OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Wednesday 6 November 2024 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

H

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined page at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

DC (PQ/SW) 341298/4

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Factorise fully.

(a) $6x^2 + 9x$

3x is the highest common factor of $6x^2$ and 9x. Bringing 3x out as a factor, dividing both terms by 3x and leaving the result in a bracket

(a) $3x(2x + 3)$ **[2]**

(b) $x^2 + 8x + 15$

Two numbers which add to the 8 and multiply to the 15 are 5 and 3. Putting these in brackets with x

(b) $(x + 5)(x + 3)$ **[2]**

2 You may use these kinematics formulae to answer these questions.

$v = u + at$

$v^2 = u^2 + 2as$

A moving particle accelerates at 2 m/s^2 for 8 seconds.
The particle's final velocity after the 8 seconds is 21 m/s.

(a) Show that the velocity of the particle at the start of the 8 seconds is 5 m/s. **[2]**

$21 = u + 2 \times 8$ ← Substituting 21 for v (final velocity), 2 for a (acceleration) and 8 for t (time) in the formula v = u + at

$21 - 2 \times 8 = u$ ← Subtracting 2 × 8 from both sides gets u on its own

$5 = u$ ← u is the initial velocity

(b) Work out the distance travelled by the particle during the 8 seconds.

$21^2 = 5^2 + 2 \times 2 \times s$ ← Substituting 21 for v (final velocity), 5 for u (initial velocity) and 2 for a (acceleration) into the formula $v^2 = u^2 + 2as$

$416 = 2 \times 2 \times s$ ← Subtracting 5^2 from both sides

Dividing both sides by 2 × 2 gets s on its own, which is the distance travelled

(b) 104 m **[3]**

3 **(a)** *N* is a number such that:

- $N = 3 \times 5 \times k$, where *k* is a prime number
- *N* is greater than 400.

Find the smallest possible value of *N*.

$3 \times 5 \times k > 400$ ← Setting the expression for N greater than 400

$k > 26.6...$ ← Dividing both sides by 3 × 5 gets k on its own

$k = 29$ ← 29 is the smallest prime number greater than 26.6... Prime numbers are only divisible by themselves and 1. Expressing a number as a product of prime factors on the calculator can check if it is prime as it will not change

$3 \times 5 \times 29$ ← Putting the value of k back into the expression of N

(a) $N =$ ....435........ **[3]**

(b) *a* and *b* are different prime numbers.

Explain why $a \times b$ is not a prime number.

It is divisible by a ....

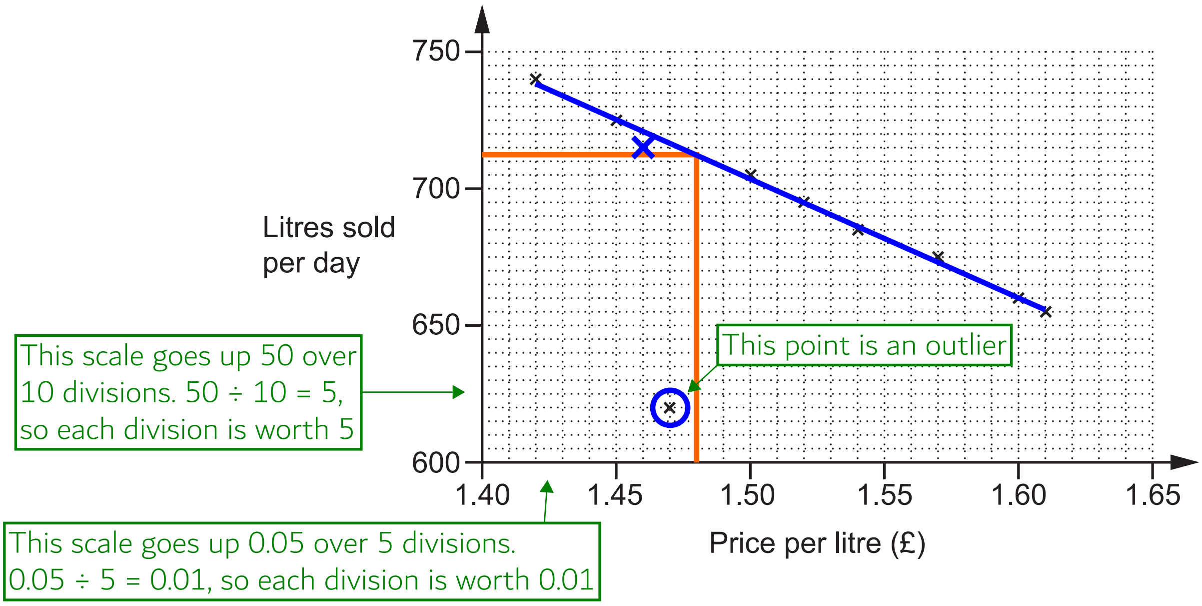
Prime numbers are only divisible by themselves and 1. As 1 is not prime, a and b are not 1. So multiplying a by b gives a number which is not a or b and is divisible by a and b, which are not 1 ... **[1]**

4 Each week the manager of a petrol station records the average daily sales, in litres, and the average price, in pounds, of a litre of petrol for that week.

The table shows their results for ten weeks.

Week	1	2	3	4	5	6	7	8	9	10
Price per litre (£)	1.42	1.45	1.47	1.50	1.54	1.60	1.57	1.52	1.61	1.46
Litres sold per day	740	725	620	705	685	660	675	695	655	715

The results for the first nine weeks are plotted on the scatter diagram.



(a) Plot the result for week 10. **[1]**

(b) Describe the type of correlation shown in the scatter diagram.

(b) ....Negative.......... **[1]**

(c) In one week, there was a delay with petrol deliveries.

Circle the most likely point on the scatter diagram for that week. **[1]**

(d) **(i)** On the scatter diagram, draw a line of best fit. **[1]**

(ii) Use the line of best fit to estimate the average daily sales when the price per litre of petrol is £1.48.

Reading up from £1.48 to the line of best fit then across

(d)(ii) ....712.5.......... litres **[1]**

(e) The manager says,

As the sales go down, the total amount of money we take stays roughly the same.

Find evidence to support this statement.

$1.42 \times 740 = 1050.80$ ← Selecting the day with the greatest sales. Multiplying the price per litre by the litres sold that day works out that the money taken is £1050.80

$1.61 \times 655 = 1054.55$ ← Selecting the day with the least sales (ignoring the outlier). Multiplying the price per litre by the litres sold that day works out that the money taken is £1054.55

£1050.80 is roughly the same as £1054.55

....

.... **[3]**

5 A person invests £8000 at a rate of 5% per year compound interest.

Calculate the total amount of **interest** earned after 3 years.

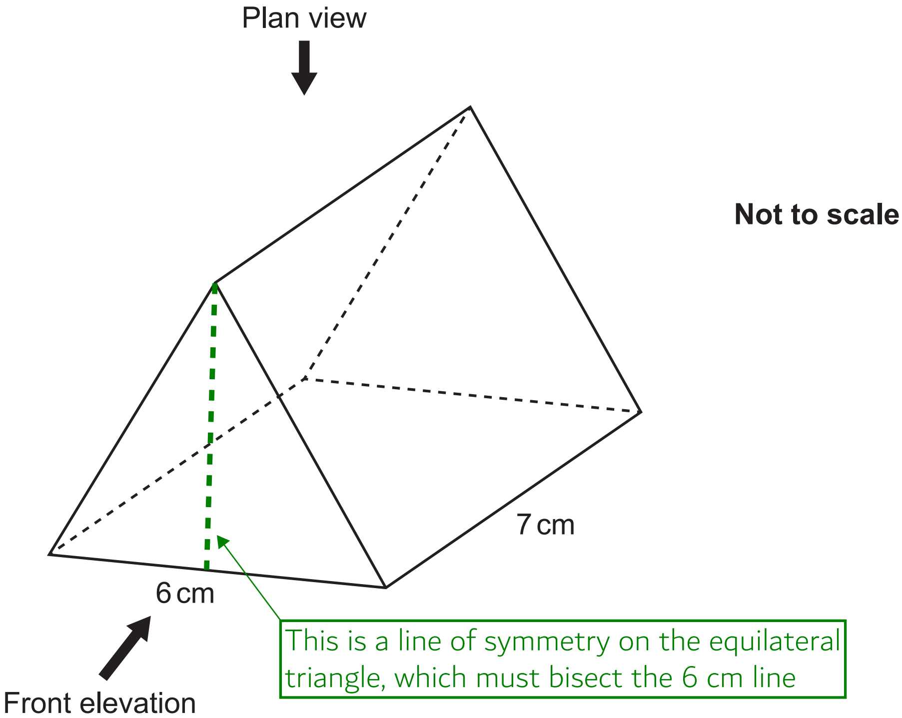
$\frac{100 + 5}{100}$ ← Adding the 5% to 100% expresses the percentage it increases to each year. Dividing this by 100 converts it to the decimal 1.05, which increases by 5% when multiplied

8000×1.05^3 ← This works out that the £8000 increased by 5% 3 times is £9261

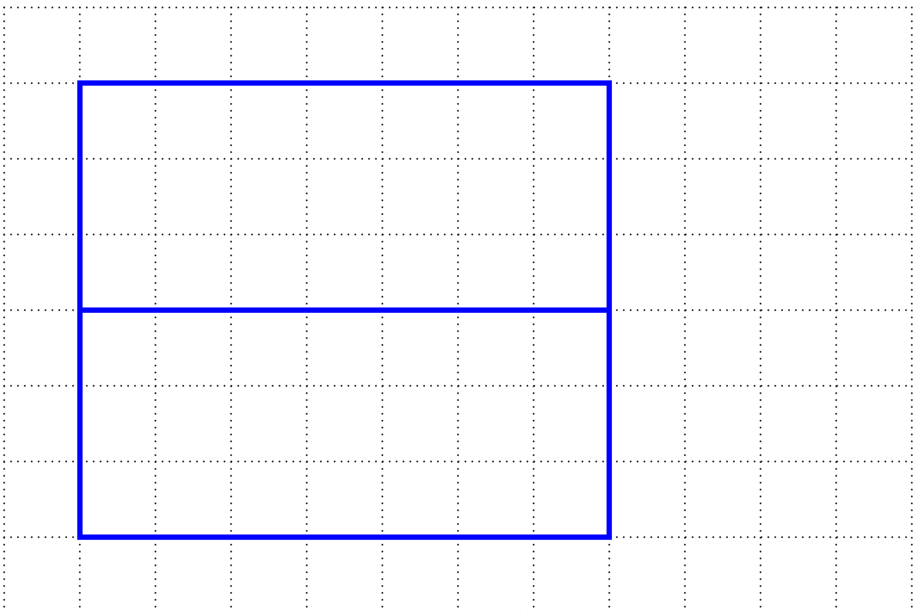
$9261 - 8000$ ← Subtracting the original £8000 from the value after 3 years works out that the interest is £1261

£ ....1261................. **[3]**

6 The diagram shows an equilateral triangular prism.
Each side of the equilateral triangle is 6 cm and the length of the prism is 7 cm.

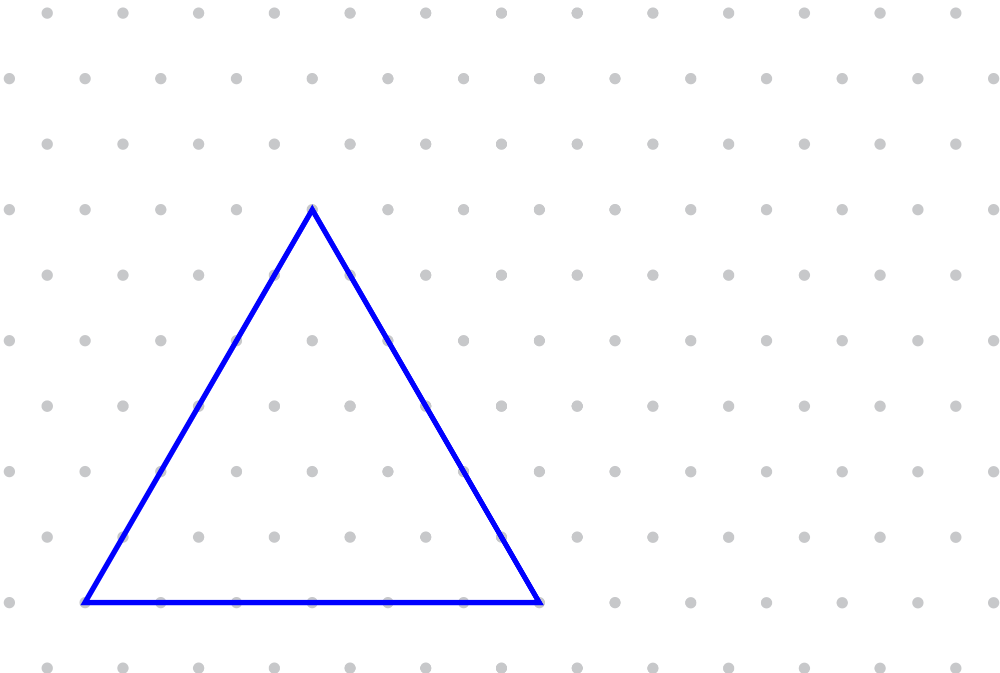


(a) Draw an accurate plan view of the prism on the one-centimetre square grid below.



[3]

(b) Draw an accurate front elevation of the prism on the one-centimetre isometric grid below.



[2]

7 A rock has a mass of 36 920 g and a volume of 14 200 cm^3.

Work out the density of the rock.
Give the units of your answer.

$d \, ^{m} \, v$ ← Writing a formula triangle for density, mass, volume

36920 ÷ 14200 ← Covering d in the formula triangle finds that density = mass ÷ volume

The mass in g was divided by the volume in cm^3

.... 2.6 .... g/cm^3 .... **[3]**

Turn over

8 There is a total of 354 balls in a bag.
There are white balls, red balls and green balls only.

The ratio of white balls to red balls is 3 : 4.
The ratio of red balls to green balls is 5 : 6.

Work out the number of green balls in the bag.

W	R	G
3	4	
	5	6
15	20	24

Combining the ratios in the ratio of white : red : green. Multiplying both sides of 3 : 4 by 5 and multiplying both sides of 5 : 6 by 4 gives the same number of parts for red, and this colour was in common to both ratios

15 + 20 + 24

Adding the numbers of parts in the combined ratio works out that there are 59 parts in total which represent the total 354 balls

354 ÷ 59

Dividing the total of 354 balls by the total 59 parts works out that 1 part of the ratio is worth 6 balls

6 × 24

Multiplying the value of 1 part of the ratio by 24 parts which represent green works out that there are 144 green balls

.................144................. **[4]**

9 A cylinder has a radius of 8.4 cm.
The ratio of the radius of the cylinder to the height of the cylinder is 2 : 5.

Find the volume of the cylinder.

$8.4 \div 2$ ← 2 parts of the ratio represent the radius. So dividing the radius by 2 works out that 1 part of the ratio is worth 4.2 cm

4.2×5 ← Multiplying the value of 1 part of the ratio by the 5 parts which represent the height works out that the height is 21 cm

$\pi \times 8.4^2 \times 21$ ← The formula for the volume of cylinder is the same as the formula for the volume of a prism. Volume of prism = area of cross section × length. The circle is the cross section and area of circle = π × radius^2. The length is the height

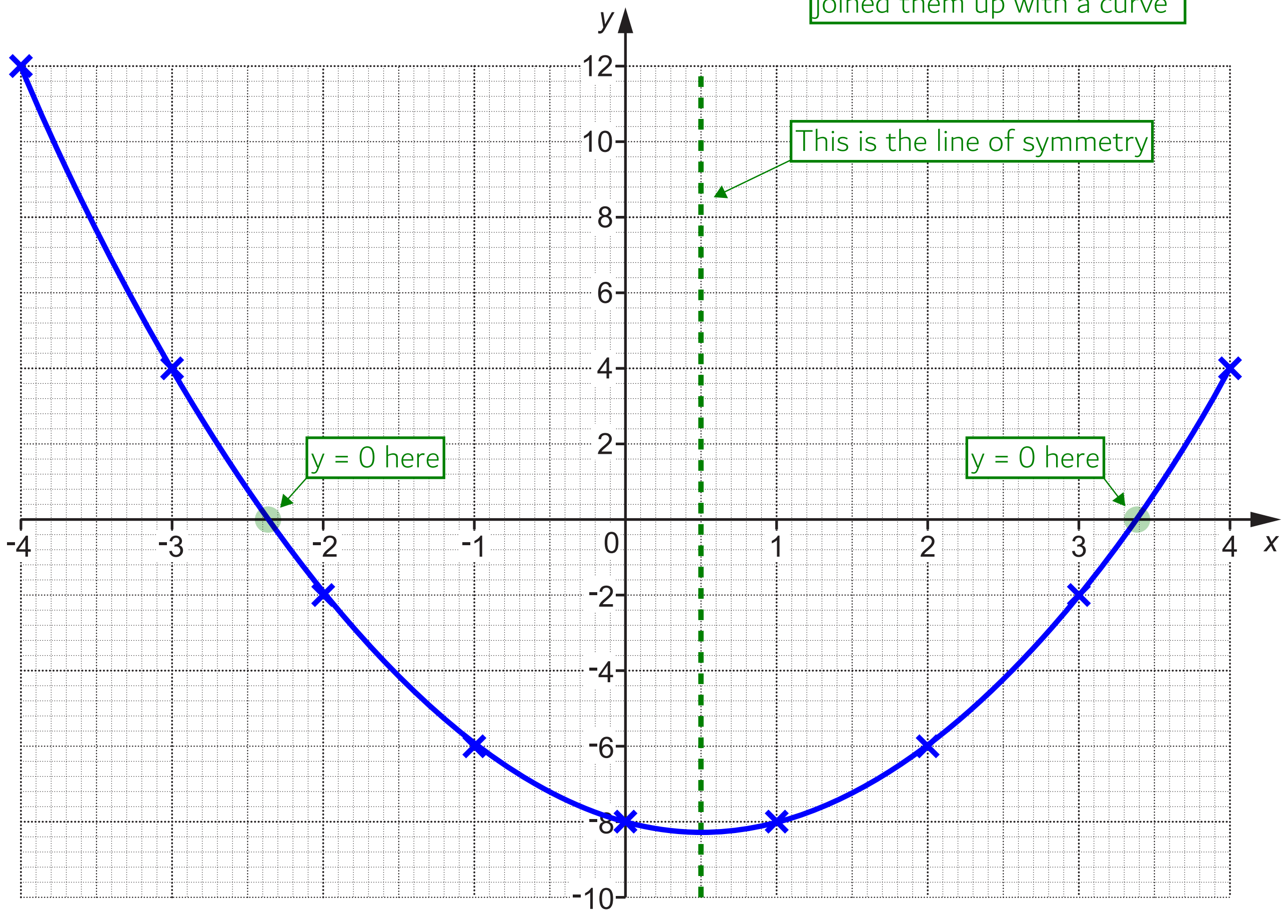
$\frac{37044}{25}\pi$ .... cm^3 **[4]**

10 Here is a table of values for $y = x^2 - x - 8$.

x	-4	-3	-2	-1	0	1	2	3	4
y	12	4	-2	-6	-8	-8	-6	-2	4

(a) Draw the graph of $y = x^2 - x - 8$ for $-4 \leqslant x \leqslant 4$.

Plotted the coordinates then joined them up with a curve



[3]

(b) Write down the equation of the line of symmetry of the graph.

All the x-coordinates on the line of symmetry are 0.5

(b) $x = 0.5$ **[1]**

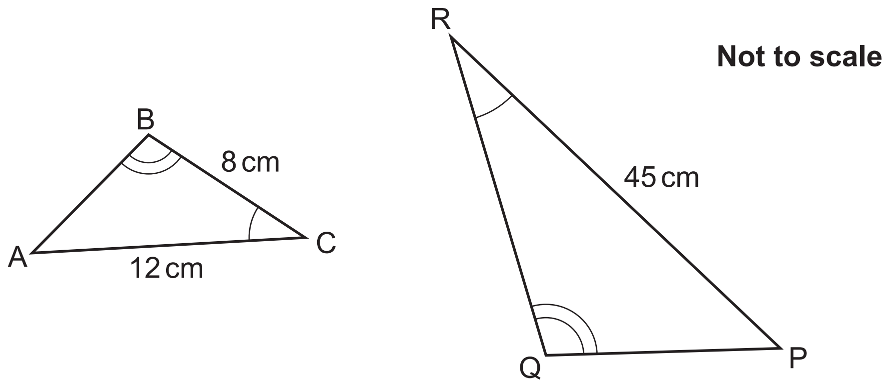
(c) Use the graph to solve the equation $x^2 - x - 8 = 0$.
Give your answers to **1** decimal place.

y has been replaced with 0 so it is basically asking for the x-coordinates where y = 0 on the curve

To the nearest small box as each small box is worth 0.1 in the x-direction

(c) $x =$ 3.4 or $x =$ -2.4 **[2]**

11 Triangles ABC and PQR are mathematically similar.
Angle ACB = Angle PRQ.
Angle ABC = Angle PQR.



The perimeter of triangle PQR is 99 cm.

Find the length of PQ.

$45 \div 12$ ← The 45 cm is the larger version of AC, as they are opposite the same angle in similar triangles. Dividing the 45 cm by the 12 cm works out that the scale factor is 3.75

8×3.75 ← Multiplying BC by the scale factor works out that RQ is 30 cm

$99 - 45 - 30$ ← Subtracting RP and RQ from the perimeter works out PQ

PQ = ...24...................... cm **[4]**

Turn over

12 y is directly proportional to the square of t.
$y = 14$ when $t = 2$.

t is directly proportional to x.
$t = 12$ when $x = 3$.

Find a formula for y in terms of x.
Give your answer in its simplest form.
You must show your working.

$y = kt^2$ ← $y \propto t^2$. The right side can be multiplied by anything and still be directly proportional. Multiplying the right side by k converts it to an equation

$k = 14 \div 2^2$ ← Dividing both sides by t^2 gets k on its own. Substituting 14 for y and 2 for t finds that k = 3.5

$y = 3.5t^2$ ← Putting the value of k into the equation

$t = cx$ ← $t \propto x$. The right side can be multiplied by anything and still be directly proportional. Multiplying the right side by c converts it to an equation

$c = 12 \div 3$ ← Dividing both sides by x gets c on its own. Substituting 12 for t and 3 for x finds that c = 4

$t = 4x$ ← Putting the value of c into the equation

$y = 3.5(4x)^2$ ← Substituting 4x for t in the 1st equation

$3.5(4)^2 = 56$ and squaring the x

$y =$ .... $56x^2$ .... **[6]**

13 A water company is laying pipes to cover a distance of 37 metres, correct to the nearest metre.
Each pipe has a length of 2.3 metres, correct to 1 decimal place.
Assume the pipes are laid end to end with no gaps or overlaps.

Work out the minimum number of pipes the water company needs to be sure of covering that distance.
You must show your working.

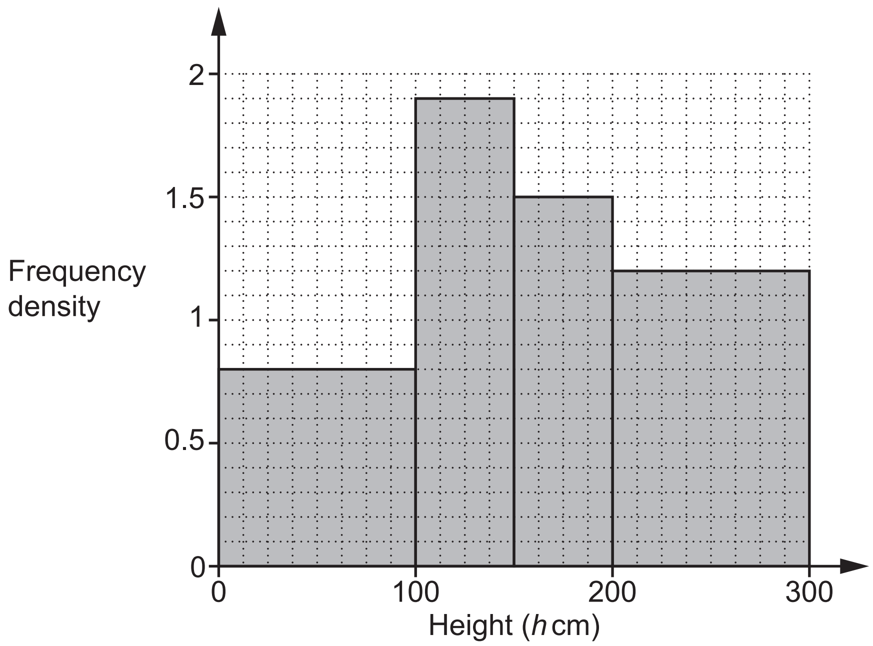
$(37 + \frac{1}{2}) \div (2.3 - \frac{0.1}{2})$

To be sure, the upper bound of the distance to be covered and the lower bound of the length of each pipe needs to be used. Adding half the resolution (which is 1 for the distance to be covered as it is to the nearest 1 metre) expresses the upper bound. Subtracting half of the resolution (which is 0.1 for the pipes as the 1st decimal place goes up in 0.1) expresses the lower bound. Dividing the distance to be covered by the length of each pipe works out how many pipes are needed

16.6... is rounded up to 17 as a whole number of pipes is needed and 16 is not enough

.... 17 .... **[4]**

14 The histogram summarises the heights, h cm, of some plants in a garden centre.



(a) Show that there are 80 plants with a height in the interval $0 < h \leqslant 100$. **[1]**

$100 \times 0.8 = 80$ ← Frequency on a histogram is the area of each bar. Area of rectangle = base × height. The base of the 1st bar is 100 and its height is 0.8

(b) The value, in pounds, of each plant depends on the plant's height.
The table below shows this information.

Height (h cm)	Value (£)
$0 < h \leqslant 100$	2.50
$100 < h \leqslant 150$	3.40
$150 < h \leqslant 200$	5.00
$200 < h \leqslant 300$	6.30

Use this information to find the **total** value of the plants represented in the histogram.

$50 \times 1.9 = 95$ ← So the number of plants for $100 < h \leq 150$ is 95

$50 \times 1.5 = 75$ ← So the number of plants for $150 < h \leq 200$ is 75

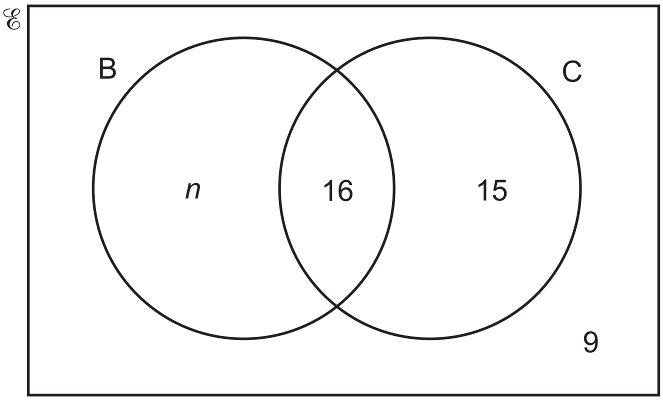
$100 \times 1.2 = 120$ ← So the number of plants for $200 < h \leq 300$ is 120

$80 \times 2.50 + 95 \times 3.40 + 75 \times 5 + 120 \times 6.30$

Multiplying the number of plants for each interval by the value of each plant of that interval expresses the total value of all the plants in that interval. Adding these totals gives an overall total value of all the plants

(b) £ ....1654........ **[4]**

15 In a survey, some students were asked whether they had travelled to school by bus (B) or by car (C) in the last week.
The Venn diagram shows some of the results.



(a) One of the students is chosen at random.
The probability that, in the last week, this student had travelled to school by bus and by car is $\frac{1}{4}$.

Find the value of n.

$n + 16 + 15 + 9$ ← Adding all the values in the Venn diagram expresses the total number of students. Collecting like terms gives n + 40

$\frac{16}{n + 40} = \frac{1}{4}$ ← Expressing the 16 students who travelled by bus and by car as a fraction of the total number of students must be equal to the value of the probability 1/4

$64 = n + 40$ ← Multiplying both sides by the denominators eliminates them

Subtracting 40 from both sides gets n on its own

(a) $n =$ ....24........ **[3]**

(b) One of the students is chosen at random.

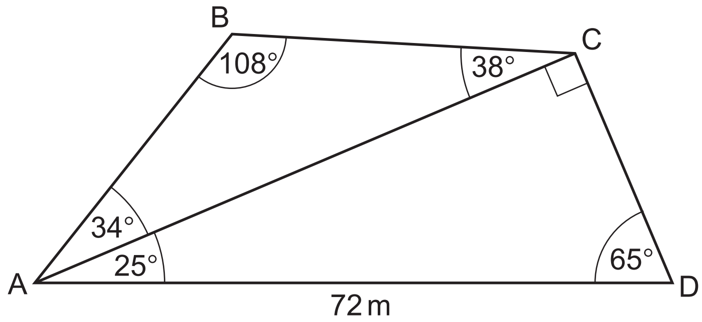
Find the probability that, in the last week, this student had travelled to school by car given that they had also travelled to school by bus.

$24 + 16$ ← Adding the values in B works out that 40 students travelled by bus

16 out of the 40 students who travelled by bus also travelled by car

(b) ....$\frac{16}{40}$........ **[2]**

16 AC is a diagonal of the quadrilateral ABCD.



Not to scale

AD = 72 m.
Angle ABC = 108°, angle BCA = 38° and angle BAC = 34°.
Angle ACD = 90°, angle CDA = 65° and angle CAD = 25°.

Find the area of ABCD.
You must show your working.

$\frac{AC}{\sin 65} = \frac{72}{\sin 90}$ ← $\frac{a}{\sin A} = \frac{b}{\sin B}$ ← The sine rule can be used in triangle ACD as there are two opposite pairs of sides and angles. Substituting in the values from the diagram

$AC = \frac{72}{\sin 90} \times \sin 65$ ← Multiplying both sides by sin65 finds that AC is 62.2... cm. The exact value can be stored as A on the calculator

$\frac{AB}{\sin 38} = \frac{62.2...}{\sin 108}$ ← $\frac{a}{\sin A} = \frac{b}{\sin B}$ ← The sine rule can be used in triangle ABC as there are two opposite pairs of sides and angles. Substituting in the values from the diagram and using the stored value of A on the calculator as b

$AB = \frac{62.2...}{\sin 108} \times \sin 38$ ← Multiplying both sides by sin38 finds that AB is 42.2... cm. The exact value can be stored as B on the calculator

$\frac{1}{2} \times 42.2... \times 62.2... \times \sin 34 = 770.6...$ ← This works out that the area of triangle ABC is 770.6... cm^2. Area of triangle = 1/2 absinC, where a and b are two sides and b is the angle between them. Storing the exact value as C

$\frac{1}{2} \times 62.2... \times 72 \times \sin 25 = 992.7...$ ← This works out that the area of triangle ACD is 992.7... cm^2. Area of triangle = 1/2 absinC, where a and b are two sides and b is the angle between them. Storing the exact value as D

$770.6... + 992.7...$ ← Adding the areas of triangles ABC and ACD gives the area of ABCD

........................1763.5........................ m^2 **[6]**

Turn over

17 An app's passcode consists of three digits. Each of the digits is a number from 0 to 9. A digit can be used more than once.

Find the fraction of the possible passcodes that contain at least one 5.

$10 \times 10 \times 10 = 1000$ ← Using the product rule for counting works out that there are 1000 possible passcodes. There are 10 digits from 0 to 9 so each digit of the passcode has 10 possibilities

5NN, N5N, NN5, 55N, 5N5, N55, 555 ← Systematically listing the possible passcodes with at least one 5. N stands for not 5

$1 \times 9 \times 9 + 9 \times 1 \times 9 + 9 \times 9 \times 1 + 1 \times 1 \times 9 + 1 \times 9 \times 1 + 9 \times 1 \times 1 + 1 \times 1 \times 1 = 271$

Using the product rule for counting works out that there are 271 possible passcodes with at least one 5. There is 1 digit which is a 5 and 9 digits which are not 5, so $1 \times 9 \times 9$ works out how many possible codes are 5NN

271 out of the 1000 possible codes have at least one 5

$\frac{271}{1000}$ .... **[4]**

18 Some sequences are defined using this term-to-term rule.

$u_{n+1} = 5u_n - 8$.

(a) If $u_3 = 22$, show that $u_4 = 102$. **[1]**

$u_4 = 5 \times 22 - 8 = 102$ ← Substituting u_4 for u_{n+1} and u_3 for u_n

(b) If $u_3 = 22$, work out u_2.

$22 = 5 \times u_2 - 8$ ← Substituting u_3 for u_{n+1} and u_2 for u_n

$30 = 5 \times u_2$ ← Adding 8 to both sides

Dividing both sides by 5

(b) ....6.......... **[3]**

(c) If $u_1 = 2$, write down the value of u_{50}.
Give a reason for your answer.

Entering 2 then pressing =. Entering 5Ans - 8 then keep pressing =.
This uses the term-to-term rule to find the next terms. They area all 2

u_{50} = ...2.... because ....all terms are 2....

.... **[2]**

19 Two ornaments, A and B, are mathematically similar.
The table shows information about the two ornaments.

	Ornament A	**Ornament B**
Height (m)	*h*	12
Surface area (m^2)	216	*A*
Volume (m^3)	240	3750

Find the value of *h* and the value of *A*.
You must show your working.

$12 \times \sqrt[3]{\frac{240}{3750}}$ ← 240/3750 expresses the volume of A as a fraction of the volume of B. Cube rooting this gives the height of A as a fraction of the height of B. Multiplying this by the height of B works out the height of A

$216 \times \left(\sqrt[3]{\frac{3750}{240}}\right)^2$ ← 3750/240 expresses the volume of B as a fraction of the volume of A. Cube rooting this gives the height of B as a fraction of the height of A. Squaring this gives the surface area of B as a fraction of the surface area of A. Multiplying this by the surfaces of A works out the surface area of B

h = 4.8

A = 1350 **[6]**

20 **(a)** Show that the equation $x^3 - 3x - 4 = 0$ has a solution between $x = 2$ and $x = 3$. **[3]**

$2^3 - 3(2) - 4 = -2$ ← Substituting in 2 to the left side gives a negative result

$3^3 - 3(3) - 4 = 14$ ← Substituting in 3 to the left side gives a positive result

Change in sign ← One result is negative and the other result is positive. So as it is a continuous function (there are no breaks in an x^3 graph), it must pass through 0 at some point between x = 2 and x = 3

(b) Use $x = 2.5$ to find a smaller interval for the solution to $x^3 - 3x - 4 = 0$.
You must show your working.

$2.5^3 - 3(2.5) - 4 = 4.125$ ← Substituting in 2.5 to the left side gives a positive result

When x = 2 the result was negative and when x = 2.5 the result is positive. So as it is a continuous function (there are no breaks in an x^3 graph), it must pass through 0 at some point between x = 2 and x = 2.5

(b) Between x = 2 and x = 2.5 **[2]**

(c) Find this solution correct to **1** decimal place.
You must show your working.

Using table mode on the calculator. Define $f(x) = x^3 - 3x - 4$. Change the table range. Start: 2. End: 2.5. Step: 0.1. This substitutes the values between 2 and 2.5 to 1 decimal place into the left side of the equation

$2.1^3 - 3(2.1) - 4 = -1.039$ ← Substituting in 2.1 to the left side gives a negative result

$2.2^3 - 3(2.2) - 4 = 0.048$ ← Substituting in 2.2 to the left side gives a positive result

So the solution must be between x = 2.1 and x = 2.2. Using table mode on the calculator. Define $f(x) = x^3 - 3x - 4$. Change the table range. Start: 2.1. End: 2.5.2. Step: 0.01. This substitutes the values between 2.1 and 2.2 to 2 decimal places into the left side of the equation

$2.19^3 - 3(2.19) - 4 = -0.06...$ ← Substituting in 2.1.9 to the left side gives a negative result

So the solution must be between x = 2.19 and x = 2.2.
All values between these round to 2.2 to 1 decimal place

(c) $x =$ 2.2 **[3]**

END OF QUESTION PAPER