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Candidate surname

Other names

Centre Number

Candidate Number

Pearson Edexcel
Level 1/Level 2 GCSE (9–1)

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Tuesday 6 November 2018

Morning (Time: 1 hour 30 minutes)

Paper Reference **1MA1/1H**

Mathematics

Paper 1 (Non-Calculator)

Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.
Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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6/7/7/7/7/7/1/C2/

.CG Maths.
Worked Solutions


Pearson

Please note that these worked solutions have neither been provided nor approved by Pearson Education and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Work out the value of $\frac{3^7 \times 3^{-2}}{3^3}$

$\frac{3^5}{3^3}$ ← $a^x \times a^y = a^{x+y}$. So adding the indices on the numerator. $7 + -2 = 5$

3^2 ← $a^x/a^y = a^{x-y}$. So subtracting the indices. $5 - 3 = 2$

$3^2 = 3 \times 3 = 9$ → 9

(Total for Question 1 is 2 marks)

2 $v^2 = u^2 + 2as$

$u = 12 \quad a = -3 \quad s = 18$

(a) Work out a value of v .

$12^2 + 2 \times -3 \times 18$ ← Substituting in the values of u , a and s into the right side of the formula

$\frac{18}{\times 6}$ ← $2 \times -3 = -6$. Then $-6 \times 18 = -108$
 $\frac{108}{4}$

$\frac{1 \cancel{4}^{14}}{-1 \ 0 \ 8}$ ← $12^2 = 12 \times 12 = 144$. Then subtracting the 108. So $v^2 = 36$
 $\frac{0 \ 3 \ 6}{0 \ 3 \ 6}$

$6^2 = 6 \times 6 = 36$
6
(2)

(b) Make s the subject of $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$ ← Subtracting u^2 from both sides gets the term involving s on its own

Dividing both sides by $2a$ gets s on its own → $s = \frac{v^2 - u^2}{2a}$

(2)

(Total for Question 2 is 4 marks)

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- 3 A bonus of £2100 is shared by 10 people who work for a company.
 40% of the bonus is shared equally between 3 managers.
 The rest of the bonus is shared equally between 7 salesmen.

One of the salesmen says,

“If the bonus is shared equally between all 10 people I will get 25% more money.”

Is the salesman correct?

You must show how you get your answer.

$$2100 \div 10 = 210$$

This works out that 10% of the £2100 is £210. It also works out that each salesman would get £210 if the bonus was shared equally between all 10 people

$$\begin{array}{r} 210 \\ \times 6 \\ \hline 1260 \end{array}$$

100% - 40% = 60%, which is the percentage the salesmen get. Multiplying the value of 10% by 6 works out that 60% of the £2100 is £1260

$$7 \overline{) 1260} \begin{array}{r} 0180 \\ \hline 1260 \\ \hline 0 \end{array}$$

Dividing the £1260 by the 7 salesmen works out that each salesman gets £180

$$4 \overline{) 180} \begin{array}{r} 045 \\ \hline 180 \\ \hline 0 \end{array}$$

25% is 1/4. So dividing the £180 by 4 works out that 25% of the £180 is £45

$$\begin{array}{r} 180 \\ + 45 \\ \hline 225 \end{array}$$

Adding the value of 25% to the £180 works out that 25% more than the £180 is £225

No

25% more money would be £225, which is not the £210 they would get if it was shared equally between all 10 people

(Total for Question 3 is 5 marks)

4 It would take 120 minutes to fill a swimming pool using water from 5 taps.

(a) How many minutes will it take to fill the pool if only 3 of the taps are used?

$$\begin{array}{r} 120 \\ \times 5 \\ \hline 600 \end{array}$$

Multiplying the 120 minutes by the 5 taps works out that 600 minutes worth of work is done

$$\begin{array}{r} 200 \\ 3 \overline{)600} \end{array}$$

Dividing the 600 minutes worth of work by the 3 taps works out that it would take 200 minutes

200 minutes
(2)

(b) State one assumption you made in working out your answer to part (a).

All taps work at the same rate

(1)

(Total for Question 4 is 3 marks)

5 A plane travels at a speed of 213 miles per hour.

(a) Work out an estimate for the number of seconds the plane takes to travel 1 mile.

$\begin{matrix} d \\ s \end{matrix} \times t$

Writing a formula triangle for distance, speed, time

$$\frac{1}{213} \times 60 \times 60$$

Covering t in the formula triangle finds that time = distance/speed. So putting the distance of 1 mile over the speed of 213 miles per hour expresses the time taken in hours (as hours was involved in the units of mile per hour). 1 hour = 60 minutes and 1 minute = 60 seconds. So multiplying by 60 converts it to minutes then multiplying by 60 again converts it to seconds

$$200 \overline{)3600}$$

$60 \times 60 = 3600$. Then dividing this by 200 instead of 213 as it is an estimate and this is easier to do

18 seconds
(3)

(b) Is your answer to part (a) an underestimate or an overestimate?
Give a reason for your answer.

Overestimate as the speed was rounded down

Dividing by less gives a larger answer

(1)

(Total for Question 5 is 4 marks)

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6 Solve the simultaneous equations

$$5x + y = 21 \leftarrow \text{1st equation}$$

$$x - 3y = 9 \leftarrow \text{2nd equation}$$

$$5x - 15y = 45 \leftarrow \text{Multiplying the 2nd equation by 5 gives the same number of x as the 1st equation. This forms the 3rd equation}$$

$$16y = -24 \leftarrow \text{Subtracting the 3rd equation from the 1st equation cancels out the x term and leaves an equation just in terms of y}$$

$$y = \frac{-24}{16} \leftarrow \text{Dividing both sides by 16 gets y on its own. Dividing both the numerator and denominator by 8 simplifies it to } -3/2$$

$$x - 3\left(\frac{-3}{2}\right) = 9 \leftarrow \text{Substituting the value of y into the 2nd equation}$$

$$x = \frac{18}{2} - \frac{9}{2} \leftarrow -3(-3/2) = 9/2. \text{ Subtracting } 9/2 \text{ from both sides gets x on its own. Writing 9 as } 18/2 \text{ so that it has the same denominator}$$

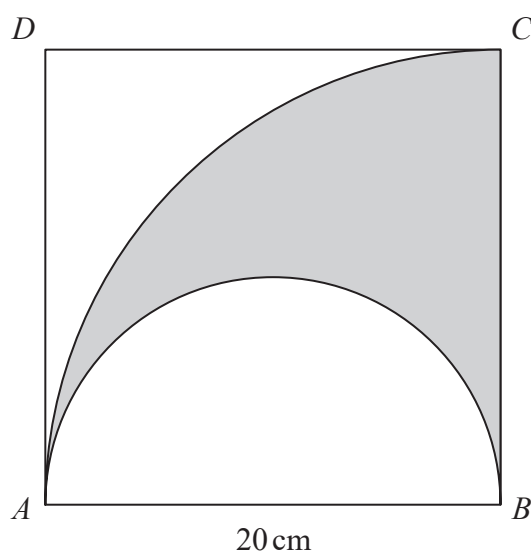
Subtracting the numerators and the denominator stays the same

$$x = \dots\dots\dots \begin{array}{r} 9 \\ 2 \\ -3 \\ \hline 2 \end{array}$$

$$y = \dots\dots\dots \begin{array}{r} 9 \\ 2 \\ -3 \\ \hline 2 \end{array}$$

(Total for Question 6 is 3 marks)

- 7 The diagram shows a square $ABCD$ with sides of length 20 cm. It also shows a semicircle and an arc of a circle.



AB is the diameter of the semicircle.
 AC is an arc of a circle with centre B .

Show that $\frac{\text{area of shaded region}}{\text{area of square}} = \frac{\pi}{8}$

$20^2 = 400$ ← Area of square = length². So the area of the square is 400 cm²

$400\pi \div 4 = 100\pi$ ← Area of circle = $\pi \times \text{radius}^2$. The radius of the quarter circle ABC is 20 cm and $20^2 = 400$. So the area of the whole circle would be 400π cm² and dividing this by 4 works out that the area of the quarter circle ABC is 100π cm²

$20 \div 2$ ← Radius is half of the diameter so the radius of the semicircle is 10 cm

$\pi \times 10^2$ ← Area of circle = $\pi \times \text{radius}^2$. So the area of the whole circle would be 100π cm²

$100\pi \div 2$ ← The semicircle is half of the whole circle so its area is 50π cm²

$100\pi - 50\pi$ ← Subtracting the area of the semicircle from the quarter circle works out that the shaded area is 50π cm²

$\frac{50\pi}{400} = \frac{\pi}{8}$ ← Putting the shaded area over the area of the square. Dividing both the numerator and denominator by 50 simplifies the fraction

(Total for Question 7 is 4 marks)

Writing the angles of 0, 30, 45, 60, 90 degrees. Writing 0, 1, 2, 3, 4 under these for the sin values. Writing 4, 3, 2, 1, 0 under these for the cos values. $\tan 45 = \sin 45 \div \cos 45$, so as the sin and cos value will be the same the tan value must be 1

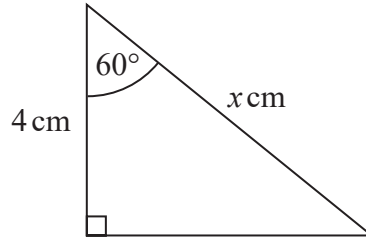
8 (a) Write down the exact value of $\tan 45^\circ$

0 30 45 60 90
 0 1 2 3 4
 4 3 2 1 0

1

(1)

Here is a right-angled triangle.



$\cos 60^\circ = 0.5$ ← $0.5 = 1/2$

(b) Work out the value of x .

S O H C A H T O A

Using right-angled trigonometry. Ticking A as the 4 cm is the adjacent and ticking H as the x cm is the hypotenuse. There are two ticks on the CAH formula triangle so this one can be used

$4 \div \frac{1}{2}$

Covering H in the CAH formula triangle finds that hypotenuse = adjacent \div cos of the angle

4×2

To divide by a fraction: keep the first number, change the division to a multiplication, flip the second fraction. $2/1$ is 2

8

(2)

(Total for Question 8 is 3 marks)

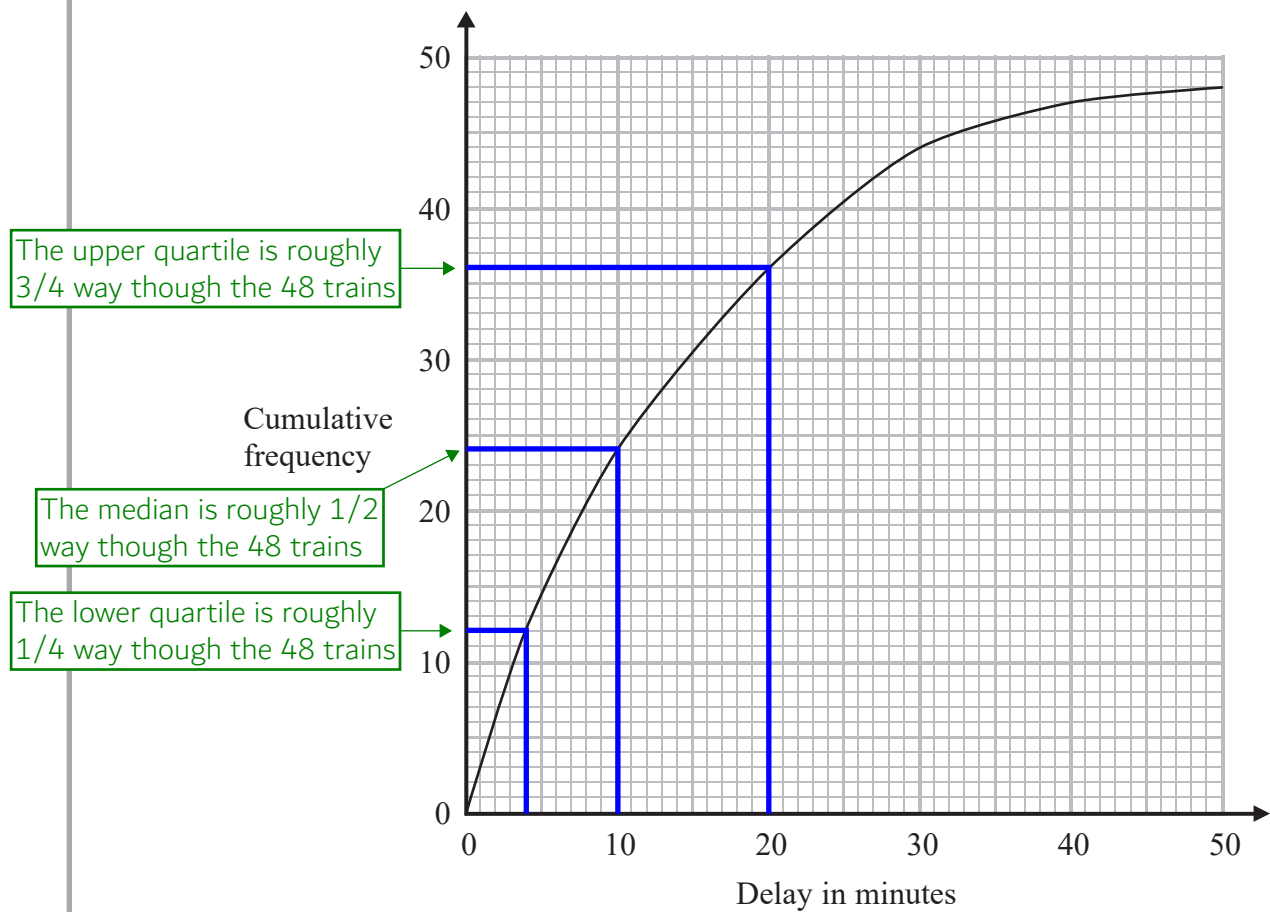
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9 The times that 48 trains left a station on Monday were recorded.

The cumulative frequency graph gives information about the numbers of minutes the trains were delayed, correct to the nearest minute.



The upper quartile is roughly 3/4 way through the 48 trains

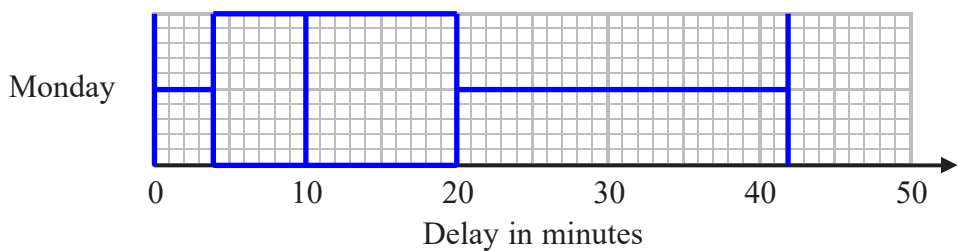
The median is roughly 1/2 way through the 48 trains

The lower quartile is roughly 1/4 way through the 48 trains

The shortest delay was 0 minutes.
The longest delay was 42 minutes.

Drawing vertical lines for the shortest, lower quartile, median, upper quartile and longest. Then joining into a box plot

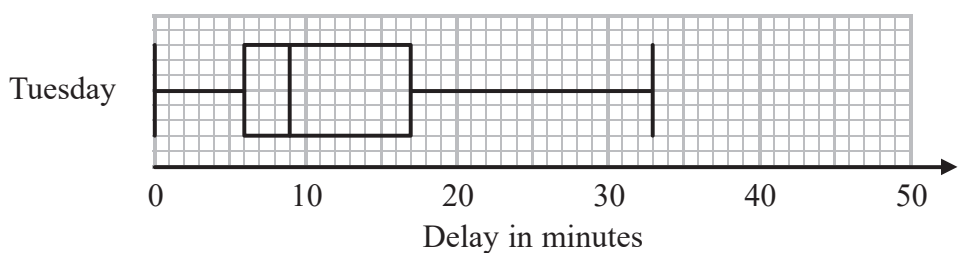
(a) On the grid below, draw a box plot for the information about the delays on Monday.



(3)

48 trains left the station on Tuesday.

The box plot below gives information about the delays on Tuesday.



- (b) Compare the distribution of the delays on Monday with the distribution of the delays on Tuesday.

The median was greater for Monday. The interquartile range was less for Monday

Interquartile range can be seen from the width of the box.
It is the difference between the lower and upper quartiles

(2)

Mary says,

“The longest delay on Tuesday was 33 minutes.

This means that there must be some delays of between 25 minutes and 30 minutes.”

- (c) Is Mary right?

You must give a reason for your answer.

No, as the next longest delay could be less than 25 minutes

The next longest delay could be anything between the upper quartile and the longest delay so could be anything between 17 minutes and 33 minutes

(1)

(Total for Question 9 is 6 marks)

10 (a) Simplify $\frac{x-1}{5(x-1)^2}$

Dividing both the numerator and denominator by $(x-1)$

$$\frac{1}{5(x-1)}$$

(1)

(b) Factorise fully $50 - 2y^2$

$$2(25 - y^2)$$

2 is the highest common factor of 50 and $-2y^2$ so bringing this out as a factor, dividing the 50 and $-2y^2$ by 2 and leaving the result in a bracket

Factorising $25 - y^2$ using difference of two squares. $A^2 - B^2 = (A + B)(A - B)$

$$2(5+y)(5-y)$$

(2)

(Total for Question 10 is 3 marks)

11 Jack and Sadia work for a company that sells boxes of breakfast cereal.

The company wants to have a special offer.

Here is Jack's idea for the special offer.

Put 25% more cereal into each box and do **not** change the price.

Here is Sadia's idea.

Reduce the price and do **not** change the amount of cereal in each box.

Sadia wants her idea to give the same value for money as Jack's idea.

By what percentage does she need to reduce the price?

$$\frac{5}{4}A = P$$

25% is $\frac{1}{4}$. Adding $\frac{1}{4}$ of the cereal to 1 lot of the cereal gives $\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$ of the cereal. Let A be the amount of cereal and P be the price. For $\frac{5}{4}$ of A it is P

$$A = P \div \frac{5}{4}$$

Dividing both sides by $\frac{5}{4}$ works out how much P is for A, which is the price for the full amount of cereal

$$= P \times \frac{4}{5}$$

To divide by a fraction: keep the first number, change the division to a multiplication, flip the second fraction. This can be read as $\frac{4}{5}$ of the price gets the full amount

$$\frac{80}{100}$$

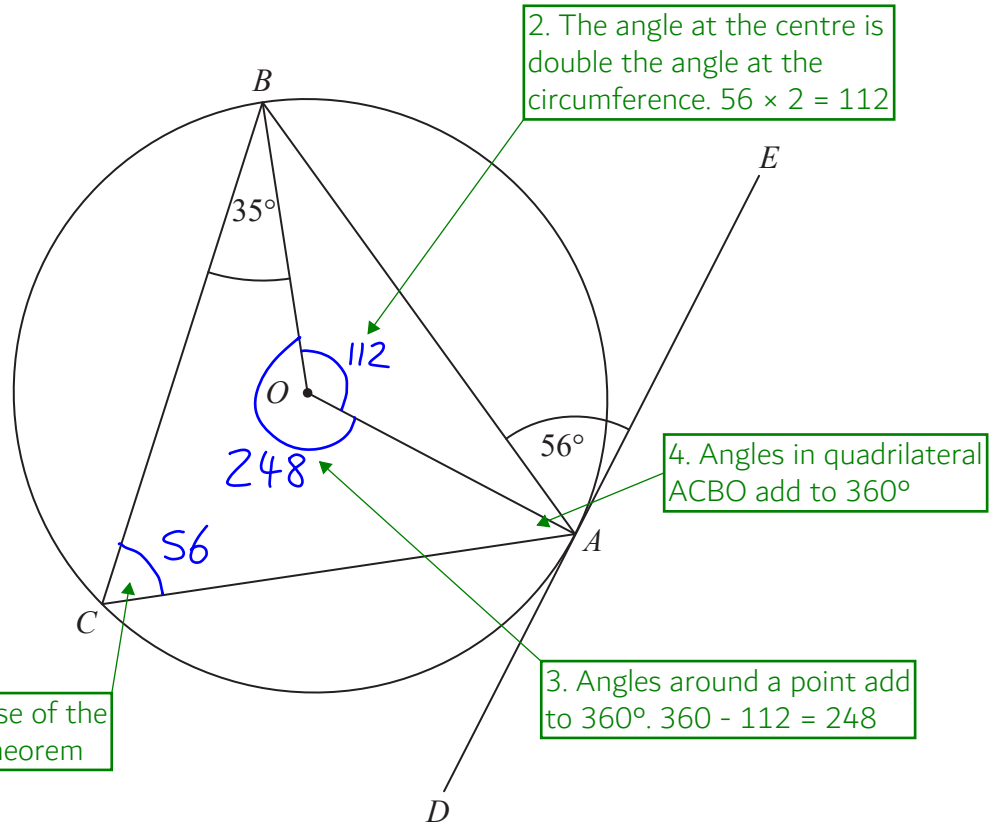
Multiplying both the numerator and denominator of $\frac{4}{5}$ by 20 gives $\frac{80}{100}$, which is equivalent to 80%

Reducing the price by 20% brings it down to 80% of the price

20

%

(Total for Question 11 is 3 marks)



A , B and C are points on the circumference of a circle, centre O .
 DAE is the tangent to the circle at A .

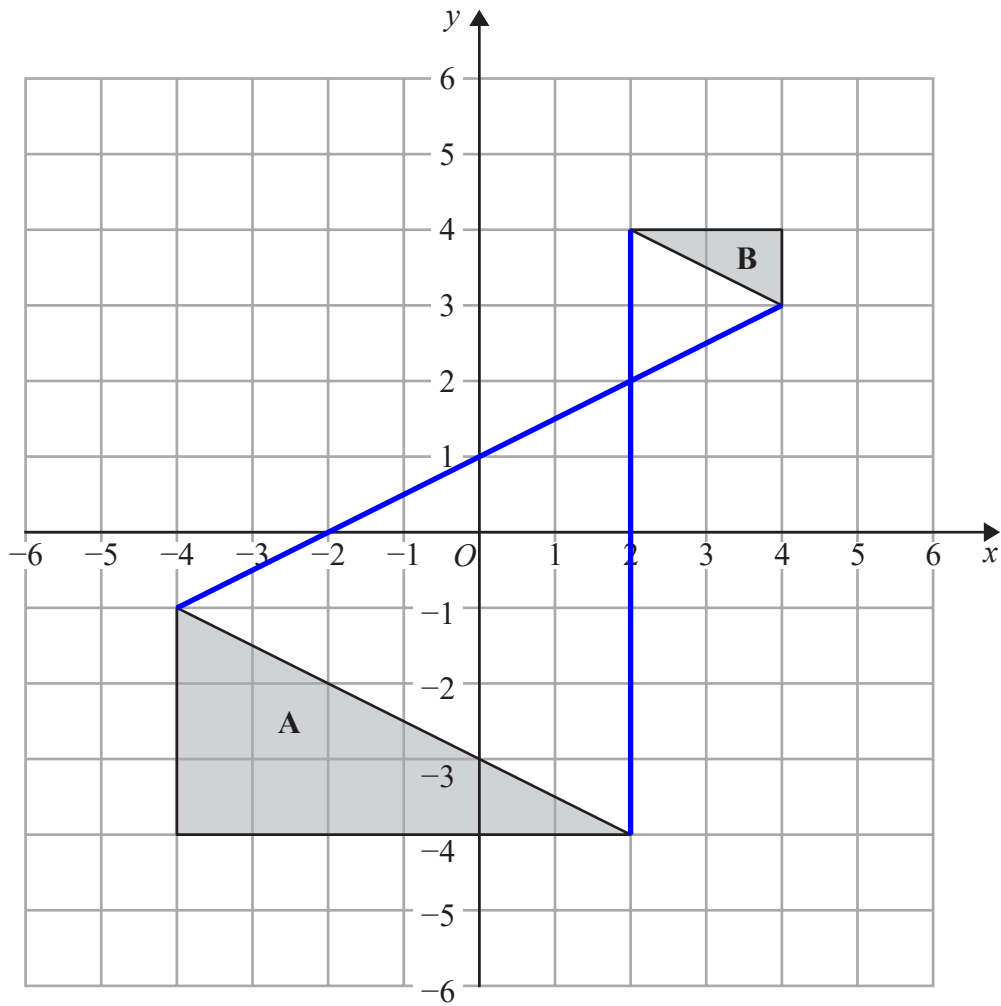
Angle $BAE = 56^\circ$
 Angle $CBO = 35^\circ$

Work out the size of angle CAO .
 You must show all your working.

$$\begin{array}{r}
 56 \\
 \times 2 \\
 \hline
 112
 \end{array}
 \quad
 \begin{array}{r}
 360 \\
 - 112 \\
 \hline
 248
 \end{array}
 \quad
 \begin{array}{r}
 248 \\
 + 56 \\
 \hline
 304 \\
 + 35 \\
 \hline
 339
 \end{array}
 \quad
 \begin{array}{r}
 360 \\
 - 339 \\
 \hline
 21
 \end{array}$$

21

(Total for Question 12 is 3 marks)



Describe fully the single transformation that maps triangle A onto triangle B.

Enlargement, scale factor $-1/3$, centre $(2, 2)$

(Total for Question 13 is 2 marks)

It is an enlargement as it changes size. The scale factor is $-1/3$ as the sides have divided by 3 and B is on the other side of the centre of enlargement. The centre can be found by drawing straight lines from two of the corners to the same corners on the other shape (where the lines meet is the centre)

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14 (a) Work out the value of $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

$\frac{2}{3}$ ← The denominator of 4 in the power means to fourth root, which can be found by square rooting twice

The numerator of 3 in the power means to cube

$\frac{8}{27}$
.....
(2)

$3^a = \frac{1}{9}$ $3^b = 9\sqrt{3}$ $3^c = \frac{1}{\sqrt{3}}$

(b) Work out the value of $a + b + c$

$-2 + 2\frac{1}{2} + (-\frac{1}{2})$

a must be -2 as $1/9 = 1/3^2$ (negative power does the reciprocal). b must be $2\frac{1}{2}$ as $9 = 3^2$ and $\sqrt{3} = 3^{1/2}$, multiplying these together adds the indices. c must be $-1/2$ as $\sqrt{3} = 3^{1/2}$ and it is the reciprocal of this

○
.....
(2)

(Total for Question 14 is 4 marks)

15 Three solid shapes A, B and C are similar.

The surface area of shape A is 4 cm^2

The surface area of shape B is 25 cm^2

The ratio of the volume of shape B to the volume of shape C is 27 : 64

Work out the ratio of the height of shape A to the height of shape C.

Give your answer in its simplest form.

| A | B | C |
|---|----|----|
| 2 | 5 | |
| | 3 | 4 |
| 6 | 15 | 20 |

Surface area of A : surface area of B = 4 : 25. The unit of area is cm^2 . So square rooting the ratio of the surface areas gives lengths of A : lengths of B = 2 : 5.

The unit of volume is cm^3 . So cube rooting the ratio of the volumes gives lengths of B : lengths of C = 3 : 4.

Combining the ratio of lengths by getting the same number of parts for B. Multiplying both sides of the 1st ratio by 3 and multiplying both sides of the 2nd ratio by 5 to get 15 parts for B

Height of B : height of C = 6 : 20. Dividing both sides of this by 2 simplifies it. It cannot go simpler as 3 and 10 cannot be divided by the same amount to get smaller whole numbers

3 : 10

(Total for Question 15 is 4 marks)

16 Prove algebraically that $0.2\dot{5}\dot{6}$ can be written as $\frac{127}{495}$

$$x = 0.2\dot{5}\dot{6} \quad \leftarrow \text{Let } x \text{ be the recurring decimal}$$

$$100x = 25.6\dot{5}\dot{6} \quad \leftarrow \text{There are 2 recurring digits so multiplying by ten 2 times rewrites the decimal with the recurring digits in the same decimal places}$$

$$99x = 25.4 \quad \leftarrow \text{Subtracting } x \text{ from } 100x \text{ cancels out the recurring digits}$$

$$x = \frac{25.4}{99} \quad \leftarrow \text{Dividing both sides by 99 gets } x \text{ (which was the recurring decimal) on its own}$$

$$= \frac{254}{990} \quad \leftarrow \text{Multiplying both the numerator and denominator by 10 eliminates the decimals in the fraction}$$

$$= \frac{127}{495} \quad \leftarrow \text{Dividing both the numerator and denominator by 2 simplifies the fraction and gets the desired fraction}$$

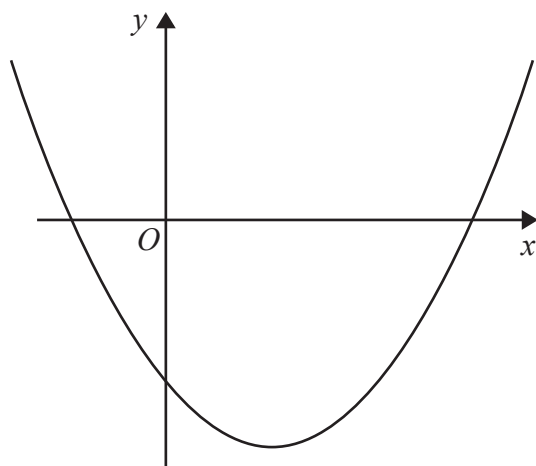
(Total for Question 16 is 3 marks)

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17 Here is a sketch of a curve.



The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.

$$-5 = 0^2 + a(0) + b \quad \leftarrow \text{Substituting the coordinates from } (0, -5) \text{ into the equation}$$

$$-5 = b \quad \leftarrow 0^2 + a(0) = 0 \text{ so can be ignored. So } b \text{ is } -5$$

$$0 = 5^2 + a(5) - 5 \quad \leftarrow \text{Substituting the coordinates from } (5, 0) \text{ into the equation. Substituting } -5 \text{ for } b$$

$$0 = 20 + 5a \quad \leftarrow 5^2 - 5 = 25 - 5 = 20. a(5) = 5a$$

$$-20 = 5a \quad \leftarrow \text{Subtracting } 20 \text{ from both sides gets the } a \text{ term on its own}$$

$$-4 = a \quad \leftarrow \text{Dividing both sides by } 5 \text{ gets } a \text{ on its own. So } a \text{ is } -4$$

$$y = x^2 - 4x - 5 \quad \leftarrow \text{Substituting } -4 \text{ for } a \text{ and } -5 \text{ for } b \text{ in the equation}$$

$$y = (x - 2)^2 - 5 - (-2)^2 \quad \leftarrow \text{Completing the square by halving the coefficient of } x \text{ (which was } -4) \text{ to get } -2, \text{ putting this in a bracket with } x, \text{ squaring the bracket and subtracting } (-2)^2 \text{ from the end}$$

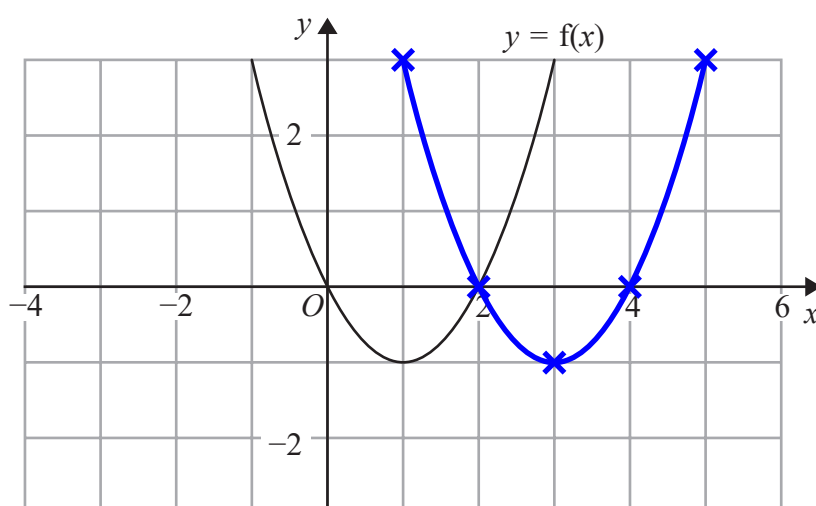
$$y = (x - 2)^2 - 9 \quad \leftarrow -5 - (-2)^2 = -5 - 4 = -9$$

The turning point occurs when the square bracket is equal to 0 (as this is the minimum value a squared value can be). $x = 2$ for this to happen. When the square bracket is 0, $y = -9$

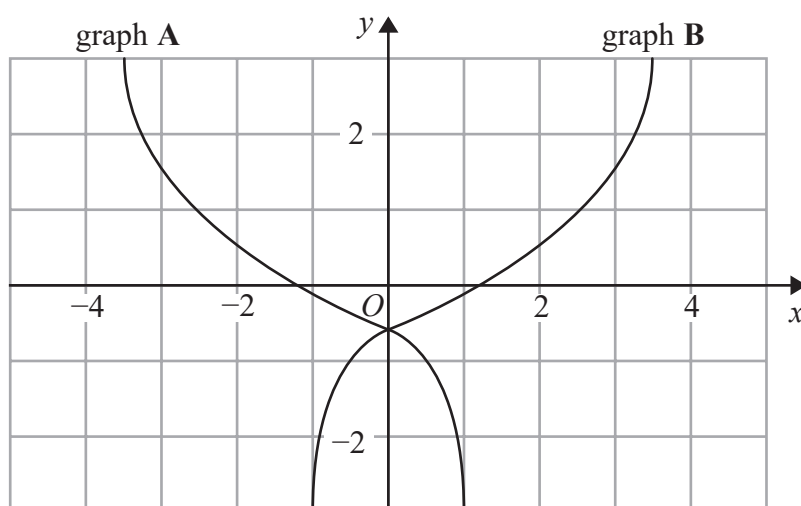
(..... 2 , -9)

(Total for Question 17 is 4 marks)

18 The graph of $y = f(x)$ is shown on the grid below.



- (a) On the grid above, sketch the graph of $y = f(x - 2)$ Subtracting 2 from x translates the graph 2 to the right (1)



On the grid, graph A has been reflected to give graph B.

The equation of graph A is $y = g(x)$

- (b) Write down the equation of graph B.

It is a reflection in the y -axis. This flips the sign of all the x -coordinates

$$y = g(-x)$$

(1)

(Total for Question 18 is 2 marks)

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19 For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(a) Show that $gf(x) = 2x(x + 2)$

$$(x + 1)(x + 1) \leftarrow (x + 1)^2 \text{ is } (x + 1) \text{ multiplied by itself}$$

$$x^2 + x + x + 1 \leftarrow \text{Expanding the brackets}$$

$$2(x^2 + 2x + 1 - 1) \leftarrow \text{Simplifying by collecting like terms and substituting for } x \text{ in } g(x) \text{ to express the composite function } gf(x)$$

$$2(x^2 + 2x) \leftarrow \text{The } +1 \text{ and } -1 \text{ cancel out}$$

$$2x(x + 2) \leftarrow \text{Bringing } x \text{ out as a factor}$$

(2)

(b) Find $g^{-1}(7)$

$$x = 2(y - 1) \leftarrow \text{Switching } g(x) \text{ for } x \text{ and } x \text{ for } y \text{ in } g(x)$$

$$y - 1 = \frac{x}{2} \leftarrow \text{Dividing both sides by } 2$$

$$y = \frac{x}{2} + 1 \leftarrow \text{Adding } 1 \text{ to both sides gets } y \text{ on its own. The right side is the inverse function } f^{-1}(x)$$

$$\frac{7}{2} + \frac{2}{2} \leftarrow \text{Substituting } 7 \text{ for } x \text{ in } f^{-1}(x). \text{ Writing } 1 \text{ as } 2/2 \text{ so that it has the same denominator}$$

The numerators can be added and the denominator stays the same

$$\frac{9}{2}$$

(2)

(Total for Question 19 is 4 marks)

20 Show that $\frac{(\sqrt{18} + \sqrt{2})^2}{\sqrt{8} - 2}$ can be written in the form $a(b + \sqrt{2})$ where a and b are integers.

$\sqrt{9}\sqrt{2}$ ← Simplifying $\sqrt{18}$ by splitting it into two square roots, one of which is the square root of a square number

$3\sqrt{2} + \sqrt{2}$ ← $\sqrt{9} = 3$. So $\sqrt{18} = 3\sqrt{2}$. Then adding the $\sqrt{2}$

$(4\sqrt{2})^2 = 32$ ← $3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$. Then squaring this gives 32 (as $4^2 = 16$ and $(\sqrt{2})^2 = 2$ then $16 \times 2 = 32$. So this is the numerator

$\sqrt{4}\sqrt{2}$ ← Simplifying $\sqrt{8}$ by splitting it into two square roots, one of which is the square root of a square number

$(2\sqrt{2} - 2)(2\sqrt{2} + 2)$ ← $\sqrt{4} = 2$. So $\sqrt{8} = 2\sqrt{2}$ and the denominator is $2\sqrt{2} - 2$. Flipping the sign in the middle and multiplying by this will rationalise the denominator

$8 + 2\sqrt{2} - 2\sqrt{2} - 4$ ← Expanding the brackets

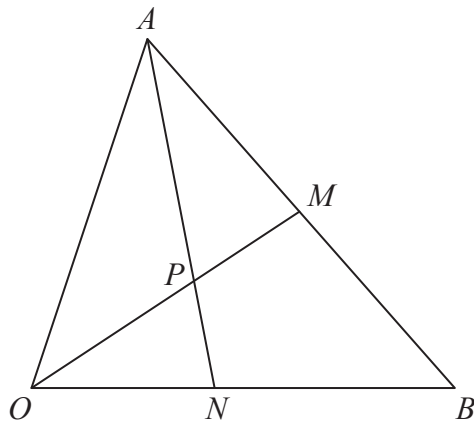
$\frac{32(2\sqrt{2} + 2)}{4}$ ← Collecting like terms works out that the new denominator is 4. Because the denominator was multiplied by $(2\sqrt{2} + 2)$, the numerator must also be multiplied by the same

$8(2 + 2\sqrt{2})$ ← $32/4 = 8$. Switching the 2 and $2\sqrt{2}$

$16(1 + \sqrt{2})$ ← Bringing out 2 as a factor and multiplying this by the 8 which is already outside the bracket

(Total for Question 20 is 3 marks)

21



OAB is a triangle.
 OPM and APN are straight lines.
 M is the midpoint of AB .

$$\vec{OA} = \mathbf{a} \quad \vec{OB} = \mathbf{b}$$

$$OP:PM = 3:2$$

Work out the ratio $ON:NB$

The method is on the next page

..... 3 : 4

(Total for Question 21 is 5 marks)

$$\vec{AB} = -a + b \leftarrow \vec{AB} = \vec{AO} + \vec{OB}. \vec{AO} = -\vec{OA} = -a. \vec{OB} = b$$

$$\vec{AM} = -0.5a + 0.5b \leftarrow \vec{AM} = 0.5\vec{AB} \text{ as } M \text{ is the midpoint of } AB$$

$$\vec{OM} = a + -0.5a + 0.5b \leftarrow \vec{OM} = \vec{OA} + \vec{AM}. \vec{OA} = a$$

$$= 0.5a + 0.5b \leftarrow a - 0.5a = 0.5a$$

$$\vec{AN} = -a + xb \leftarrow \vec{AN} = \vec{AO} + \vec{ON}. \vec{AO} = -\vec{OA} = -a. \vec{ON} = x\vec{OB} = xb, \text{ where } x \text{ is a fraction of } OB$$

$$= y\left(-a + \frac{3}{5}\left(\frac{1}{2}a + \frac{1}{2}b\right)\right) \leftarrow \vec{AN} = y\vec{AP} \text{ (where } y \text{ is a constant) as } APN \text{ is a straight line so } \vec{AN} \text{ is in the same direction as } \vec{AP} \text{ but is longer. } \vec{AP} = \vec{AO} + \vec{OP}. \vec{AO} = -\vec{OA} = -a. \vec{OP} = \frac{3}{5}\vec{OM}, \text{ as } 3 + 2 = 5 \text{ parts in total in the ratio and } OP \text{ is } 3 \text{ of these. } 0.5 \text{ is written as } \frac{1}{2} \text{ to make it easier to multiply}$$

$$= y\left(-\frac{10}{10}a + \frac{3}{10}a + \frac{3}{10}b\right) \leftarrow \text{Expanding the inside bracket and writing } -a \text{ as } -\frac{10}{10}a \text{ so that it has the same denominator as the } \frac{3}{10}a$$

$$= -\frac{7}{10}ya + \frac{3}{10}yb \leftarrow -\frac{10}{10} + \frac{3}{10} = -\frac{7}{10}. \text{ Then expanding the brackets}$$

$$-\frac{7}{10}y = -1 \leftarrow \vec{AN} \text{ was expressed in two different ways and these must be equivalent. Equating the coefficients of } a. \text{ The coefficient of } -a \text{ is } -1 \text{ and the coefficient of } -\frac{7}{10}ya \text{ is } -\frac{7}{10}y$$

$$y = -1 \div -\frac{7}{10} \leftarrow \text{Dividing both sides by } -\frac{7}{10} \text{ gets } y \text{ on its own}$$

$$= -1 \times -\frac{10}{7} \leftarrow \text{To divide by a fraction: keep the 1st number, change the division to a multiplication, flip the 2nd fraction. So } y = \frac{10}{7}$$

$$x = \frac{3}{10} \times \frac{10}{7} \leftarrow \vec{AN} \text{ was expressed in two different ways and these must be equivalent. Equating the coefficients of } b. \text{ The coefficient of } xb \text{ is } x \text{ and the coefficient of } \frac{3}{10}yb \text{ is } \frac{3}{10} \times \frac{10}{7} \text{ as } y \text{ is } \frac{10}{7}$$

$$= \frac{3}{1} \times \frac{1}{7} \leftarrow \text{Simplifying by dividing the numerators and denominators by } 10. \text{ So } x = \frac{3}{7}$$

$$\vec{ON} = a - a + \frac{3}{7}b \leftarrow \vec{ON} = \vec{OA} + \vec{AN}. \vec{OA} = a. \text{ Substituting } \frac{3}{7} \text{ for } x \text{ in } -a + xb \text{ for } \vec{AN}$$

$$\frac{3}{7}b : \frac{4}{7}b \leftarrow a - a \text{ cancels out so } \vec{ON} = \frac{3}{7}b. \text{ Subtracting this from } b \text{ (which is } \vec{OB}) \text{ finds that } \vec{NB} = \frac{4}{7}b. \text{ Expressing these as a ratio}$$

$$\text{Multiplying both sides by } 7 \text{ and dividing both sides by } b \text{ gives } 3 : 4$$

22 There are only green pens and blue pens in a box.

There are three more blue pens than green pens in the box.

There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.

The probability that Simon will take two pens of the same colour is $\frac{27}{55}$

Work out the number of green pens in the box.

$G + G + 3$ ← Let G be the number of green pens. The number of blue pens must be $G + 3$ as there are 3 more blue pens than green pens. Adding the number of green pens and the number of blue pens expresses the total number of pens. Collecting like terms gives $2G + 3$

$$\frac{G}{2G+3} \times \frac{G-1}{2G+2} + \frac{G+3}{2G+3} \times \frac{G+2}{2G+2}$$

Expressing the probability of taking two pens of the same colour in terms of G . Green AND green OR blue AND blue. AND means to multiply the probabilities. OR means to add the probabilities. There is one fewer green pen after the 1st pick so is $G - 1$ for the 2nd pick. There is one fewer blue pen after the 1st pick so is $G + 2$ for the 2nd pick. There is one fewer counter in total after the 1st pick so is $2G + 2$ for the 2nd pick

$$\frac{G^2 - G}{4G^2 + 4G + 6G + 6} + \frac{G^2 + 2G + 3G + 6}{4G^2 + 4G + 6G + 6}$$

← Expanding the numerators and denominators like expanding brackets

$$\frac{2G^2 + 4G + 6}{4G^2 + 10G + 6} = \frac{27}{55}$$

← Collecting like terms and adding the fractions. The numerators can be added and the denominator stays the same. Setting equal to the value of the probability of taking two pens of the same colour

$$55(2G^2 + 4G + 6) = 27(4G^2 + 10G + 6)$$

← Multiplying both sides by the denominators to eliminate them

$$110G^2 + 220G + 330 = 108G^2 + 270G + 162$$

← Expanding the brackets

$$2G^2 - 50G + 168 = 0$$

← Bringing into the quadratic form by subtracting everything on the right from both sides

$$G^2 - 25G + 84 = 0$$

← Simplifying by dividing all terms by 2

$$(G - 21)(G - 4) = 0$$

← Factorising the left. -21 and -4 add to the -25 and multiply to the 84

$$G - 21 = 0$$

← One of the two brackets must be 0

$$G = 21$$

← Adding 21 to both sides

$$G - 4 = 0$$

← One of the two brackets must be 0

$$G = 4$$

← Adding 4 to both sides

$$4 + 4 + 3 = 11$$

← Substituting 4 into $G + G + 3$ (the expression of the total number of pens used at the start) gives 11 pens in total, which is not more than 12 pens in total. So the number of green pens cannot be 4

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(Total for Question 22 is 6 marks)

TOTAL FOR PAPER IS 80 MARKS