AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

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Forename(s) ______________________

Candidate signature ______________________
I declare this is my own work.

Level 2 Certificate
FURTHER MATHEMATICS

Paper 2 Calculator

Wednesday 19 June 2024 Morning Time allowed: 1 hour 45 minutes

Materials
For this paper you must have:
- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

J U N 2 4 8 3 6 5 2 0 1

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided.

1 A straight line

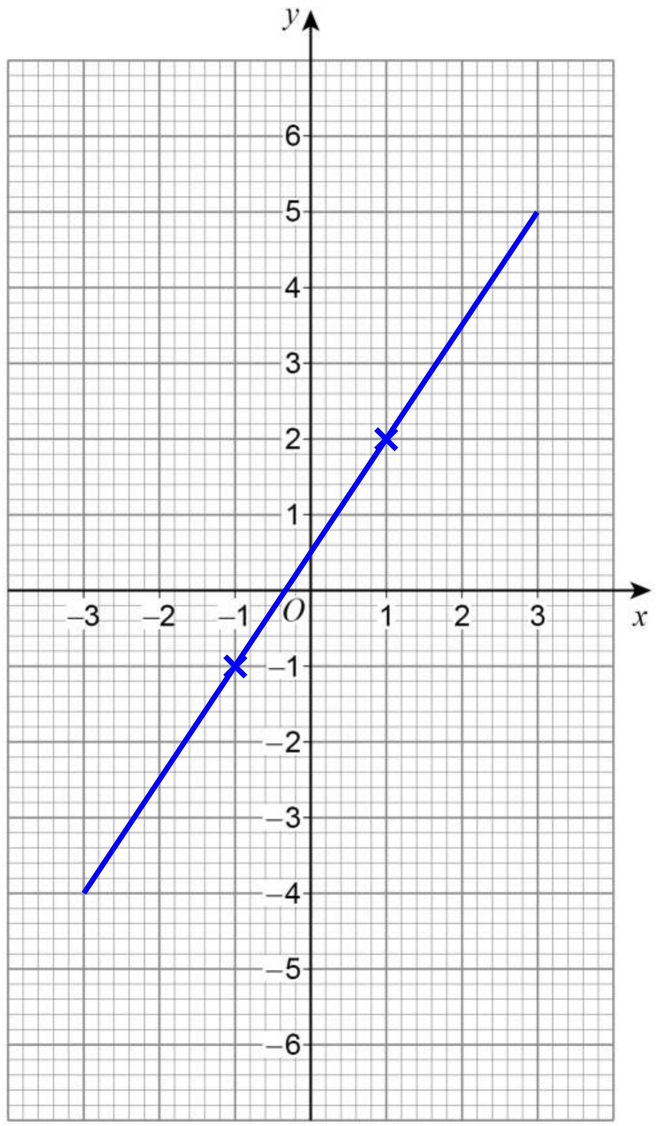
passes through the point (–1, –1)

and

has gradient $\frac{3}{2}$

Draw the line for values of x from –3 to 3

[3 marks]



Plotting (-1, -1). Gradient = (change in y)/(change in x). So when y changes 3, x changes 2. Going 2 to the right and up 3 from (-1, -1) gives another point on the line. Drawing a straight line through these two points starting where x is -3 and ending where x is 3

2 $5n^2 + 2 < 38$

Work out all the possible **integer** values of n.

[2 marks]

$5n^2 < 36$ ← Subtracting 2 from both sides to get the n^2 term on its own

$n^2 < 7.2$ ← Dividing both sides by 5 gets n^2 on its own

$-2.6... < n < 2.6...$ ← Doing the square root of both sides to get n on its own. Also doing the negative square root of 7.2 and flipping the inequality symbol for this solution

Answer -2, -1, 0, 1, 2

This is all of the integers greater than -2.6... and less than 2.6...

3 The equation of a curve is $y = x^4 - 3kx^2$ where k is a constant.

When $x = 2$ the rate of change of y with respect to x is 23

Work out the value of k.

[3 marks]

$4x^3 - 6kx$ ← Differentiating by multiplying each term by the power and then subtracting 1 from the power. This gives an expression of the rate of change of y with respect to x (the gradient)

$4(2)^3 - 6k(2)$ ← Substituting 2 for x expresses the ratio of change of y with respect to x when x = 2

$32 - 12k = 23$ ← Simplifying the expression and setting equal to the 23

$9 = 12k$ ← Adding 12k to both sides to get the k term positive. Then subtracting 23 from both sides to get the k term on its own

$k =$ 0.75

Dividing both sides by 9 gets k on its own

8

4 Here is some information about three linear sequences, A, B and C.

nth term of C = nth term of A + nth term of B

The nth term of C is $42 - 3n$

The first four terms of B are 14 22 30 38

Work out the 20th term of A.

[4 marks]

$8n + 6$ ← Expressing the nth term of B. It goes up 8 between each term so must involve 8n. Going backward in the sequence find that the 0th term (the term before the 1st term) would be 6. So the nth term of B is 8n + 6

$42 - 3n = A + 8n + 6$ ← Let A be the nth term of A. Substituting the nth term of C and nth term of B into the equation

$36 - 11n = A$ ← Subtracting 8n and 6 from both sides gets A on its own. So the nth term of A is 36 - 11n

$36 - 11(20)$ ← Substituting 20 for n in the nth term of A finds the 20th term of A

Answer -184

5 (a) $$\begin{pmatrix} c & 1 \\ 5d & 3 \end{pmatrix}\begin{pmatrix} -2 & 0 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 0 & 9 \end{pmatrix}$$

Work out the values of c and d.

[3 marks]

$-2c + 7 = -5$ ← $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$ ← Only dealing with the multiplication which leads to the -5 in the top left

$-2c = -12$ ← Subtracting 7 from both sides gets the c term on its own. Then dividing both sides by -2 gives c = 6

$-10d + 21 = 0$ ← $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$ ← Only dealing with the multiplication which leads to the 0 in the bottom left

$-10d = -21$ ← Subtracting 21 from both sides gets the d term on its own. Then dividing both sides by -10 gives d = 2.1

$c =$ 6 $d =$ 2.1

5 (b) $$\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix}\mathbf{M} = \begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix}$$

Write down matrix **M**

[1 mark]

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This is the identity matrix. Multiplying by it gives the same matrix as the answer

8

Turn over ▶

6 The function f is given by $f(x) = 3x^2 + 2$ with domain $-1 \leqslant x \leqslant 4$

Work out the range of the function.

[2 marks]

The minimum value of x^2 is 0 and this happens when x = 0, which is within the domain of the function. $3(0)^2 + 2 = 2$ so f(x) can be as low as 2

$3(4)^2 + 2$ ←

x^2 has the greatest value when x has the greatest magnitude (value ignoring negatives). The greatest magnitude of x within the domain is 4, so substituting this into f(x) gives the greatest possible value of f(x)

Answer 2 $\leqslant f(x) \leqslant$ 50

7 The equation of a curve is $y = 4 - (x - 3)^2$

7 (a) Circle the coordinates of the point where the curve crosses the y-axis.

[1 mark]

(–5, 0) [✓ (0, –5)] (–13, 0) (0, –13)

x = 0 when the curve is crossing the y-axis. $y = 4 - (0 - 3)^2 = -5$

7 (b) Write down the coordinates of the maximum point of the curve.

[1 mark]

Answer (3 , 4)

The minimum value of a squared bracket (which is needed as subtracting the minimum value from 4 gives the greatest result) is 0. x = 3 for the squared bracket to equal to 0. When the squared bracket equals 0, y = 4

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8 $y = \frac{1}{2}x^2 + \frac{3}{4x^4}$

Work out $\frac{d^2y}{dx^2}$

Give your answer in the form $a + bx^n$ where a, b and n are integers.

[3 marks]

$\frac{1}{2}x^2 + \frac{3}{4}x^{-4}$ ← Rewriting the right side of the equation without x as a denominator. Dividing by x^4 is the same as multiplying by x^{-4}

$x - 3x^{-5}$ ← Differentiated by multiplying each term by the power and then subtracting 1 from the power

Answer $1 + 15x^{-6}$

Differentiated again by multiplying each term by the power and then subtracting 1 from the power. $x = x^1$ so differentiating this becomes $1x^0$, which is 1. $x^0 = 1$ as anything to the power of 0 is 1 then 1 × 1 = 1

9 A set of 4-digit integers each have

a first digit **greater than** 6

and

a second digit **less than** 8

What is the greatest possible number of integers in the set that are multiples of 5 ?

[3 marks]

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ← Listing out the single digits in numerical order

3 × 8 × 10 × 2 ← Using the product rule for counting. There are 3 possibilities for the first digit (7, 8, 9). There are 8 possibilities for the second digit (0, 1, 2, 3, 4, 5, 6, 7). There are 10 possibilities for the third digit as it could be any single digit. There are 2 possibilities for the fourth digit as multiples of 5 end in 0 or 5

Answer 480

10

Turn over ►

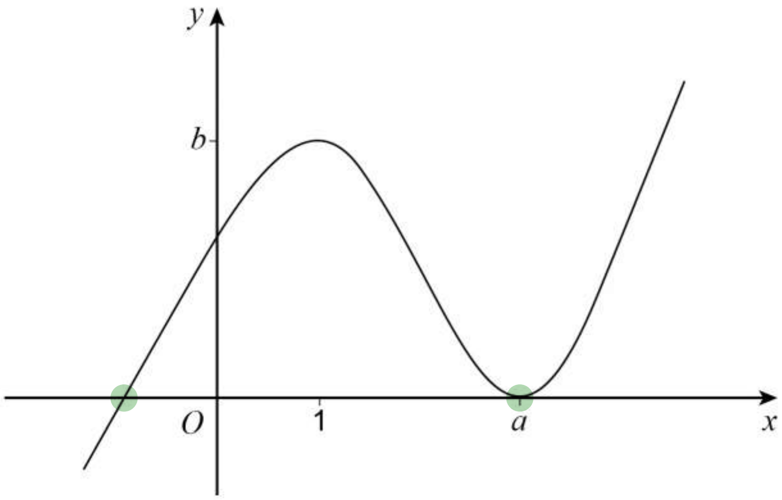
10 f is a cubic function.

The curve $y = \text{f}(x)$ has

a minimum point at $(a, 0)$

and

a maximum point at $(1, b)$



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Tick one box for each statement.

[2 marks]

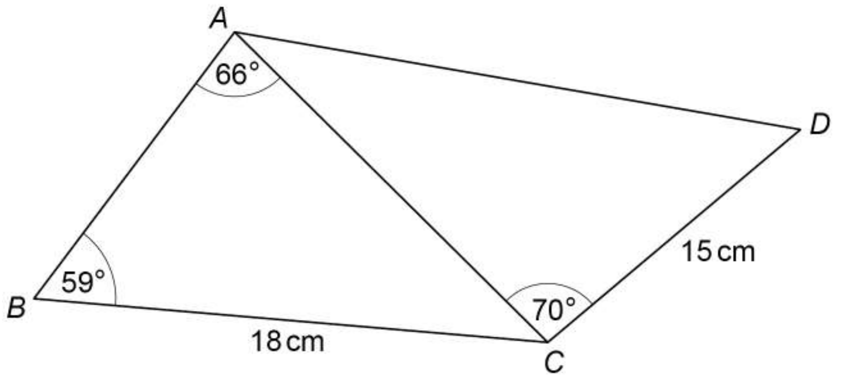
	True	**False**
The tangent to the curve at $(1, b)$ is parallel to the x-axis	☑	☐
There are **three different** values of x for which $y = 0$	☐	☑
The function is increasing for $0 < x < 1$	☑	☐

1st statement: true, as (1, b) is a stationary point so its gradient is 0. The x-axis also has a gradient of 0. Straight lines with the same gradient are parallel.

2nd statement: false, as there are only two different values for x for which y = 0, which is where the curve intersects the x-axis. The points are highlighted in green on the graph.

3rd statement: true, as the gradient is positive for 0 < x < 1.

11 *ABC* and *ACD* are triangles.



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Work out the area of triangle *ACD*.

[4 marks]

$\frac{AC}{\sin 59} = \frac{18}{\sin 66}$ ← $\boxed{\frac{a}{\sin A} = \frac{b}{\sin B}}$ ← Using the sine rule in triangle ABC to work out side AC. Side a is opposite angle A and side b is opposite angle B

AC = 16.8... ← Multiplying both sides by sin59 gets AC on its own

$\frac{1}{2} \times 16.8... \times 15 \times \sin 70$ ← Area of triangle = 1/2 absinC, where a and b are two sides and C is the angle between them

Answer 119.0 cm^2

6

Turn over ▶

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12 The equation of a circle is $(x-2)^2+(y+3)^2=16$

The equation of a line is $y=4-x$

The circle and line intersect at two points, *A* and *B*.

12 (a) Show that the x-coordinates of *A* and *B* satisfy the equation $2x^2-18x+37=0$

[3 marks]

$4 - x + 3$ ← Substituting 4 - x for y in y + 3. This simplifies to 7 - x

$(x-2)^2 + (7-x)^2$ ← Writing the left side of the equation of the circle in terms of only x

$x^2 - 4x + 4 + 49 - 14x + x^2 = 16$ ← Expanding the square brackets by squaring the first term, doubling the product of the two terms, squaring the last term. Now including the = 16

$2x^2 - 18x + 37 = 0$ ← Subtracting 16 from both sides and collecting like terms

Do not write outside the box

12 (b) For *A*, the x-coordinate and y-coordinate are both **positive**.

Work out the coordinates of *A*.

Give each coordinate to 2 decimal places.

[3 marks]

$$\frac{--18 \pm \sqrt{(-18)^2 - 4 \times 2 \times 37}}{2 \times 2} \leftarrow \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \leftarrow$$

The equation was in the form $ax^2 + bx + c = 0$ so can be solved using the quadratic formula. a = 2, b = -18, c = 37

4 - 5.8... ← One of the solutions of x was 5.8... but y = 4 - x and this gives a negative y-coordinate

4 - 3.1... ← The other solution of x was 3.1... and y = 4 - x. This works out the y-coordinate

Answer (3.18 , 0.82)

Turn over for the next question

6

Do not write outside the box

13 Expand and simplify fully $(3x - 1)(4 - x)(2x + 5)$

[3 marks]

$12x - 3x^2 - 4 \ + x$ ← Expanding the first two brackets

$(-3x^2 + 13x - 4)(2x + 5)$ ← Collecting like terms then writing it multiplied by the third bracket

$-6x^3 - 15x^2 + 26x^2 + 65x - 8x - 20$ ← Expanding these two brackets

Answer $-6x^3 + 11x^2 + 57x - 20$

↑ Collecting like terms

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Do not write outside the box

14 P is the point on the graph $y = 3^{-x}$ that has x-coordinate –2

Q is a point on the graph $y = 1152 \times \left(\frac{1}{2}\right)^{x}$

y-coordinate of P = y-coordinate of Q

Work out the x-coordinate of Q.

[3 marks]

$y = 3^{--2}$ ← Substituting the -2 for x in the first equation works out that the y-coordinate of P is 9, so the y-coordinate of Q is also 9

$9 = 1152 \times \left(\frac{1}{2}\right)^{x}$ ← Substituting 9 for y in the second equation

$\frac{1}{128} = \frac{1}{2^x}$ ← Dividing both sides by 1152. Raising both the numerator and denominator of 1/2 to the power of x. 1 to the power of anything is 1

2, 4, 8, 16, 32, 64, 128 ← 2^x = 128. Listing out powers of 2 until 128 is reached

$x =$ 7

$2^7 = 128$

Turn over for the next question

6

Turn over ▶

15 Simplify fully $\dfrac{2x^2 + 9x - 18}{12x^2 - 8x - 15}$

[3 marks]

$2x^2 + 12x - 3x - 18$ ← Factorising the numerator. It is in the form $ax^2 + bx + c$. Multiplying a by c gives -36. Two numbers which multiply to this -36 and add to b are 12 and -3. Splitting the middle x-term into these numbers of x

$2x(x + 6) - 3(x + 6)$ ← Factorising the left two terms separately to the right two terms

$(2x - 3)(x + 6)$ ← Bringing together the 2x and -3 and writing the (x + 6) once

$12x^2 + 10x - 18x - 15$ ← Factorising the denominator. It is in the form $ax^2 + bx + c$. Multiplying a by c gives -180. Two numbers which multiply to this -180 and add to b are 10 and -18. Splitting the middle x-term into these numbers of x

$2x(6x + 5) - 3(6x + 5)$ ← Factorising the left two terms separately to the right two terms

$(2x - 3)(6x + 5)$ ← Bringing together the 2x and -3 and writing the (6x + 5) once

Answer $\dfrac{x + 6}{6x + 5}$

↑ Cancelling out the (2x - 3) from the numerator and denominator

16 Simplify $\dfrac{\sqrt{x^3}\left(\sqrt{x^3}+x^3\right)}{\sqrt{x}}$

Give your answer in the form $x^{\frac{a}{b}}+x^c$ where a, b and c are integers.

[3 marks]

$\dfrac{x^{\frac{3}{2}}(x^{\frac{3}{2}}+x^3)}{x^{\frac{1}{2}}} \leftarrow$ Rewriting the expression with powers of x instead of the roots. Square root as a power is over 2

$x^1(x^{\frac{3}{2}}+x^3) \leftarrow$ $a^w/a^y = a^{w-y}$. 3/2 - 1/2 = 1

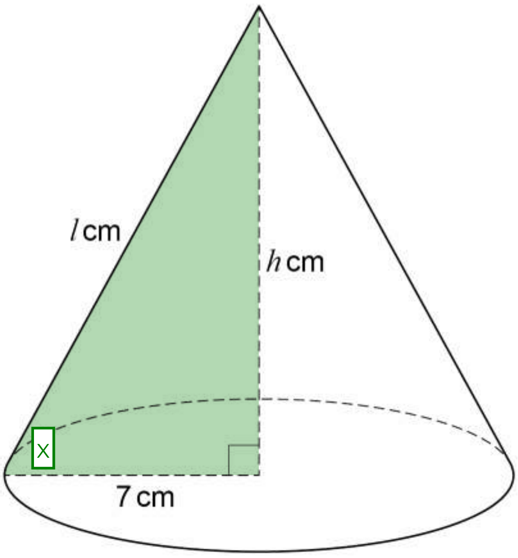
Answer $x^{\frac{5}{2}}+x^4$

$\uparrow$ $a^w \times a^y = a^{w+y}$. 3/2 + 1 = 5/2 and 3 + 1 = 4

Turn over for the next question

6

17 (a) A solid cone has base radius 7 cm, perpendicular height h cm and slant height l cm



Curved surface area of a cone = $\pi r l$

The **total** surface area of the cone is 224π cm^2

Work out the size of the angle between the slant height and the base.

[4 marks]

$\pi \times 7 \times l + \pi \times 7^2 = 224\pi$ ← Substituting 7 for r in the formula for the curved surface area of the cone. Adding the area of the circle (area of circle = π × radius^2). This gives the total surface area of the cone so is equal to 224π

$\pi \times 7 \times l = 175\pi$ ← Subtracting π × 7^2 from both sides

$l = 25$ ← Dividing both sides by π × 7

$\overset{O\checkmark}{S\ H}\quad \overset{\checkmark}{\overset{A\checkmark}{C\ H}}\quad \overset{O\checkmark}{T\ A}$ ← Using right-angled trigonometry to find the angle x in the green right-angled triangle. The 7 cm is the adjacent so ticking A. The 25 cm is the hypotenuse so ticking H. There are two ticks on the CAH formula triangle so this one can be used

$\cos x = \frac{7}{25}$ ← Covering C in the CAH formula triangle find that cos of the angle = adjacent/hypotenuse

Doing the inverse cos of both sides gets x on its own

↓

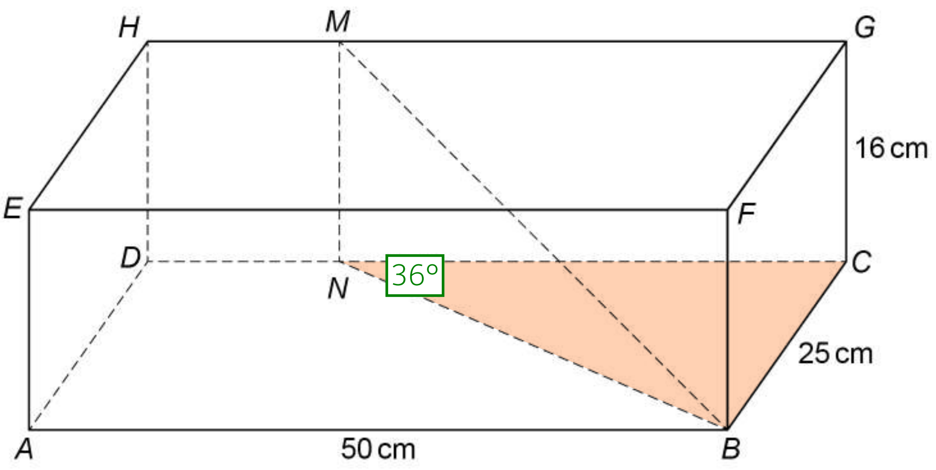
Answer 73.7 °

17 (b) *ABCDEFGH* is a cuboid.

ABCD is horizontal.

MN is vertical.

The acute angle between the planes *MNB* and *HDCG* is 36°



Work out the length *CN*.

[2 marks]

$\genfrac{}{}{0pt}{}{\overset{\checkmark}{O}}{S\quad H}\qquad\genfrac{}{}{0pt}{}{\overset{\checkmark}{A}}{C\quad H}\qquad\genfrac{}{}{0pt}{}{\overset{\checkmark}{O}}{T\quad \overset{\checkmark}{A}}$

Using right-angled trigonometry to find CN in the orange right-angled triangle. The 25 cm is the opposite so ticking O. CN is the adjacent so ticking A. There are two ticks on the TOA formula triangle so this one can be used

$\frac{25}{\tan 36}$

Covering A in the TOA formula triangle finds that adjacent = opposite/(tan of the angle)

Answer 34.4 cm

6

Do not write outside the box

18 Rearrange $m = \frac{2k^3 - 7}{5m - 3k^3}$ to make k the subject.

[4 marks]

$5m^2 - 3k^3m = 2k^3 - 7$

Multiplying both sides by 5m - 3k^3 to eliminate the denominator which involves k. The left becomes m(5m - 3k^3) which becomes 5m^2 - 3k^{3}m when the bracket is expanded

$5m^2 + 7 = 2k^3 + 3k^3m$

Adding 3k^{3}m and 7 to both sides to get all the terms involving k on the same side and all the other terms on the other side

$= k^3(2 + 3m)$

Bringing k^3 out as a factor on the right to get all the k out of the terms

$k^3 = \frac{5m^2 + 7}{2 + 3m}$

Dividing both sides by 2 + 3m to get k^3 on its own

Answer $k = \sqrt[3]{\frac{5m^2 + 7}{2 + 3m}}$

Cube rooting both sides gets k on its own

19 In the expansion of $(3 + ax)^5$ where a is a non-zero constant

$$\boxed{8 \times \text{coefficient of } x^2 = \text{coefficient of } x^4}$$

Work out the possible values of a.

[4 marks]

$1(3)^5(ax)^0 + 5(3)^4(ax)^1 + 10(3)^3(ax)^2 + 10(3)^2(ax)^3 + 5(3)^1(ax)^4 + 1(3)^0(ax)^5$

Working out the coefficients for each term by doing ${}^5C_0$, ${}^5C_1$, ${}^5C_2$, ${}^5C_3$, ${}^5C_4$, ${}^5C_5$ on the calculator. The power of 3 decreases by 1 each term and the power of ax increases by 1 each term

$8 \times 270a^2 = 15a^4$

Setting 8 × the coefficient of x^2 equal to the coefficient of x^4. $10(3)^3(ax)^2 = 270a^2x^2$ and $5(3)^1(ax)^4 = 15a^4x^4$

$2160 = 15a^2$

Dividing both sides by a^2 to get all the a on one side

$144 = a^2$

Dividing both sides by 15 to get a^2 on its own

Answer $a = \pm 12$

Doing the positive and negative square root of 144 eliminates the power of 2 and gets a on its own

Turn over for the next question

8

20 AB is the tangent to the circle $(x-3)^2+(y-4)^2=29$ at the point (5, 9)

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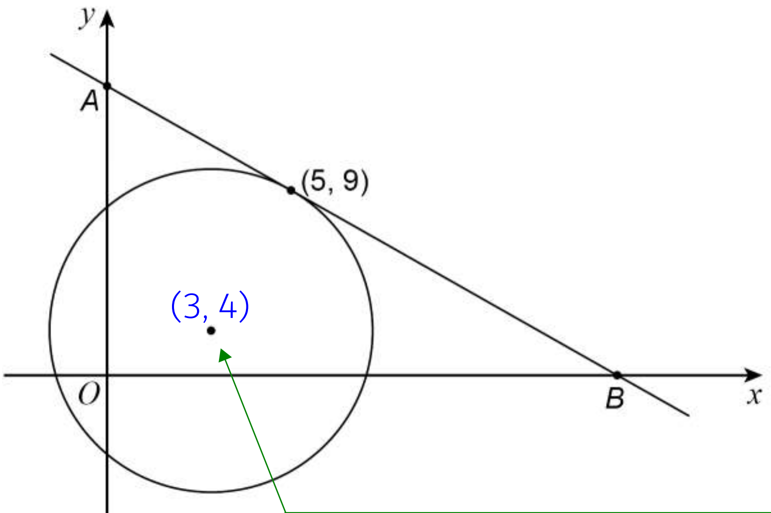

The general equation of a circle, centre (a, b), radius r, is $(x - a)^2 + (y - b)^2 = r^2$. So the centre of the circle is (3, 4)

Work out the area of triangle AOB.

[5 marks]

$\frac{9-4}{5-3}$	Expressing the gradient of the radius from (3, 4) to (5, 9). Gradient = (change in y)/(change in x). So the gradient of the radius is 5/2
$y - 9 = -\frac{2}{5}(x - 5)$	The general equation of a straight line passing through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$. $x_1 = 5$ and $y_1 = 9$ and m = -2/5 (as the tangent is perpendicular to the radius so its gradient is the negative reciprocal of 5/2
$y = -\frac{2}{5}(0 - 5) + 9$	Finding the y-coordinate of A. Rearranging to make y the subject by adding 9 to both sides. Substituting 0 for x as the x-coordinate of A is 0
$= 11$	The y-coordinate of A is 11
$(0 - 9) \div -\frac{2}{5} + 5 = x$	Finding the x-coordinate of B. Rearranging to make x the subject by dividing both sides by -2/5 then adding 5 to both sides. Substituting 0 for y as the y-coordinate of B is 0
$x = 27.5$	The x-coordinate of B is 27.5
$\frac{1}{2} \times 27.5 \times 11$	Area of triangle = 1/2 × base × height. The base OB is 27.5 and the height OA is 11

Answer 151.25 square units

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21 $f(x) = \dfrac{2-x}{3}$

$g(x) = 18x^2 + 15x$

$f^{-1}(x) + gf(x)$ simplifies to $ax^2 + bx + c$

Work out the values of a, b and c.

[6 marks]

$x = \dfrac{2-y}{3}$ ←

Working out the inverse function $f^{-1}(x)$ by switching f(x) with x and x with y. Then rearranging to make y the subject

$3x = 2 - y$ ←

Multiplying both sides by 3 to eliminate the denominator

$y = 2 - 3x$ ←

Adding y to both sides to make it positive and subtracting 3x from both sides to get y on its own. So $f^{-1}(x) = 2 - 3x$

$\left(\dfrac{2-x}{3}\right)^2$ ←

Working out the composite function gf(x) by substituting f(x) for x in g(x). First expressing the x^2

$18\left(\dfrac{4 - 4x + x^2}{9}\right) + 15\left(\dfrac{2-x}{3}\right)$ ←

Squaring the fraction by squaring the numerator and squaring the denominator. Squaring the numerator by squaring the first term, doubling the product of the two terms, squaring the last term. Now writing all of gf(x)

$2(4 - 4x + x^2) + 5(2 - x)$ ←

18/9 = 2 and 15/3 = 5

$2 - 3x + 8 - 8x + 2x^2 + 10 - 5x$ ←

Writing $f^{-1}(x)$ and adding all of gf(x) with the brackets expanded

$2x^2 - 16x + 20$ ←

Collecting like terms to put it into the form $ax^2 + bx + c$

$a =$ 2 $b =$ -16 $c =$ 20

22 A, B and C are points on a circle.

PAC is a straight line.

PB is a tangent to the circle.

$AC = BC$

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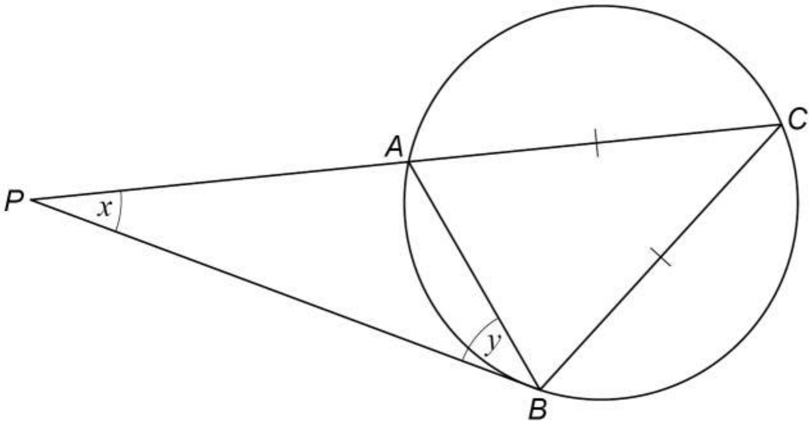

Prove that $x = 90 - \frac{3}{2}y$

[5 marks]

Angle ACB = y due to the alternate segment theorem

The angle between tangent PB and chord AB is equal to the interior opposite angle ACB

Angle ABC = $\frac{(180 - y)}{2}$ as there are 180° in a triangle and base angles of an isosceles triangle are equal

Triangle ABC is isosceles as two of its sides are equal (AC = BC). Subtracting angle ACB from 180° leaves the total of angles BAC and ABC, which are both equal, so dividing by 2 expresses angle ABC

$x + y + \frac{(180 - y)}{2} + y = 180$ as there are 180° in a triangle

Adding angles APB, PBA, ABC, BCA expresses the total of the angles in triangle PBC

$x = 180 - \frac{180}{2} + \frac{y}{2} - y - y$

Splitting the (180 - y)/2 into 180/2 and -y/2. Then rearranging to make x the subject

$x = 90 - \frac{3}{2}y$

Simplifying to give what we are trying to prove

2 2

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Do not write outside the box

END OF QUESTIONS

5