Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

I declare this is my own work.

GCSE MATHEMATICS

H

Higher Tier   Paper 3 Calculator

Monday 13 November 2023   Morning   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The first four terms of a linear sequence are

6 13 20 27

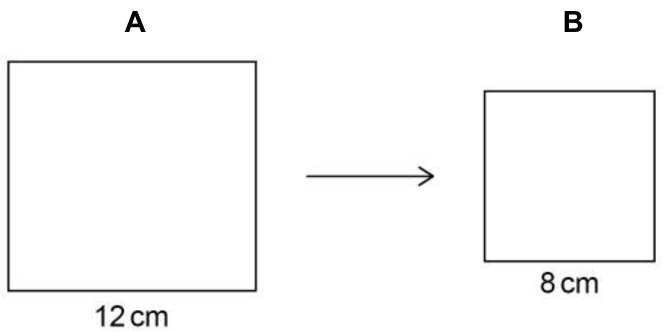
Write down the expression for the nth term.

[1 mark]

It increases by 7 between each term so must involve 7n. Going backward in the sequence finds that the 0th term is -1. So the expression for the nth term is 7n - 1

Answer $7n-1$

2 Square A is enlarged to square B.



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Write down the scale factor of the enlargement as a fraction.

[1 mark]

Answer $\frac{8}{12}$

8 is 8/12 of 12

Do not write outside the box

3 The length of a line is 8 cm to the nearest centimetre.

Complete the error interval.

[2 marks]

$8 \pm \frac{1}{2}$ ← Adding and subtracting half of the resolution works out the upper and lower bounds of the measurement. The resolution is 1 cm as it is to the nearest 1 cm. Putting this over 2 halves it. Adding and subtracting it from the 8

Answer 7.5 cm ⩽ length < 8.5 cm

4 At what point does the graph $y = x^3 - 1$ cross the y axis?

[1 mark]

All points on the y-axis have an x-coordinate of 0. When x = 0, y = 0^3 - 1 = -1

Answer (0 , -1)

x-coordinate   y-coordinate

Turn over for the next question

5

Do not write outside the box

5 Carly's total annual pay = salary + bonus

	Salary	Bonus
Last year	£26 000 +	£4000 = 30000
This year	6% increase	9% decrease

Work out the percentage change in her total annual pay.

State whether it is an increase or a decrease.

[4 marks]

Adding the salary and bonus for last year works out that the total annual pay for last year was £30000

$\frac{6}{100} \times 26000 = 1560$

Putting the 6% over 100 converts it into a fraction which when multiplied by the £26000 works out that 6% of £26000 is £1560

$\frac{9}{100} \times 4000 = 360$

Putting the 9% over 100 converts it into a fraction which when multiplied by the £4000 works out that 9% of £4000 is £360

$\frac{1560 - 360}{30000} \times 100$

Expressing the change in her total annual pay by subtracting the £360 decrease from the £1560 increase. Putting this over the original salary expresses the change as a fraction. Multiplying this by 100 converts the fraction to a percentage

Answer 4% increase

It is an increase as the percentage change is positive

Do not write outside the box

6 An exhibition

was open for 240 hours

and

had 29 760 visitors.

For $\frac{2}{5}$ of the time the exhibition was open, there were 172 visitors per hour.

For the remaining time, how many visitors per hour were there?

[4 marks]

$\frac{2}{5} \times 240$ ← This works out that 2/5 of the 240 hours was 96 hours

$240 - 96 = 144$ ← Subtracting the 96 hours from the total 240 hours works out that the remaining time is 144 hours

96×172 ← Multiplying the 96 hours by the 172 visitors per hour works out that there were 16512 visitors during this time

$29760 - 16512$ ← Subtracting the 16512 visitors from the total 29760 visitors works out that there were 13248 visitors during the remaining time

$13248 \div 144$ ← Dividing the 13248 visitors by the 144 remaining hours works out that there were 92 visitors per hour during this time

Answer 92

7 The first two cube numbers are 1 and 8

Show that

the 3rd cube number can be written as the sum of three different prime numbers.

[3 marks]

27 = 3 + 5 + 19

See next page for an explanation

11

$3^3 = 27$, so this is the 3rd cube number. It is best to systematically go through the prime numbers starting with the smallest. The first prime number is 2, however all of the other prime numbers are odd and even + odd + odd will give an even number. 27 is odd. So 2 cannot be one of the prime numbers. The next two prime numbers are 3 and 5. Subtracting these from 27 works out that 19 must be added to these to get 27. 19 is a prime number as it is only divisible by itself and 1. So the three different prime numbers could be 3, 5 and 19

Do not write outside the box

8 Circle the largest number.

[1 mark]

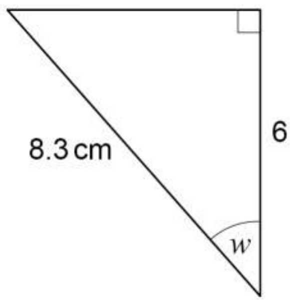
$5.30\dot{4}$ 5.344 5.34 $5.3\dot{4}$

5.3044
5.3440
5.3400
5.3444

Writing all the numbers to 4 decimal places makes it easier to spot the largest number

9 Use trigonometry to work out the size of angle w.

[3 marks]



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S O H C A H T O A

Writing SOH CAH TOA as formula triangles. Ticking O and H as we have the opposite and hypotenuse

$\cos w = \frac{6.7}{8.3}$

There are two ticks on the CAH formula triangle so this one can be used. Covering C finds that cos of the angle = adjacent/hypotenuse

$w = \cos^{-1}\left(\frac{6.7}{8.3}\right)$

Doing the inverse cos of both sides cancels out the cos on the left and finds w

$w =$ 36.2 °

Do not write outside the box

10 Two bags contain only green discs and yellow discs.

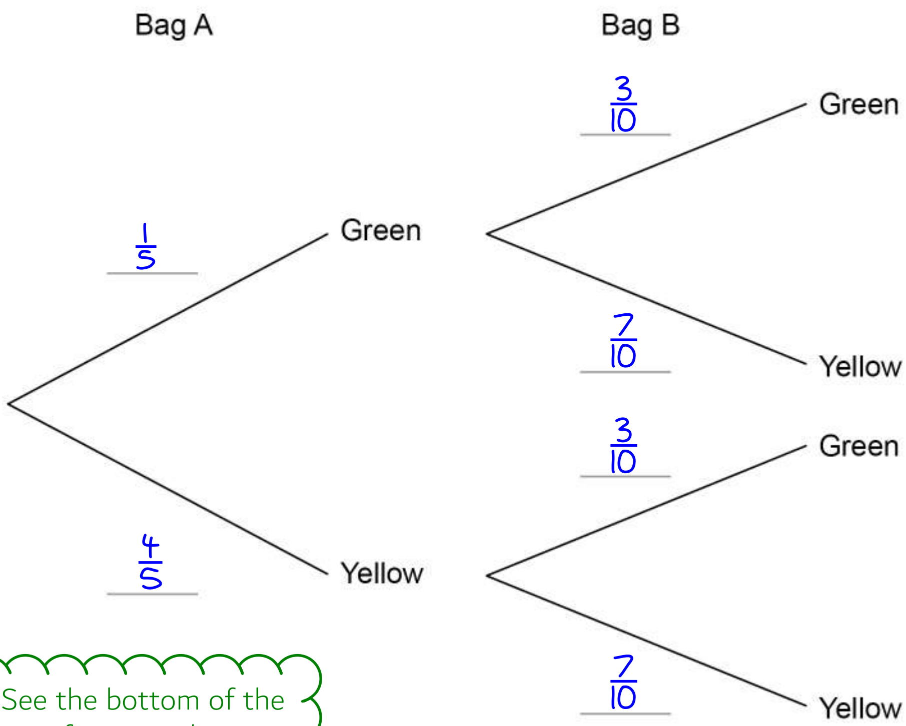
Bag A contains 1 green disc and 4 yellow discs.

Bag B contains 3 green discs and 7 yellow discs.

One disc is picked at random from each bag.

10 (a) Complete the tree diagram with the correct probabilities.

[2 marks]



See the bottom of the page for an explanation

10 (b) Work out the probability that **both** discs are green.

[2 marks]

$\frac{1}{5} \times \frac{3}{10}$ ← Green AND green. AND means to multiply the probabilities

Answer $\frac{3}{50}$

1 + 4 = 5, so there are 5 discs in total in Bag A. 1 out of these 5 is green and 4 out of these 5 are yellow. 3 + 7 = 10, so there are 10 discs in total in Bag B. 3 out of these 10 are green and 7 out of these 10 are yellow

8

Turn over ▶

Do not write outside the box

11 Solve these simultaneous equations.

$7x + 2y = 100$ ← First equation

$3x + 2y = 48$ ← Second equation

[3 marks]

$4x = 52$ ← The number of y is the same in both equations. Subtracting the second equation from the first equation cancels out the y terms. 7x - 3x = 4x. 2y - 2y = 0. 100 - 48 = 52

Dividing both sides by 4 cancels out the 4 on the left and finds that x = 13

$7 \times 13 + 2y = 100$ ← Substituting 13 for x in the first equation

$2y = 9$ ← Subtracting 7 x 13 from both sides to get the y term on its own

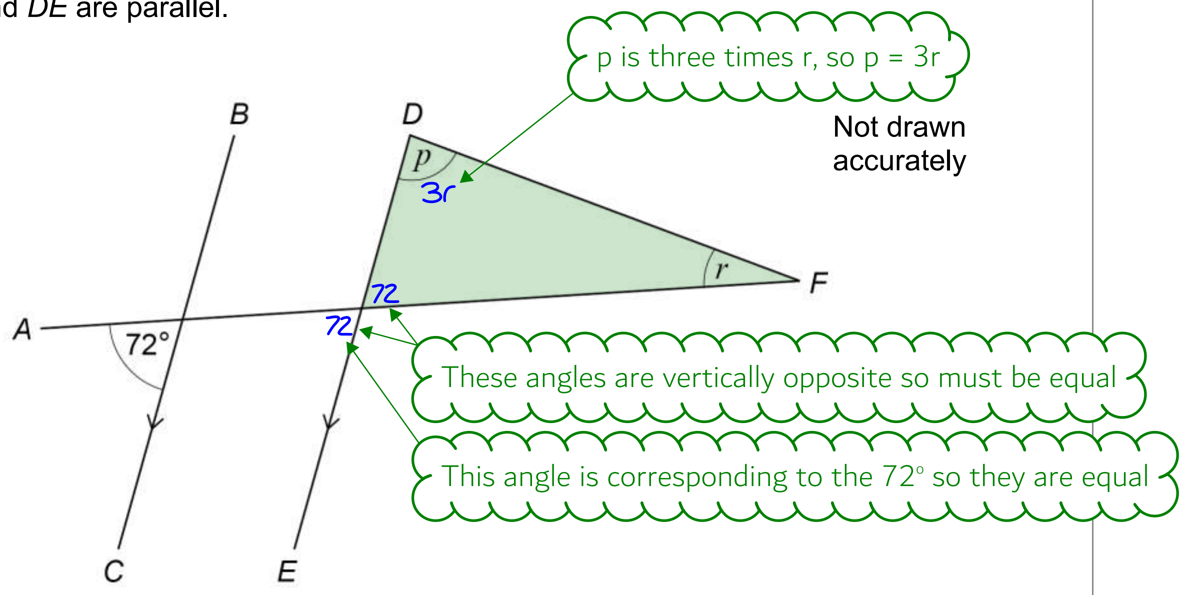
Dividing both sides by 2 cancels out the 2 on the left and finds that y = 9/2

$x =$ 13 $y =$ $\frac{9}{2}$

Do not write outside the box

12 *AF*, *BC*, *DE* and *DF* are straight lines.

BC and *DE* are parallel.



p is three times r.

Work out the size of angle p.

[3 marks]

72+4r=180 ← Adding all of the angles in the highlighted triangle gives 72 + 4r, which must equal to 180 as there are 180 degrees in total in a triangle

4r=108 ← Subtracting 72 from both sides gets the r term on its own

r=27 ← Dividing both sides by 4 gets r on its own

3×27 ← Multiplying the value of r by 3 finds p

$p =$ 81 °

6

Turn over ►

.CG Maths.

13 100 people were asked about the distance they travel from home to work.

The table shows information about the results.

Distance, d (miles)	**Frequency**
$0 \leqslant d < 5$	21
$5 \leqslant d < 10$	24
$10 \leqslant d < 20$	37
$20 \leqslant d < 40$	18

13 (a) Write down the **greatest** possible number of people who work from home.

[1 mark]

Answer 21

The distance they travel from home to work is 0 miles if they work from home. All of the people in the $0 \leqslant d < 5$ interval could have travelled a distance of 0 miles

13 (b) One person is chosen at random.

Work out the probability that the person travels **at least** 10 miles.

[1 mark]

37+18

The people in the $10 \leqslant d < 20$ and $20 \leqslant d < 40$ intervals travelled at least 10 miles. Adding the frequencies works out that 55 people travelled at least 10 miles

Answer $\frac{55}{100}$

55 out of the 100 people travelled at least 10 miles

Do not write outside the box

13 (c) The table is repeated.

Distance, d (miles)	Frequency
$0 \leqslant d < 5$	21
$5 \leqslant d < 10$	24
$10 \leqslant d < 20$	37
$20 \leqslant d < 40$	18

From 0 to 5 is a width of 5

4.2 ← 21 ÷ 5 = 4.2

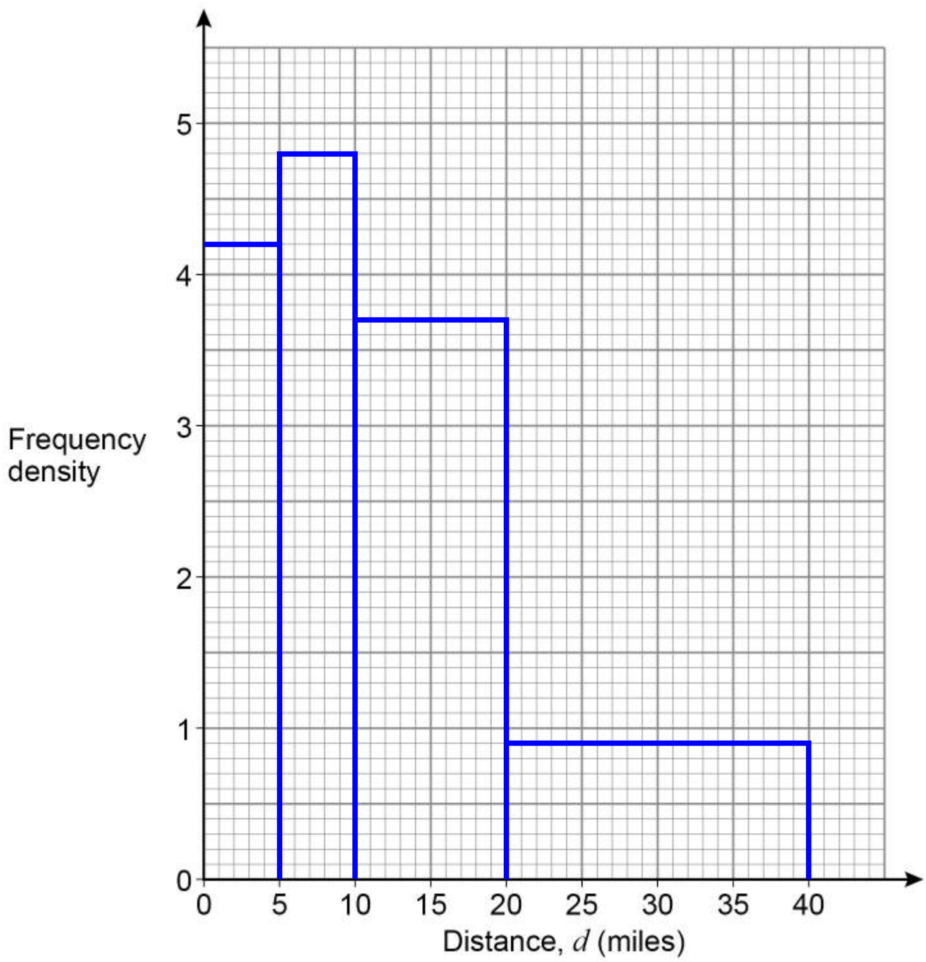
4.8 ← 24 ÷ 5 = 4.8

3.7 ← 37 ÷ 10 = 3.7

0.9 ← 18 ÷ 20 = 0.9

Draw a histogram to represent the results.

[3 marks]



It is the area of each box which gives the frequencies on a histogram. Area of rectangle = base x height, so height = area ÷ base. So dividing the frequencies by the class widths (how wide the bar would be on the histogram) gives the frequency densities

5

Turn over ▶

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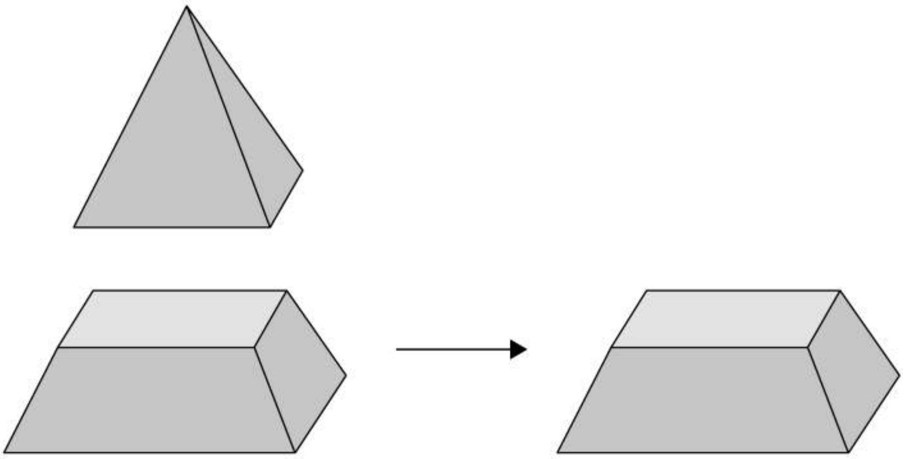
14 A solid trophy consists of a stand and a player.

Trophy

The stand is made by removing a small pyramid from a large pyramid.

Large pyramid	Square base, edge 8 cm	Perpendicular height 16 cm
Small pyramid	Square base, edge 5 cm	Perpendicular height 10 cm



Volume of a pyramid $= \frac{1}{3} \times$ area of base $\times$ perpendicular height

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14 (a) Show that the volume of the **stand** is 258 cm^3

[2 marks]

$\frac{1}{3} \times 8^2 \times 16 - \frac{1}{3} \times 5^2 \times 10 = 258$

Subtracting the volume of the small pyramid from the volume of the large pyramid leaves the volume of the stand

Volume of the small pyramid is expressed using the formula. The area of the base is 5^2 as the area of a square = $length^2$

Volume of the large pyramid is expressed using the formula. The area of the base is 8^2 as the area of a square = $length^2$

14 (b) The trophy is made from a metal of density 7.5 grams per cm^3

The **total** mass of the trophy is 2340 grams.

Work out the volume of the **player**.

[2 marks]

d m v

Writing the density, mass, volume formula triangle

$2340 \div 7.5$

Covering v finds that volume = mass ÷ density. So the volume of the whole trophy is 312 cm^3

$312 - 258$

Subtracting the volume of the stand from the volume of the trophy leaves the volume of the player

Answer 54 cm^3

4

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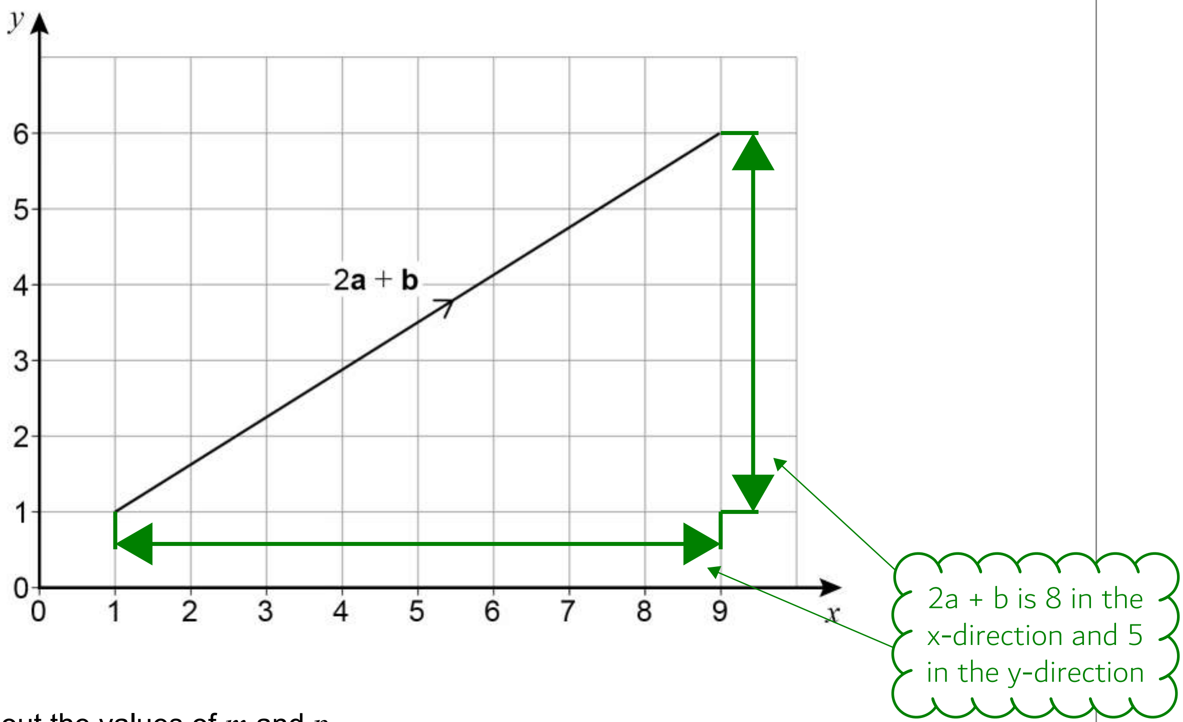
15 $\mathbf{a} = \begin{pmatrix} m \\ 3 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -4 \\ p \end{pmatrix}$

x-component

y-component

The diagram shows the vector $2\mathbf{a} + \mathbf{b}$



Work out the values of m and p.

[4 marks]

$2m-4=8$

Expressing the x-component of 2a + b by substituting in the x-component of a and the x-component of b. This must be equal to the 8 in the x-direction

$2m=12$

Adding 4 to both sides to get the m term on its own

Dividing both sides by 2 finds that m = 6

$2\times3+p=5$

Expressing the y-component of 2a + b by substituting in the y-component of a and the y-component of b. This must be equal to the 5 in the 5-direction

Subtracting 2 x 3 from both sides finds that p = -1

$m =$ 6 $p =$ -1

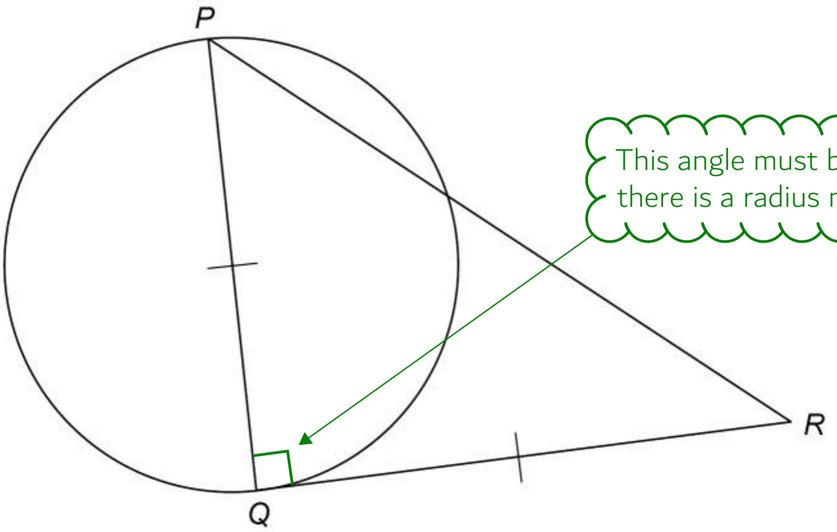
Do not write outside the box

16 *PQ* is a diameter of a circle.

QR is a tangent to the circle.

$PQ = QR$

$PR = 10$ cm



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Work out the **radius** of the circle.

Give your answer as a decimal.

[3 marks]

$d^2 + d^2$ ← Let d be the diameter PQ. QR is also d. Pythagoras' Theorem can be used in the right-angled triangle PQR. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. So a and b are both d. Substituting these into the left side of the Theorem

$2d^2 = 10^2$ ← Simplifying the left and substituting 10 for c in the right side

$d^2 = 50$ ← Dividing both sides by 2

$d = \sqrt{50}$ ← Square rooting both sides

$\sqrt{50} \div 2$ ← The radius is half of the diameter

Answer 3.5 cm

7

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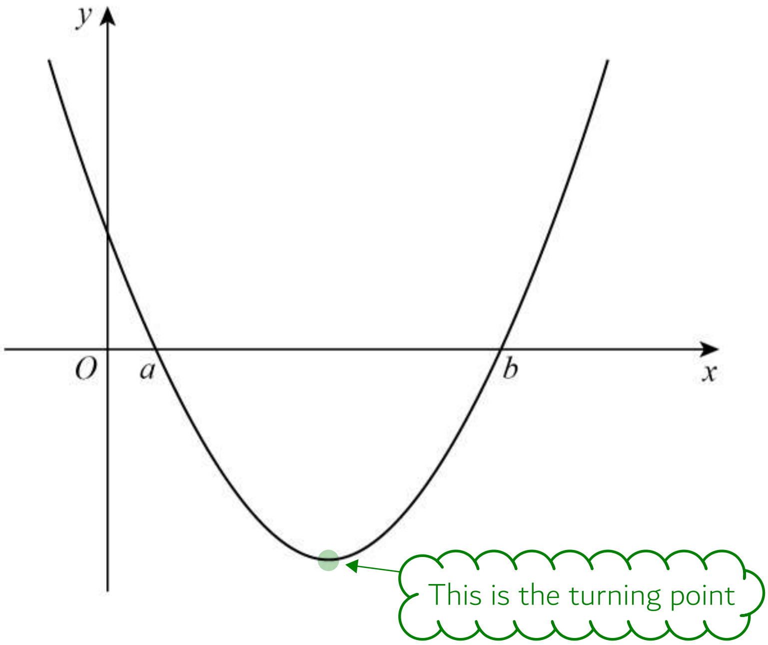
17 Here is a sketch of the quadratic graph $y = f(x)$

The graph crosses the x-axis at $x = a$ and $x = b$



Write an expression for the x-coordinate of the turning point.

[1 mark]

Answer $\frac{a+b}{2}$

Quadratic graphs are symmetrical so the x-coordinate of the turning point is halfway between a and b. Doing the mean of a and b expresses halfway between a and b. Mean = total/number so adding them together expresses the total then putting them over 2 as there are 2 values

Do not write outside the box

18 Simplify $\dfrac{2(x+4)^5}{(x+4)^3}$

Give your answer in the form $ax^2 + bx + c$ where a, b and c are integers.

[3 marks]

$2(x+4)^2$ ← Let A = x + 4. $2A^5/A^3 = 2A^2$

$2(x^2+8x+16)$ ← Expanding the square bracket by squaring the first term, doubling the product of the two terms, squaring the last term

Answer $2x^2+16x+32$

Expanding the new bracket with the 2

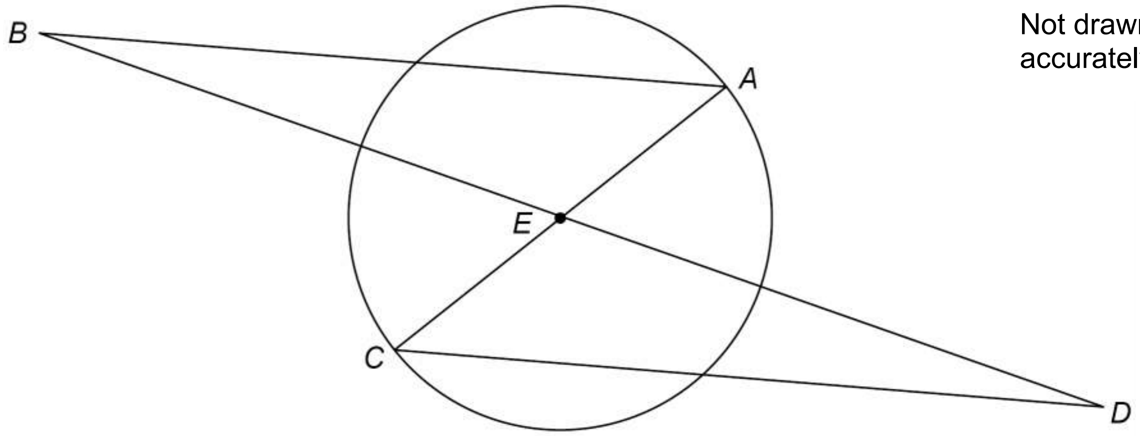
Turn over for the next question

4

Do not write outside the box

19 *AC* is a diameter of a circle, centre *E*.

E is the midpoint of *BD*.



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Prove that triangle *ABE* is congruent to triangle *CDE*.

[4 marks]

BE = ED as E is the midpoint of BD

AE = EC as they are both radii ← A radius is a straight line from the centre of the circle (E) to the outside and they are all the same length

Angles AEB = CED as they are vertically opposite

SAS ← Side-angle-side. Two sides and the angle between them are the same in both triangles so they are congruent (the same shape and size)

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20 Solve $2x(x+10) = 5x - 18$

[4 marks]

$2x^2 + 20x = 5x - 18$ ← Expanding the bracket on the left

$2x^2 + 15x + 18 = 0$ ← Putting in the quadratic from $ax^2 + bx + c = 0$ by subtracting 5x from both sides and adding 18 to both sides

$x = \frac{-15 \pm \sqrt{15^2 - 4 \times 2 \times 18}}{2 \times 2}$ ← $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← Solving using the quadratic formula. a = 2, b = 15, c = 18

Answer -1.5 , -6

Turn over for the next question

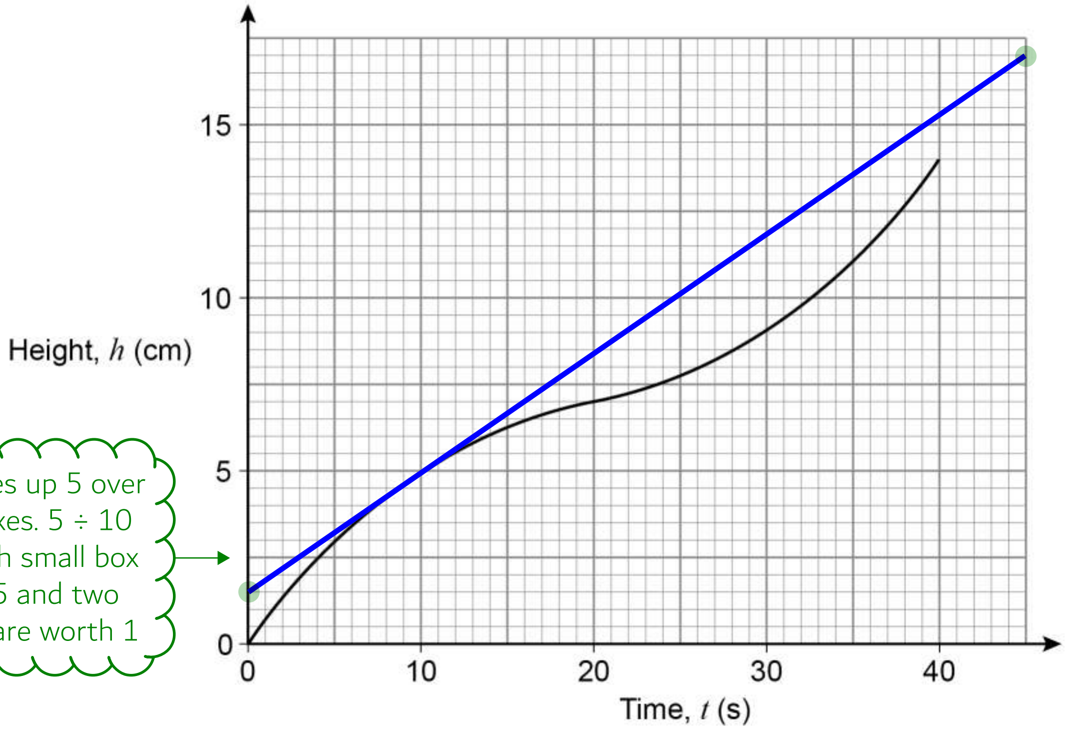
8

Do not write outside the box

21 Water flows from a tap at a constant rate.

A container is filled with water from the tap in 40 seconds.

The graph shows the height, h centimetres, of the water after time, t seconds.

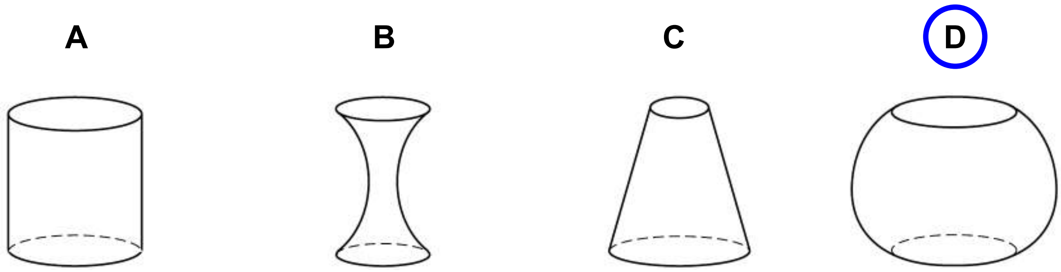


The scale goes up 5 over 10 small boxes. 5 ÷ 10 = 0.5, so each small box is worth 0.5 and two small boxes are worth 1

21 (a) The container is one of these shapes.

Circle the letter of the correct shape.

[1 mark]



The gradient of the curve represents the rate in which the height of the water increases. It must be D as the rate decreases then increases again, which would happen as the shape gets wider then narrower when going up the shape

Do not write outside the box

21 (b) By drawing a tangent on the graph,

estimate the rate at which the height is increasing when $t = 10$

[2 marks]

$\frac{17-1.5}{45-0}$ ← Gradient = (change in y)/(change in x). Picking the two end points of the tangent. Change in y = 17 - 1.5 and change in x = 45 - 0

Answer 0.34 cm/s

22 Write $\frac{7}{2a^2} - \frac{3}{5a}$ as a single fraction in its simplest form.

[2 marks]

$\frac{35}{10a^2} - \frac{6a}{10a^2}$ ← To subtract fractions the denominators need to be the same. Multiplying both the numerator and denominator of the first fraction by 5 and both the numerator and denominator of the second fraction by 2a makes both the denominators the same

Answer $\frac{35-6a}{10a^2}$

Once the denominators are the same, the numerators can be subtracted

5

Turn over ▶

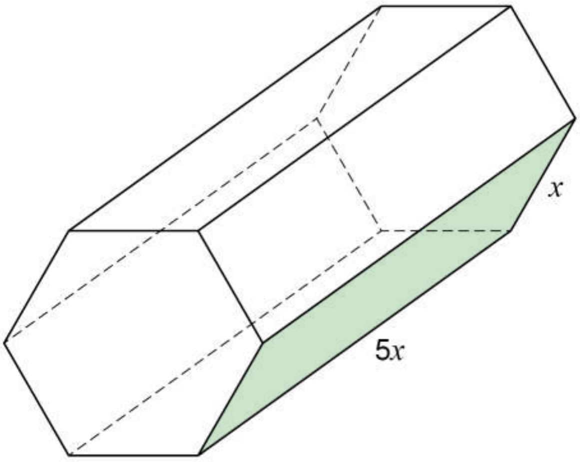
Do not write outside the box

23 A chocolate box in the shape of a prism is being designed.

All lengths are in centimetres.

The cross section is a regular hexagon with side x

The length is $5x$



An expression for the area of the cross section, in cm^2, is $\frac{3\sqrt{3}}{2}x^2$

The **total** surface area of the box must be less than 650 cm^2

Work out the largest possible **integer** value of x.

You **must** show your working.

[4 marks]

$5x^2 \times 6 = 30x^2$ ← Area of rectangle = length x width. So the area of one of the rectangular faces (highlighed in green) must be $5x \times x = 5x^2$. Multiplying this by 6 as there are 6 of these rectangular faces

$\frac{3\sqrt{3}}{2}x^2 \times 2$ ← Multiplying the area of one of the hexagons (which is the shape of the cross section) by 2 as there are 2 of these hexagons on the surface

$(3\sqrt{3}+30)x^2 < 650$ ← Adding the coefficients of the x^2 and leaving the x^2 outside of the bracket expresses the surface are of the shape in terms of x. This needs to be less than 650 cm^2

$x^2 < 18.4...$ ← Dividing both sides by the bracket

$x < 4.2...$ ← Square rooting both sides

Answer 4

4 is the largest integer less than 4.2...

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24 Work out the area of triangle *ABC*.

[4 marks]

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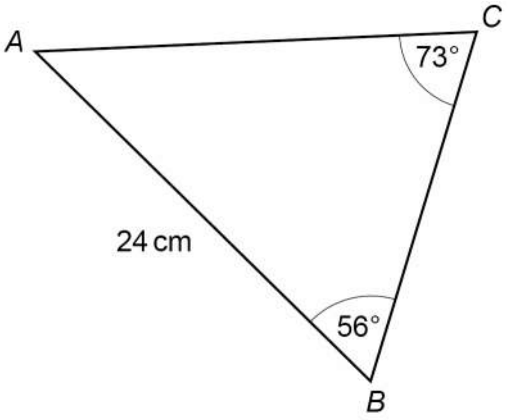

$\frac{AC}{\sin 56} = \frac{24}{\sin 73}$ ← Using the sine rule as there are opposite pairs of sides and angles. a/sinA = b/sinB. Substituting AC for a, 56 for A, 24 for b, 73 for B

AC=20.8…Ⓐ ← Multiplying both sides by sin56 finds AC. Storing the exact value as A on the calculator

180−56−73 ← There are 180 degrees in a triangle. So subtracting the other angles from 180 finds that angle BAC is 51 degrees

$\frac{1}{2} \times 20.8\ldots \times 24 \times \sin 51$ ← Area of triangle = 1/2 absinC, where a and b are two sides and C is the angle between them. Using the exact stored value of A on the calculator for 20.8...

Answer 194.0 cm^2

8

Do not write outside the box

25 a is three quarters of c

$6b = 5c$

Work out the ratio $a : b : c$

Give your answer in its simplest form, where a, b and c are integers.

[3 marks]

$\frac{9}{2}:5:6$ ← b could be 5 while c is 6 as 6 x 5 = 5 x 6. If c is 6, a is 3/4 x 6 = 9/2

Answer 9 : 10 : 12

Multiplying all sides of the ratio by 2 eliminates the fractions and cannot be simplified further as 9, 10 and 12 cannot be divided by the same amount to get smaller whole numbers

Do not write outside the box

26 In a game, these numbered tiles are in a bag.

5	5	5	5
3	3	2	1

To play the game

Choose tiles at random one at a time and do not replace the tiles.

You win if at any stage the total of the numbers on your tiles is 10

Amber plays the game once.

Work out the probability that she wins.

[4 marks]

5&5 or 5&3&2

Listing out the possible numbers on the tiles which will result in a win. 5 & 2 & 2 & 1 is not possible as there is only one 2 and the tiles are not replaced

$\frac{4}{8}\times\frac{3}{7} + \frac{4}{8}\times\frac{2}{7}\times\frac{1}{6}\times3\times2\times1$

AND means to multiply the probabilities. OR means to add the probabilities. The probability of the first tile being 5 is is 4/8 as 4 out of the 8 tiles are the number 5. The probability of the second tile being 5 is 3/7 as there is 1 fewer 5 and 1 fewer tile in total. The probabilities for 5, 3, and 2 are multiplied. However there are multiple combinations which can result in getting these numbers (for example: 5, 3, 2 or 5, 2, 3). Using the product rule for counting expresses how many different combinations there are. There are 3 possibilities for the first tile, then 2 for the second, then 1 for the third. Multiplying the probability of 5 & 3 & 2 by 3 x 2 x 1 therefore works out the probability of all the combinations as the numerators will be the same and denominators will be the same for each combination. Multiplying the numerators in different orders will give the same result

Answer $\frac{5}{14}$

7

Do not write outside the box

27 (a) The graph of $y = x^3$ is translated to the graph of $y = (x - 2)^3$

Write down the translation vector.

[1 mark]

Answer $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Subtracting 2 for x moves the graph +2 in the x-direction as it gets to the same value 2 later

27 (b) The graph of $y = 5x + 4$ is reflected in the y-axis.

Write down the equation of the reflected graph.

[1 mark]

Answer $y=5(-x)+4$

Flipping the sign on all the x reflects the graph in the y-axis

END OF QUESTIONS

2