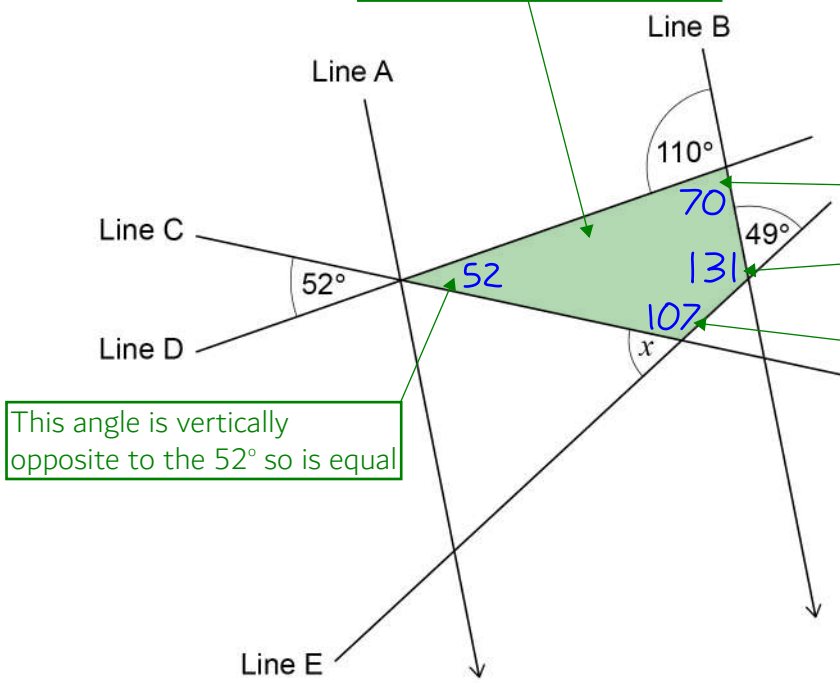
6

Do not write outside the box

6 Lines A, B, C, D and E intersect as shown.

Lines A and B are parallel.

This shape is a quadrilateral as it has 4 sides



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Angles around a point on a straight line add up to 180°

Angles in a quadrilateral add up to 360°

This angle is vertically opposite to the 52° so is equal

Work out the size of angle x.

[3 marks]

Answer 73 degrees

Angles around a point on a straight line add up to 180°

Do not write outside the box

7 102 boys and 85 girls took a test.

The table shows information about the mean marks.

	Boys	Girls
Number of students	102	85
Mean mark	68.5	72.4

The pass mark for the test was 70

Was the mean mark for **all** of these students greater than the pass mark?

You **must** show your working.

[3 marks]

$m^t n$ ← Writing a formula triangle for mean, total, number. Total is the total height of all the students and number is the number of students

$102+85=187$ ← Adding the number of boys and the number of girls works out that there are 187 students

$68.5\times102=6987$ ← Covering t in the formula triangle finds that total = mean x number. Multiplying the mean for the boys by the number of boys works out that the total mark for the boys was 6987

72.4×85 ← Covering t in the formula triangle finds that total = mean x number. Multiplying the mean for the girls by the number of girls works out that the total mark for the girls was 6154

$6987+6154$ ← Adding the totals for the boys and girls works out that the total mark for all students was 13141

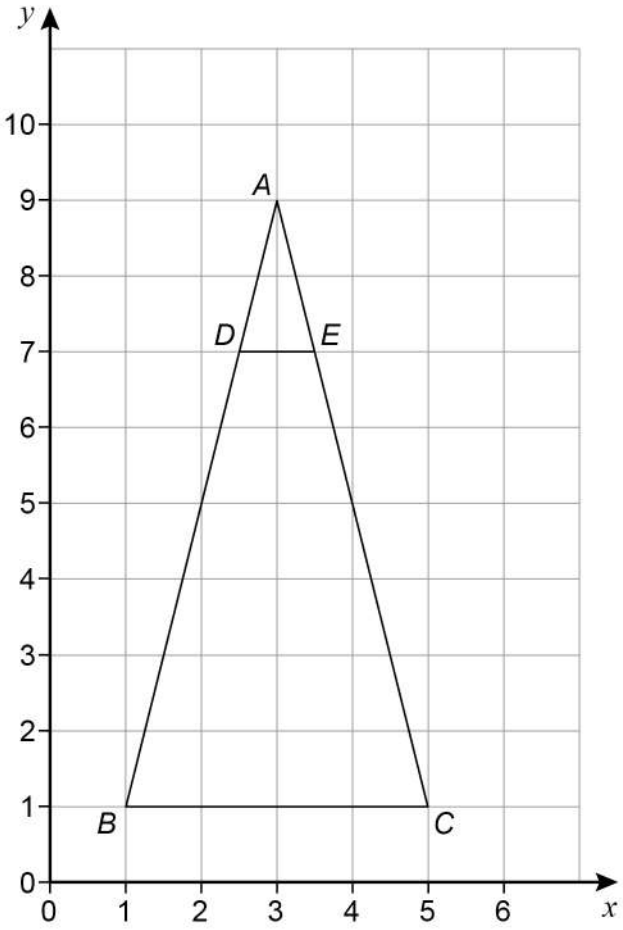
$13141\div187=70.2\dot{7}$ ← Covering m in the formula triangle finds that mean = total ÷ number. So dividing the total mark of all students by the number of students works out the mean for all the students

Yes ← The mean mark of 70.2... is greater than the pass mark of 70

6

Do not write outside the box

8



Describe fully the **single** transformation that maps triangle *ABC* to triangle *ADE*.

[3 marks]

Enlargement by scale factor 1/4 centre (3, 9)

ADE is 1/4 of the size of ABC and it is moving toward (3, 9)

Do not write outside the box

9 A ball contains 5000 cm^3 of air.

More air is pumped into the ball at a rate of 160 cm^3 per second.

The ball is full of air when it becomes a sphere with radius 15 cm

Volume of a sphere $= \frac{4}{3}\pi r^3$ where r is the radius

Does it take **less than** 1 minute to fill the ball?

You **must** show your working.

[4 marks]

$\frac{4}{3}\pi \times 15^3$ ← Substituting the radius of 15 cm into the formula for the volume of a sphere works out that the volume of the ball when full is 4500π cm^3

$4500\pi - 5000$ ← Subtracting the volume of air already in the ball from the volume of the ball when full works out that 9137.1... cm^3 of air needs to be pumped into the ball to make it full

$9137.1... \div 160 = 57.1...$ ← Dividing the volume of air needed to be pumped into the ball to make it full by the 160 cm^3 per second works out that it will take 57.1... seconds to fill it

Yes ← There are 60 seconds in a minute so 57.1... seconds is less than a minute

7

Turn over ►

Do not write outside the box

10 p is a positive number.

n is a negative number.

For each statement, tick the correct box.

[4 marks]

	Always true	Sometimes true	Never true
$p + n$ is positive	☐	✓	☐
$p - n$ is positive	✓	☐	☐
$p^2 + n^2$ is positive	✓	☐	☐
$p^3 \div n^3$ is positive	☐	☐	✓

p + n could be positive as 2 + -1 = 1 but it could also be not positive as 1 + -2 = -1.

p - n is always positive as subtracting a negative is a double negative so it becomes a positive and a positive add a positive must be positive.

p^2 + n^2 must be positive as p^2 and n^2 are positive, as squaring means to multiply by itself and a positive multiplied by positive is positive and a negative multiplied by a negative is double negative so becomes a positive and positive add a positive must be positive.

$p^3 \div n^3$ must always be negative as a positive cubed is positive and a negative cubed is negative and dividing a positive by a negative gives a negative

Do not write outside the box

11 250 trains arrived at a station.

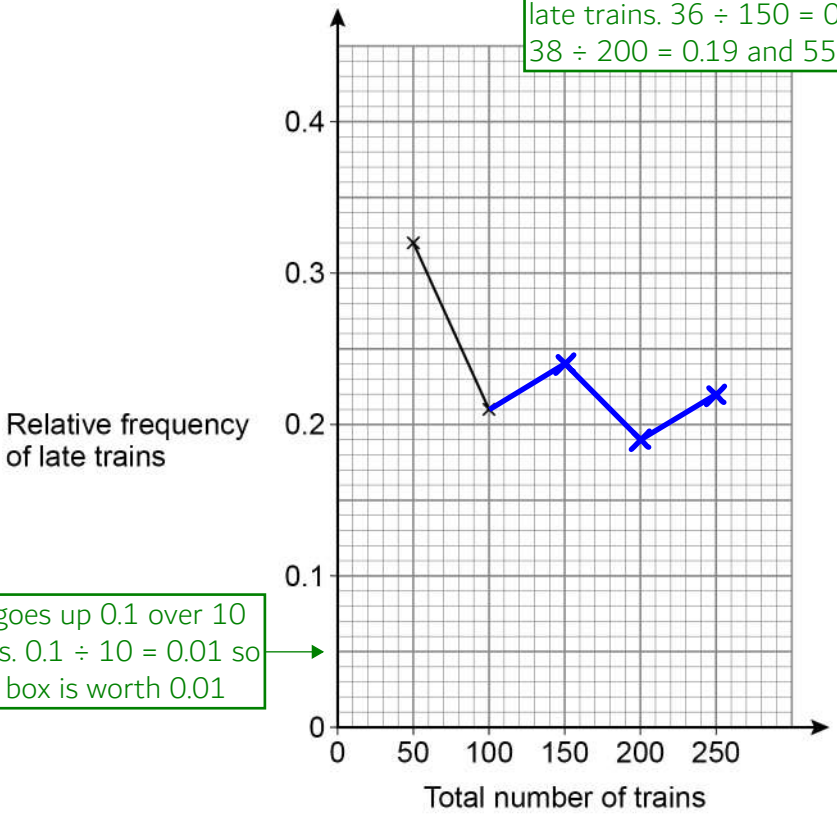
The number of trains that were late was recorded after every 50 trains.

The table shows some information about the results.

Total number of trains	50	100	150	200	250
Total number of late trains	16	21	36	38	55
Relative frequency of late trains	0.32	0.21	0.24	0.19	0.22

11 (a) Complete the relative frequency graph. **[3 marks]**

Dividing the total number of late trains by the total number of trains works out the relative frequency of late trains. 36 ÷ 150 = 0.24 and 38 ÷ 200 = 0.19 and 55 ÷ 250 = 0.22



The scale goes up 0.1 over 10 small boxes. 0.1 ÷ 10 = 0.01 so each small box is worth 0.01

11 (b) Write down the best estimate of the probability that a train arriving at the station is late. **[1 mark]**

Answer 0.22

Each relative frequency is an estimate of the probability but the best one is the one based on the most trains

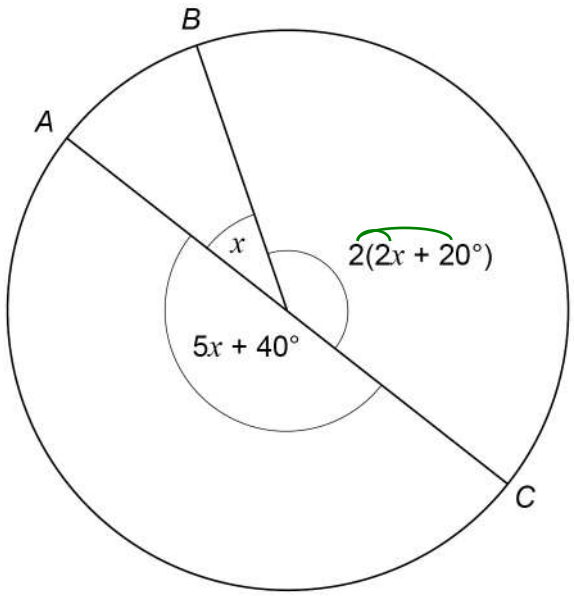
8

Do not write outside the box

12 *A*, *B* and *C* are three points on a circle.

The radii from *A*, *B* and *C* are shown.

Not drawn accurately



Is *AC* a diameter of the circle?

You **must** show your working.

[3 marks]

$x+4x+40$ ← Adding together both of the angles above AC and expanding 2(2x + 20)

$5x+40$ ← Simplifying by collecting like terms

Yes ← The angles above line AC are the same as the angle below it meaning both must be 180° in order to add up to 360°. Therefore AC must be a diameter as the radii from A and C are 180° apart and so form a straight line going through the centre

Do not write outside the box

13 A straight line

has gradient 6

and

passes through the point (3, 19)

Work out the equation of the line.

Give your answer in the form $y = mx + c$

[3 marks]

$19 = 6 \times 3 + C$ ← Substituting 19 for y, 6 for m and 3 for x into the general equation of a straight line y = mx + c, where m is the gradient and c is the y-intercept

$C = 1$ ← Subtracting 6 x 3 from both sides gets c on its own

Answer $y = 6x + 1$

↑ Substituting 6 for m and 1 for c in the general equation of a straight line y = mx + c

Turn over for the next question

6

Do not write outside the box

14 The population of butterflies in a park is 4200

14 (a) Assume that the population increases by 12% each day.

Show that after 20 days the population would be greater than 40 000

[2 marks]

$4200 \times \left(\frac{100+12}{100}\right)^{20} = 40514$

100 + 12 expresses the percentage it rises to each day. Dividing this by 100 converts it into a fraction. Raising it to the power of 20 as it needs to be multiplied by 20 times. Multiplying 4200 by this increases it by 12% 20 times. Rounded to the nearest whole number, it is greater than 40000

14 (b) In fact, the population

increases by 13% each day for 19 days

then

decreases by 8% for 1 day.

After the 20 days, is the actual population greater than 40 000 ?

Tick a box.

☐ Yes ☑ No

Show working to support your answer.

[2 marks]

$4200 \times \left(\frac{100+13}{100}\right)^{19} \times \left(\frac{100-8}{100}\right) = 39403$

Same method as above but using an increase of 13% 19 times and then a decrease of 8% once. Rounded to the nearest whole number, it is less than 40000

Do not write outside the box

14 (c) The expected number of visitors to the park each day depends on the temperature.

Temperature	Expected number of visitors each day
Less than 21°C	700
21°C or more	900

On each of the 30 days in June

the park is open

the probability that the temperature is less than 21°C is 0.4

Work out the **total** number of expected visitors to the park in June.

[3 marks]

$0.4 \times 30 = 12$ ← Multiplying the probability of the temperature being less than 21°C by the 30 days in June works out that 12 of the days in June are expected to have temperature less than 21°C

$30 - 12 = 18$ ← Subtracting the 12 days expected to have temperature less than 21°C from the 30 days in June works out that 18 of the days in June are expected to have temperature of 21°C or more

$12 \times 700 = 8400$ ← Multiplying the 12 days expected for the temperature to be less than 21°C by the 700 visitors expected on each of those days works out that 8400 visitors are expected on the days when the temperature is less than 21°C

18×900 ← Multiplying the 18 days expected for the temperature to be 21°C or more by the 900 visitors expected on each of those days works out that 16200 visitors are expected on the days when the temperature is 21°C or more

$8400 + 16200$ ← Adding the 8400 visitors expected on the days when the temperature is less than 21°C and the 16200 visitors expected on the days when the temperature is 21°C or more works out the total number of expected visitors in June

Answer 24600

7

Do not write outside the box

15 L is directly proportional to D^2

$L = 85$ when $D = 10$

15 (a) Work out an equation connecting L and D.

[3 marks]

$L = kD^2$ ← $L \propto D^2$ will be true whatever D^2 is multiplied by. So multiplying it by k, which represents a value which needs to be found

$k = \frac{85}{10^2}$ ← Rearranged the equation to make k the subject by dividing both sides by D^2. Then substituting L for 85 and D for 10 to work out that k is 0.85

Answer $L = 0.85D^2$

↑ Substituting 0.85 for k in the original equation

15 (b) Work out the value of L when $D = 5$

[2 marks]

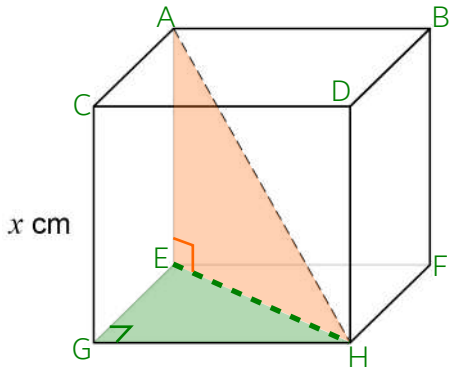
0.85×5^2 ← Substituting 5 for D in the right side of the equation found in part (a)

Answer 21.25

Do not write outside the box

16 Here is a cube with edge length x cm

One diagonal is shown.



16 (a) Circle the length, in centimetres, of the diagonal.

[1 mark]

$\sqrt{3}\,x$ $\sqrt[3]{3x^2}$ $\sqrt{x^3}$ $\sqrt[3]{3}\,x$

See next page

16 (b) The total length, in centimetres, of the edges of the cube is a multiple of 18

Circle the correct statement.

[1 mark]

x is a whole number | x is not a whole number | x might be a whole number

There are 12 edges on the cube so dividing the total length of the edges by 12 would work out x. Not all multiples of 18 are multiples of 12 so dividing the multiple of 18 by 12 might give a decimal. But there are common multiples of 18 and 12 so it could be a whole number when the multiple of 18 is divided by 12

Turn over for the next question

7

$x^2+x^2=c^2$

Using Pythagoras' Theorem in the green right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. All the edges of the cube are x so both EG and GH are x. Substituting x for a and b. EH is c

$c^2=2x^2$

$x^2 + x^2 = 2x^2$

$c=\sqrt{2x^2}$

Square rooting both sides finds c, which is EH

$x^2+(\sqrt{2x^2})^2=d^2$

Using Pythagoras' Theorem in the orange right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. All the edges of the cube are x so AE is x. Substituting x for a and EH for b. AH is c and is labelled as d

$d^2=3x^2$

The square root and square cancel out leaving $2x^2$ then $x^2 + 2x^2 = 3x^2$

Square rooting both sides finds that $d = \sqrt{3}$ x. d is AH, which is the diagonal of the cube

17 20 people were asked which device they used more often, laptop or phone.

The table shows the results.

	Laptop	Phone
Male	2	9
Female	4	5

17 (a) One male and one female are chosen at random.

Work out the probability that **exactly** one of them said laptop.

[3 marks]

$\frac{2}{11} \times \frac{5}{9} + \frac{9}{11} \times \frac{4}{9}$

Male laptop AND female phone OR male phone AND female laptop. AND means to multiply the probabilities. OR means to add the probabilities. There are 11 males in total (2 + 9) and 9 females in total (4 + 5). 2 out of the 11 males have a laptop so the probability of the male having a laptop is 2/11

Answer $\frac{46}{99}$

17 (b) Two males are chosen at random.

Work out the probability that they **both** said phone.

[2 marks]

$\frac{9}{11} \times \frac{8}{10}$

Male phone AND male phone. AND means to multiply the probabilities. 9 out of the 11 males chose phone for the first pick. On the second pick there is one fewer male in total and one fewer who chose phone so there is now 8 out of 10

Answer $\frac{36}{55}$

Do not write outside the box

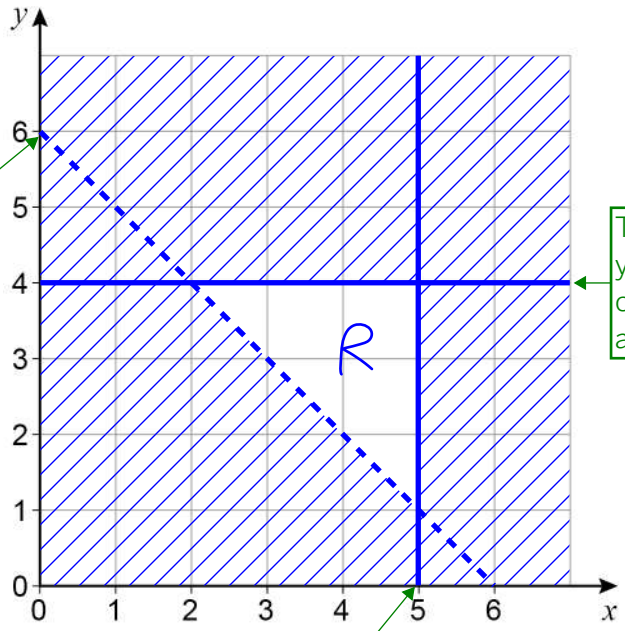
18 On the grid, identify the region represented by

$x \leqslant 5$ $y \leqslant 4$ $x + y > 6$

$y > 6 - x$ ← Rearranged to make y the subject by subtracting x from both sides

Label the region R.

[3 marks]



The line of y = 6 - x. It is dashed as y cannot be equal to 6 - x. Crossing out everything below the line as the region is above the line

The line of y = 4. It is solid as y can be equal to 4. Crossing out everything above the line as the region is below the line

The line of x = 5. It is solid as x can be equal to 5. Crossing out everything to the right of the line as the region is on the left of the line

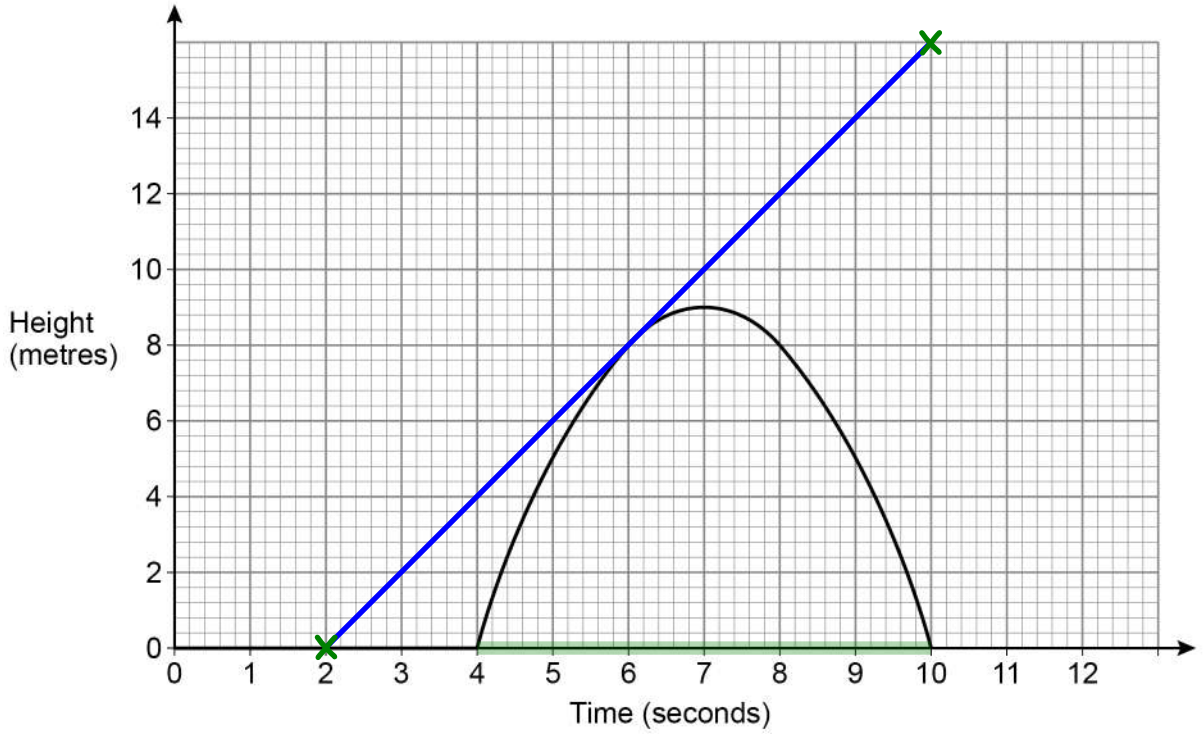
The region R must be where it is not crossed out

Turn over for the next question

8

Do not write outside the box

19 The graph shows the height above ground of a toy rocket for 10 seconds.



19 (a) For how long is the rocket in the air?

Circle your answer.

[1 mark]

10 seconds 9 seconds 6 seconds 4 seconds

The rocket took off at 4 seconds on the time axis and landed at 10 seconds on the time axis. The difference between 4 and 10 is 6 so it must have been in the air for 6 seconds

Do not write outside the box

19 (b) Using the graph, estimate the speed of the rocket after 6 seconds.

State the units of your answer.

[3 marks]

$\frac{16-0}{10-2}$ ← A tangent is drawn at 6 seconds to estimate the gradient of the curve at that point. Gradient = (change in y)/(change in x). Change in y is found by 16 - 0 and change in x is found by 10 - 2 using the points indicated with crosses on the tangent

Answer 2 m/s

↑ The unit is m/s as the change in y in metres was divided by the change in x in seconds

20 A square has an area of 0.25 square metres.

Circle the length, in **centimetres**, of one side of the square.

[1 mark]

0.5 cm 5 cm (50 cm) 500 cm

$\sqrt{0.25}$ ← Area of square = length^2. So square rooting the area finds that the side length is 0.5 m

0.5×100 ← There are 100 cm in 1 m so multiplying the 0.5 m by 100 converts it to 50 cm

Turn over for the next question

5

Do not write outside the box

21 x is an integer.

Prove that $35 + (3x + 1)^2 - 2x(4x - 3)$ is a square number.

[4 marks]

$35+9x^2+6x+1-8x^2+6x$ ← Expanding the square bracket by squaring the first term, doubling the product of the two terms, squaring the last term. Expanding the other bracket

$x^2+12x+36$ ← Collecting like terms

$(x+6)^2$ ← Factorising by finding two numbers which multiply to 36 and add to 12 and putting these in brackets with x. 6 and 6 do this so it would be (x + 6)(x + 6). This can be written as a square bracket and shows that it is a square number

Do not write outside the box

22 Liam is trying to remember a 3-digit code.

He knows the rule that

the first digit is a cube number

the second digit is a factor of 16

the third digit is an odd number.

Liam tries at random a code that matches the rule.

Work out the probability that this is the correct code.

[4 marks]

1,8 ← Listing out the single digit cube numbers

1,2,8,4 ← Listing out the single digit factors of 16

1,3,5,7,9 ← Listing out the single digit odd numbers

2X4X5 ← Using the product rule for counting. There are 2 possibilities for the first digit, 4 for the second and 5 for the third. Multiplying these numbers together works out that there are 40 possible codes

Answer $\frac{1}{40}$

↑ 1 out of the 40 possible codes is correct

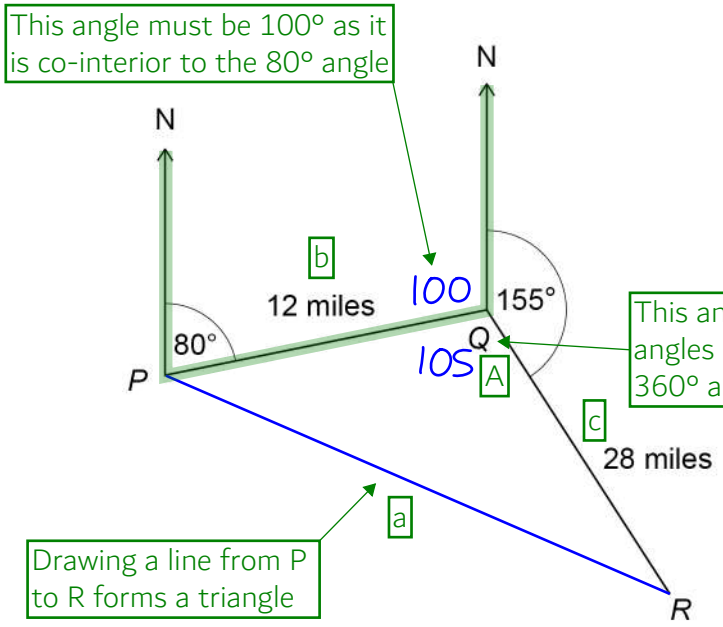
8

Do not write outside the box

23 A ship sails from *P* to *Q* and then from *Q* to *R*.

Q is 12 miles from *P*, on a bearing of 080°

R is 28 miles from *Q*, on a bearing of 155°



Work out the direct distance from *P* to *R*.

[4 marks]

$PR^2 = 12^2 + 28^2 - 2 \times 12 \times 28 \times \cos 105$

It is not a right angled triangle so Pythagoras' Theorem can't be used. There aren't two pairs of opposite sides and angles so the sine rule can't be used. Therefore the cosine rule needs to be used. $a^2 = b^2 + c^2 - 2bc\cos A$. Substituting PR for a, 12 for b, 28 for c and 105 for A

$PR = \sqrt{1101.9...}$ ← Square rooting both sides finds PR

Answer 33.2 miles

Do not write outside the box

24 The flight of a plane was in two stages.

The table shows information about the flight.

	Distance (miles)	Speed (mph)	Time (hours)
1st stage	731	x	$\frac{731}{x}$
2nd stage	287	$x - 24$	$\frac{287}{x - 24}$

In total, the flight lasted 2 hours.

Work out the value of x.

[5 marks]

$\frac{731}{x} + \frac{287}{x-24} = 2$ ← The distances and speeds are irrelevant so can be ignored. Adding the times must give 2

$731(x-24) + 287x = 2x(x-24)$ ← Multiplying both sides by the denominators to cancel them out

$731x - 17544 + 287x = 2x^2 - 48x$ ← Expanding the brackets

$2x^2 - 1066x + 17544 = 0$ ← Bringing into the quadratic form $ax^2 + bx + c = 0$

$x = \frac{-(-1066) \pm \sqrt{(-1066)^2 - 4 \times 2 \times 17544}}{2 \times 2}$ ← $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← Solving using the quadratic formula

Answer 516

The other solution of x is 17 but is ignored as this would give a negative time for the 2nd stage

9

Do not write outside the box

25 The equation of a curve is $y = x^2 + 14x + 52$

By completing the square, work out the coordinates of the turning point.

You **must** show your working.

[3 marks]

$y=(x+7)^2+52-7^2$

Completing the square by halving the coefficient of x, putting this in a bracket with x and squaring it. Subtracting 7^2 as when expanding the square bracket this will be in addition to the $x^2 + 14x$ and if it wasn't subtracted this would change the value of the right side

Answer (-7 , 3)

The turning point occurs when the square bracket is equal to 0 as this is the smallest a squared number can be. x = -7 for this to happen. When the bracket is 0, y = $52 - 7^2 = 3$

END OF QUESTIONS

3