AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number	☐☐☐☐☐	Candidate number	☐☐☐☐

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE
MATHEMATICS

H

Higher Tier Paper 3 Calculator

Tuesday 13 June 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

J U N 1 7 8 3 0 0 3 H 0 1

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** questions in the spaces provided

1 **a** = $\begin{pmatrix}-4\\-1\end{pmatrix}$ and **b** = $\begin{pmatrix}3\\-1\end{pmatrix}$

x-component

y-component

Circle the vector **2a** + **b**

[1 mark]

[✓ $\begin{pmatrix}-5\\-3\end{pmatrix}$] $\begin{pmatrix}-11\\-3\end{pmatrix}$ $\begin{pmatrix}-5\\-1\end{pmatrix}$ $\begin{pmatrix}-11\\-1\end{pmatrix}$

Substituting in the x-components: 2 x -4 + 3 = -5.
Substituting in the y-components: 2 x -1 + -1 = -3

2 Which of these values of n makes 2.7×10^n a cube number?

Circle your answer.

[1 mark]

0 [✓ 1] 2 3

2.7×10^1 gives 27, which is 3^3

3 Rearrange $2x = \frac{y}{w}$ to make w the subject.

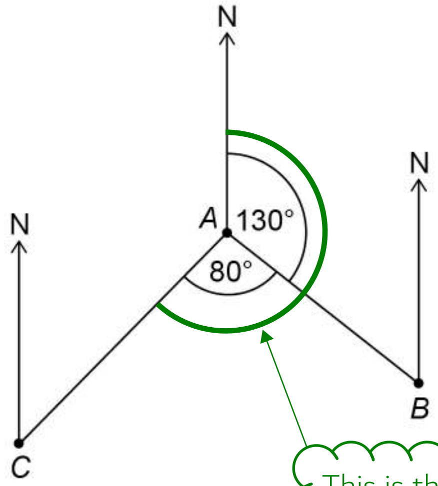
Circle your answer.

[1 mark]

$w = \frac{2y}{x}$ $w = \frac{2x}{y}$ [✓ $w = \frac{y}{2x}$] $w = \frac{x}{2y}$

Multiplying both sides by w gives 2xw = y.
Dividing both sides by 2x gets w on its own.

4



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This is the angle we are looking for. Turning clockwise from north at A until facing C

Work out the bearing of *C* from *A*.

Circle your answer.

[1 mark]

030°   130°   150°   [✓ 210°]

Adding the 130 degree angle and the 80 degree angle gives the bearing of C from A

Turn over for the next question

5 A coin lands on Tails 200 times.

The relative frequency of Tails is 0.4

Work out the number of times the coin was thrown.

[2 marks]

$0.4x=200$

Let x be the number of times the coin was thrown. Multiplying x by the relative frequency of Tails must give the 200

$x=200\div0.4$

Dividing both sides by 0.4 finds that x = 500

Answer 500

6 How are the whole number solutions to A and B different?

A Solve $3 \leqslant 3x < 18$

B Solve $3 < 3x \leqslant 18$

[2 marks]

$1\leq x<6$

$1<x\leq6$

Solving both inequalities by dividing all sides by 3. For A: x is greater than equal to 1 but less than 6. For B: x is greater than 1 but less than or equal to 6

A is 1,2,3,4,5

B is 2,3,4,5,6

A can be equal to 1 so includes 1.
B cannot be equal to 1 so does not include 1.
A cannot be equal to 6 so does not include 6.
B can be equal to 6 so includes 6

7 (a) The length of a pipe is 6 metres to the nearest metre.

Complete the error interval for the length of the pipe.

[2 marks]

$6 \pm \frac{1}{2}$ ← The resolution is 1 m as it is to the nearest metre. Halving this then adding and subtracting this from 6 gives the upper and lower bound

Answer 5.5 m ⩽ length < 6.5 m

7 (b) The length of a different pipe is 4 metres to the nearest metre.

Olly says,

“The total length of the two pipes is 11 metres to the nearest metre.”

Give an example to show that he could be correct.

[2 marks]

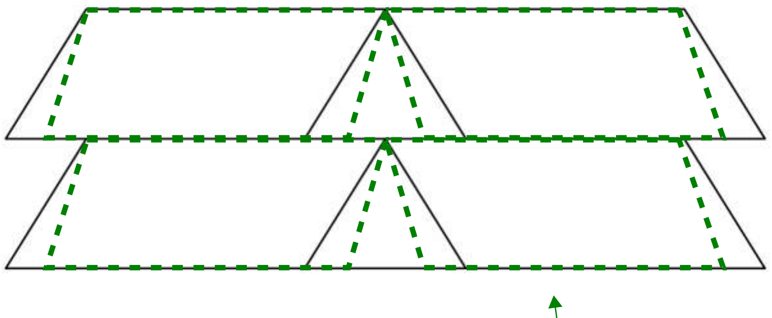
$4 + \frac{1}{2}$ ← The resolution is 1 m as it is to the nearest metre. Halving this then adding this to the 4 works out that the upper bound is 4.5 m

$6.4 + 4.4 = 10.8$ ← 6.4 m is a possible length for the first pipe. 4.4 m is a possible length for the second pipe. Adding these gives a total length of 10.8 m, which rounds to 11 m to the nearest metre. 6.5 m and 4.5 m cannot be used as these are not possible lengths. The first pipe must be less than 6.5 m and the second pipe must be less than 4.5 m

Turn over for the next question

8

8 This shape is made from two triangles and four congruent parallelograms.



Not drawn accurately

For each statement, tick the correct box.

8 (a) The triangles are equilateral.

[1 mark]

☐ Must be true

☑ Could be true

☐ Must be false

The parallelograms could be like this. The triangles aren't equilateral. But they could be equilateral as they are in the original diagram.

8 (b) The triangles are congruent.

[1 mark]

☑ Must be true

☐ Could be true

☐ Must be false

All three sides in both triangles must be the same as they are defined by the parallelograms. Regardless of how the parallelograms are drawn, the triangles will be the same shape and size so must be congruent.

9 There are 720 boys and 700 girls in a school.

The probability that a boy chosen at random studies French is $\frac{2}{3}$

The probability that a girl chosen at random studies French is $\frac{3}{5}$

9 (a) Work out the number of students in the school who study French.

[3 marks]

$\frac{2}{3} \times 720 + \frac{3}{5} \times 700$

Doing 2/3 of the 720 boys and 3/5 of the 700 girls and adding these together works out that there are 900 students in the school who study French

Answer 900

9 (b) Work out the probability that a student chosen at random from the whole school does **not** study French.

[2 marks]

$720 + 700$

Adding the 720 boys and the 700 girls works out that there are 1420 students in total in the school

$1420 - 900$

Subtracting the 900 who study French from the 1420 total students works out that 520 students do not study French

Answer $\frac{520}{1420}$

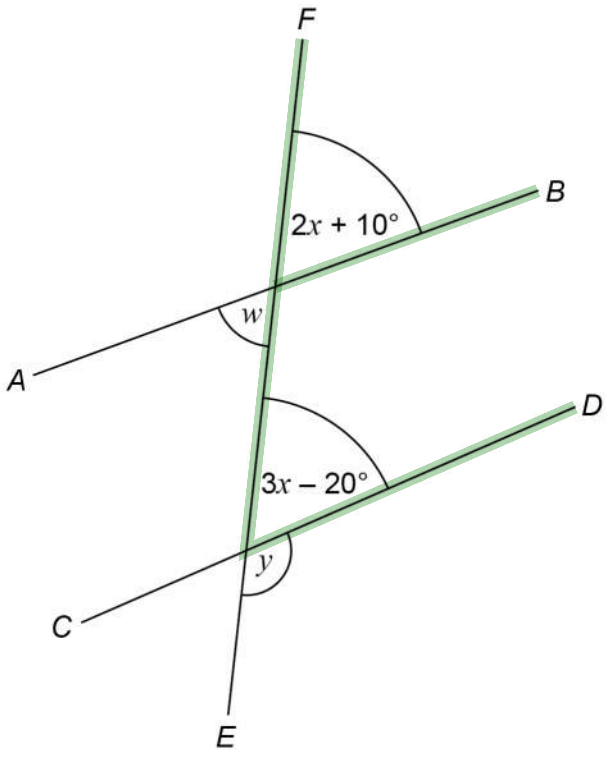
520 out of the 1420 students do not study French

Turn over for the next question

7

Turn over ►

10 *AB*, *CD* and *EF* are straight lines.



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10 (a) Ava assumes that *AB* and *CD* are parallel.

What answer should she get for the size of angle y?

[4 marks]

$3x-20=2x+10$ ← Angles 3x - 20 and 2x + 10 are corresponding (as shown on the diagram with the F-shape) so must be equal

$x-20=10$ ← Subtracting 2x from both sides gets all the x on the same side

$x=30$ ← Adding 20 to both sides gets x on its own

$3\times30-20$ ← Substituting 30 for x in 3x - 20 works out that it is worth 70 degrees

$180-70$ ← Angles around a point on a straight line add up to 180 degrees. So subtracting the 70 degrees from 180 works out that y is 110 degrees

Answer 110 degrees

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10 (b) In fact,

AB and *CD* are **not** parallel

angle w is 60°

What effect does this have on the size of angle y?

Tick a box.

☑ y is bigger

☐ y is the same

☐ y is smaller

Show working to support your answer.

[3 marks]

The angles are no longer corresponding so the working in (a) is no longer relevant

$2x+10=60$ ← w is vertically opposite to 2x + 10 so they must be equal

$2x=50$ ← Subtracting 10 from both sides gets the x term on its own

$x=25$ ← Dividing both sides by 2 gets x on its own

$3\times25-20$ ← Substituting 25 for x in 3x - 20 works out that it is worth 55 degrees

$180-55=125$ ← Angles around a point on a straight line add up to 180 degrees. So subtracting the 55 degrees from 180 works out that y is 125 degrees

Turn over for the next question

y is now 125 degrees which is bigger than the 110 degrees worked out in (a)

7

11 Purple paint is made by mixing red paint and blue paint in the ratio 5 : 2

Yan has 30 litres of red paint and 9 litres of blue paint.

What is the **maximum** amount of purple paint he can make?

[3 marks]

See next page for an explanation

Answer 31.5 litres

12 $\left(ar^{b}\right)^{4} = 16r^{20}$ where a and b are positive integers.

Work out a and b

[2 marks]

a^4r^{4b} ← Raising both the a and the r^b to the power of 4. $(a^x)^y = a^{xy}$ so the b and 4 are multiplied

$\sqrt[4]{16}$ ← a^4 must be 16. Fourth rooting the 16 finds that a = 2

$20 \div 4$ ← The power of 4b must be equal to the power of 20. Dividing the 20 by 4 works out that b = 5

a = 2 b = 5

30÷5	Assuming that all 30 litres of red paint is used, 30 litres of red paint is represented by 5 parts of the ratio. So dividing the 30 litres by the 5 parts works out that 1 part of the ratio would be worth 6 litres
6×2	Multiplying the value of 1 part of the ratio by the 2 parts which represent the blue works out that 12 litres of blue paint would be needed. There is not this much so not all the 30 litres of red paint can be used
9÷2	Assuming that all 9 litres of blue paint is used, 9 litres of blue paint is represented by 2 parts of the ratio. So dividing the 9 litres by the 2 parts works out that 1 part of the ratio would be worth 4.5 litres
4.5×5	Multiplying the value of 1 part of the ratio by the 5 parts which represent the red works out that 22.5 litres of red paint would be needed. There is enough red paint for this
22.5+9	Adding the 22.5 litres of red and the 9 litres of blue works out that the maximum amount of purple paint he can make is 31.5 litres

13 In a class of 28 students

the mean height of the 12 boys is 1.58 metres

the mean height of all 28 students is 1.52 metres.

Work out the mean height of the girls.

[4 marks]

$\overset{t}{m\ \ n}$

Mean = total ÷ number, where total is the total height and number is the number of students. Writing this as a formula triangle

$28-12=16$

Subtracting the 12 boys from the 28 students works out that there are 16 girls

$1.58\times12=18.96$

Covering t in the formula triangle finds that total = mean x number. So the total height of the boys is 18.96 m

1.52×28

Covering t in the formula triangle finds that total = mean x number. So the total height of the students is 42.56 m

$42.56-18.96$

Subtracting the total height of the boys from the total height of the students works out that the total height of the girls is 23.6 m

$23.6\div16$

Mean = total ÷ number, so dividing the total height of the girls by the number of girls works out the mean height of the girls

Answer 1.475 metres

14 $xy = c$ where c is a constant.

Circle the correct statement.

[1 mark]

y is directly proportional to x

[✓ y is directly proportional to $\frac{1}{x}$]

These all mean the same thing. Increasing x will increase y

y is inversely proportional to $\frac{1}{x}$

x is directly proportional to y

y must be directly proportional to 1/x as increasing x must decrease y for it to be equal to a constant when they are multiplied

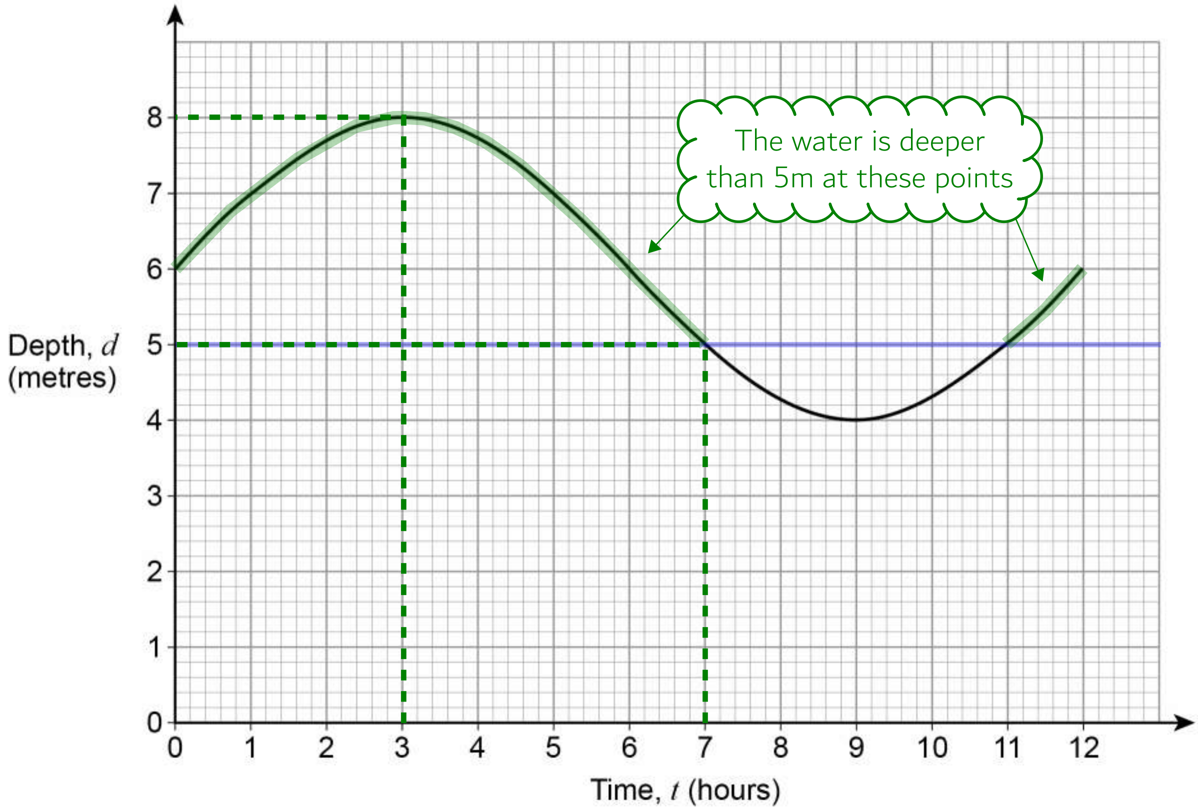
Turn over for the next question

10

15 The graph shows the depth of water in a harbour for 12 hours.

d is the depth of water in a harbour in metres

t is the number of hours after 9 am



15 (a) For how many of the 12 hours is the depth more than 5 metres?

[1 mark]

Answer 8

Drawing a line across at the depth of 5 m. Everything above this line is when the depth was more than 5 m

15 (b) By how much does the depth change between 12 noon and 4 pm?

[1 mark]

Answer 3 metres

12 noon is represented by t = 3 as it is 3 hours after 9am. 4pm is when t = 7. Reading up from t = 3 to the line and across finds that the depth at 12 noon is 8 m. Reading up from t = 7 to the line then across finds that the depth at 4 pm is 5 m. The difference between 8 m and 5 m is 8 - 5 = 3 m

16 The value of a new car is £18 000

The value of the car decreases by

25% in the first year

12% in each of the next 4 years.

Work out the value of the car after 5 years.

[3 marks]

$18000 \times \frac{100-25}{100} \times \left(\frac{100-12}{100}\right)^4$

100% - 25% expresses the percentage the value of the car decreases to in the first year. Putting this over 100 converts it to a fraction, which when multiplied by the £18000 decreases it by 25%.

100% - 12% expresses the percentage the value of the car decreases to in each of the next 4 years. Putting this over 100 converts it to a fraction, which when multiplied by decreases by 12%. Raising the fraction to the power of 4 as it is decreased by 12% 4 times

Answer £ 8095.89

Rounding 8095.887... to the nearest pence

Turn over for the next question

5

Turn over ►

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17 Liam drives his car.

He drives the first 9 miles in 9 minutes.

He then drives at an average speed of 70 miles per hour for 1 hour 36 minutes.

He finds this information about his car.

Average speed	**Miles travelled per gallon**
65 miles per hour or less	50
More than 65 miles per hour	40

Use the information to show that his car uses less than 3 gallons of petrol for the drive.

[5 marks]

$s \, ^{d} \, t$ ← Writing the distance, speed, time formula triangle

$9 \div 0°9° = 60$ ← Covering s in the formula triangle finds that speed = distance ÷ time. Putting the time into the calculator as a sexagesimal. This works out that the average speed for the first 9 miles is 60 miles per hour

$70 \times 1°36°$ ← Covering d in the formula triangle finds that distance = speed x time. Putting the time into the calculator as a sexagesimal. This works out that the distance in the second part of the journey is 112 miles

$\frac{9}{50} + \frac{112}{40} = 2.98$ ← If 50 miles can be done with one gallon when travelling at 60 miles per hour, 9/50 gallons would be used for the 9 miles. If 40 miles can be done with one gallon when travelling at 70 miles per hour, 112/40 gallons would be used for the 114 miles. Adding these works out that 2.98 gallons of petrol are used for the drive, which is less than 3 gallons

18 Nick sketches the graph of $y = 0.5^x$ for $x \geqslant 0$

Make **one** criticism of his sketch.

[1 mark]

$0.5^0=1$ When x = 0, y = 1, not 0.5

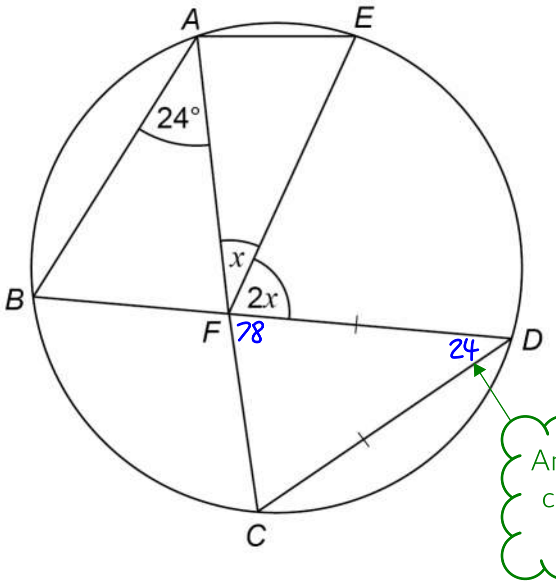
Turn over for the next question

6

19 *A*, *B*, *C*, *D* and *E* are points on a circle.

BFD and *AFC* are straight lines.

DC = *DF*



Not drawn accurately

Angles in the same segment from a common chord are equal. Both angles BAC and BDC are in the same segment from chord BC

Work out the size of angle x.

You **must** show your working which may be on the diagram.

[4 marks]

180-24 ← There are 180 degrees in total in a triangle. Subtracting the 24 degrees in triangle FDC works out that there are 156 degrees left over

156÷2 ← Triangle FDC is an isosceles triangle as DC = DF meaning that the base angles are equal. So dividing the 156 degrees by 2 works out that angle CFD is 78 degrees

180-78 ← Angles around a point on a straight line add up to 180 degrees. So subtracting the 78 degrees from 180 degrees works out that x + 2x must be 102 degrees

102÷3 ← x + 2x = 3x = 102. Dividing 102 by 3 finds that x = 34

Answer 34 degrees

20 This sign shows when a lift is safe to use.

Total mass of people must be 450 kg or less

Ben and some other people are in the lift.

Their total mass is 525 kg to the nearest 5 kg

Ben gets out.

He has a mass of 78 kg to the nearest kg

Is the lift now safe to use?

You **must** show your working.

[4 marks]

$525+\frac{5}{2}=527.5$ ← 525 kg is to the nearest 5 kg. The resolution is 5. Adding half of this to the 525 works out that the upper bound of their total mass is 527.5 kg

$78-\frac{1}{2}$ ← 78 kg is to the nearest 1 kg. The resolution is 1. Subtracting half of this from the 78 works out that the lower bound of Ben's mass is 77.5 kg

$527.5-77.5=450$ ← Subtracting the lower bound of Ben's mass from the upper bound of their total mass works out that the upper bound of the mass left in the lift is 450 kg

Answer Yes

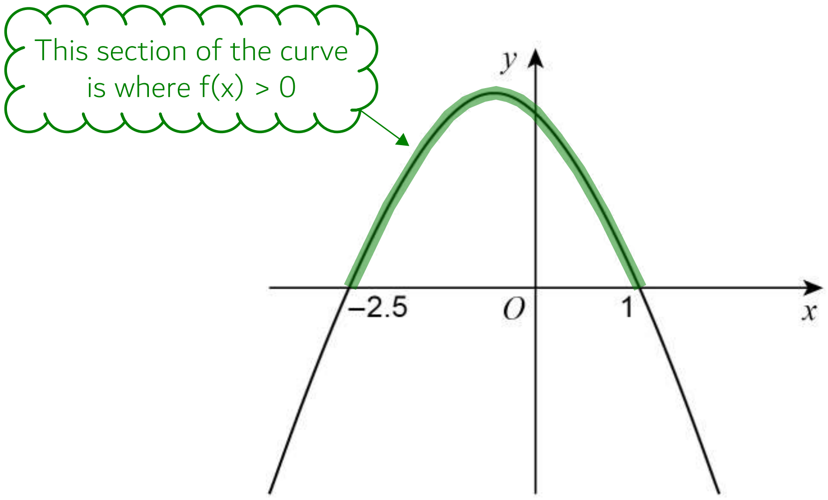
450 kg is safe as it can be 450 kg or less

Turn over for the next question

8

21 Here is a sketch of $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic function.

The graph intersects the x-axis where $x = -2.5$ and $x = 1$



This section of the curve is where f(x) > 0

Not drawn accurately

Circle the solution of $\mathrm{f}(x) > 0$

[1 mark]

$x < -2.5$ or $x > 1$

$x > -2.5$ or $x > 1$

[✓ $-2.5 < x < 1$]

$x > -2.5$ or $x < 1$

x must be both greater than -2.5 and less than 1

22 Work out an expression for the nth term of the quadratic sequence

The first differences are 15 then 23

2 15 17 23 40 71

8

The second difference is 8 as this is the difference between 15 and 23

Give your answer in the form $an^2 + bn + c$ where a, b and c are constants.

[3 marks]

$4n^2$: 4, 16

Halving the second difference of 8 finds that the coefficient of x^2, a, must be 4. Listing the sequence of $4n^2$

-2, 1 : 3n-5

Listing what needs to be added to the terms in $4n^2$ to get the original sequence. This is a linear sequence bn + c. It must involve 3n as it goes up by 3.Going back in the sequence finds that the 0th term would be -5. So it must be 3n - 5

Answer $4n^2+3n-5$

Adding the sequences of $4n^2$ and 3n - 5 gives the original sequence

Turn over for the next question

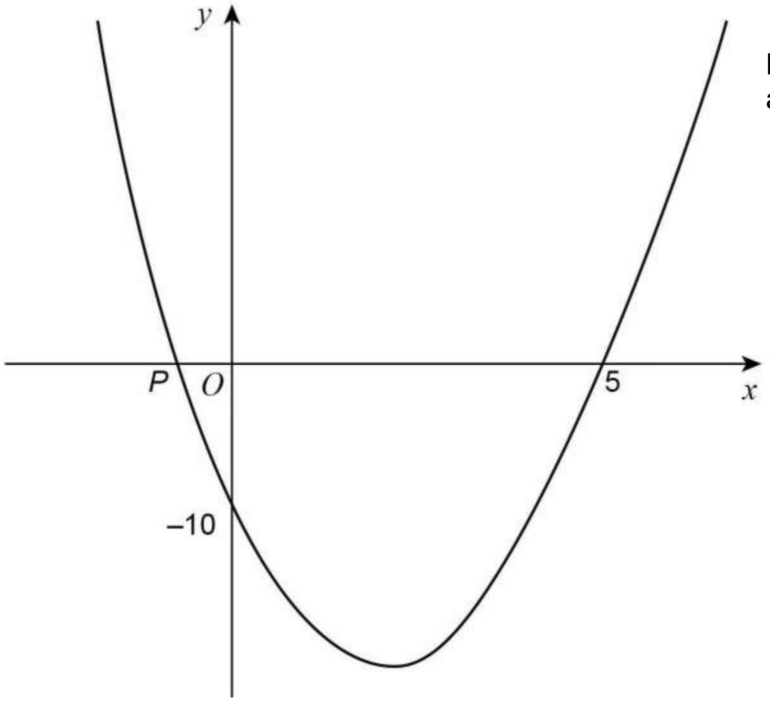
4

23 Here is a sketch of $y = x^2 + bx + c$

The curve intersects

the x-axis at (5, 0) and point P

the y-axis at (0, –10)



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Work out the x-coordinate of the turning point of the graph.

[4 marks]

$c=-10$ ← c is the y-intercept which is -10

$0=5^2+5b-10$ ← Using the coordinates of (5, 0) so substituting 0 for y, 5 for x and -10 for c

$-15=5b$ ← Subtracting 5^2 and adding 10 to both sides to get the x term on its own

$-3=b$ ← Dividing both sides by 5 gets b on its own

$y=x^2-3x-10$ ← Writing the equation and substituting in the values of b and c

$=(x-1.5)^2-10-1.5^2$ ← Completing the square by halving the coefficient of x (which was -3), putting the result in a bracket with x, squaring the bracket and subtracting the 1.5^2

Answer 1.5

The minimum value a squared bracket can be is 0. When the bracket is equal to 0, x must be 1.5

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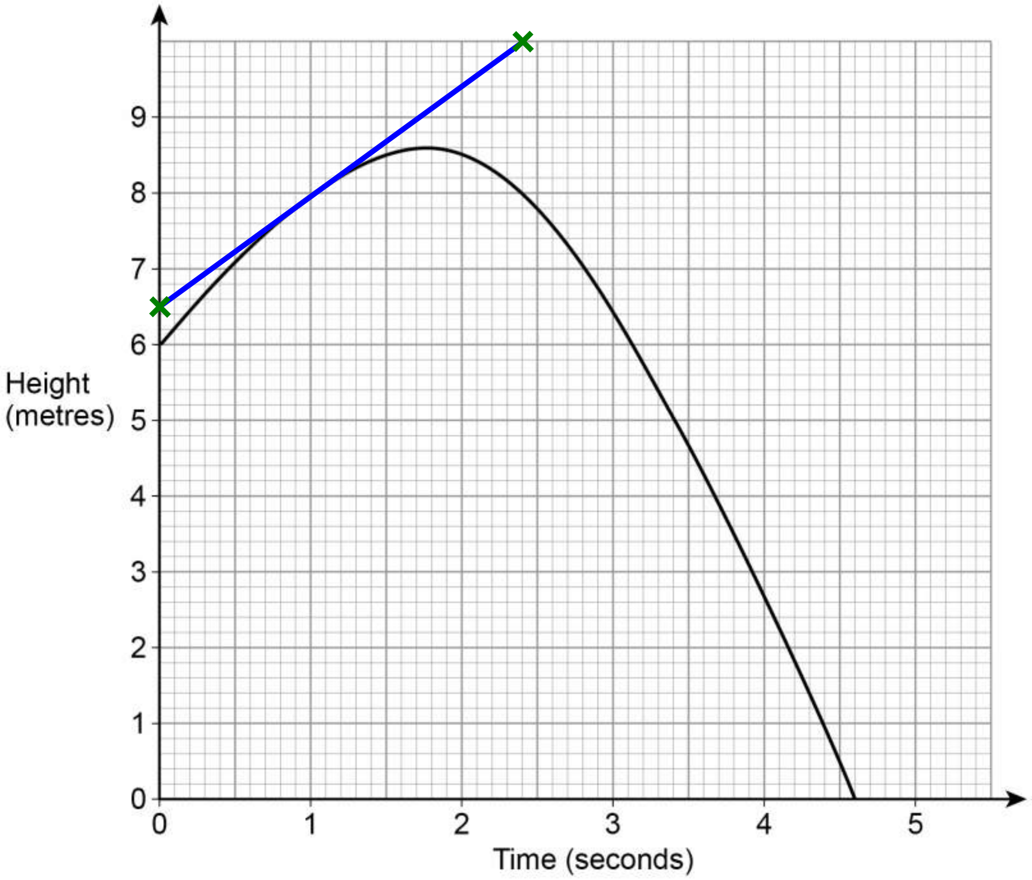
24 A ball is thrown from a point 6 metres above the ground.

The graph shows the height of the ball above the ground, in metres.



Estimate the speed of the ball, in m/s, after 1 second.

You **must** show your working.

[2 marks]

The gradient on a distance-time graph is the speed so drawing a tangent at the time of 1 seconds and working out its gradient estimates the speed

Gradient = (change in y)/(change in x).
Using the points (0, 6.5) and (2.4, 10).
Change in y = 10 - 6.5
Change in x = 2.4 - 0

$\frac{10-6.5}{2.4-0}$

Answer 1.46 m/s

6

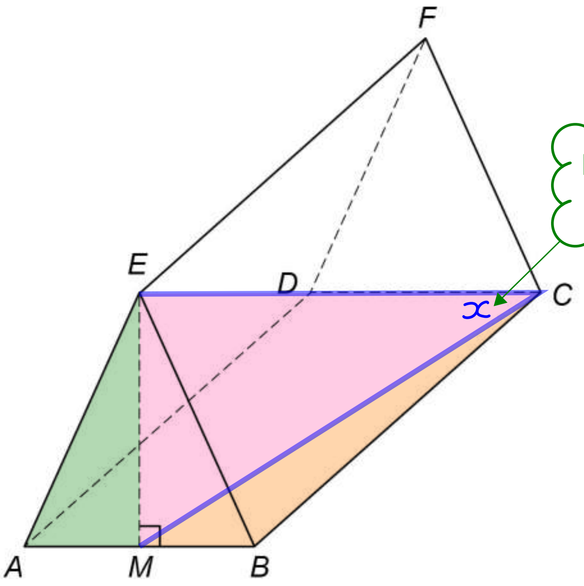
Turn over ▶

25 Rectangle *ABCD* is the horizontal base of a triangular prism *ABCDEF*.

AE = *BE*

E is vertically above *M*, the midpoint of *AB*.

AB = 16 cm *AE* = 17 cm *BC* = 30 cm



Drawing on angle ECM and labelling it as x

25 (a) Show that *EM* = 15 cm

[2 marks]

$16 \div 2$ ← M is the midpoint of AB so dividing AB by 2 works out that AM is 8 cm

$8^2 + EM^2 = 17^2$ ← Using Pythagoras' Theorem in the right-angled triangle highlighted in green. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting AM for a, EM for b and AE for c

$EM^2 = 225$ ← Subtracting 8^2 from both sides

$EM = 15$ ← Square rooting both sides

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25 (b) Work out the size of angle *ECM*.

[4 marks]

$8^2+30^2=MC^2$ ← Using Pythagoras' Theorem in the right-angled triangle highlighted in orange. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting BM (which must also be 8 cm) for a, BC for b and MC for c

$MC=\sqrt{964}$ ← Square rooting both sides

$=31.0...$ ← The result is a long decimal which can be stored on the calculator as A

$S\overset{\checkmark}{O}H\ C\overset{\checkmark}{A}H\ T\overset{\checkmark}{O}\overset{\checkmark}{A}$ ← Using right-angled trigonometry on the right-angled triangle highlighted in pink. Writing SOH CAH TOA as formula triangles. Ticking O as we have the opposite and A as we have the adjacent. There are two ticks on TOA so this formula triangle can be used

$\tan x=\frac{15}{31.0...}$ ← Covering T in the TOA formula triangle finds that tan of the angle = opposite/adjacent. The opposite is EM and the adjacent is MC. Using the exact value of MC

$x=\tan^{-1}(0.4...)$ ← Doing the inverse tan of both sides gets x on its own

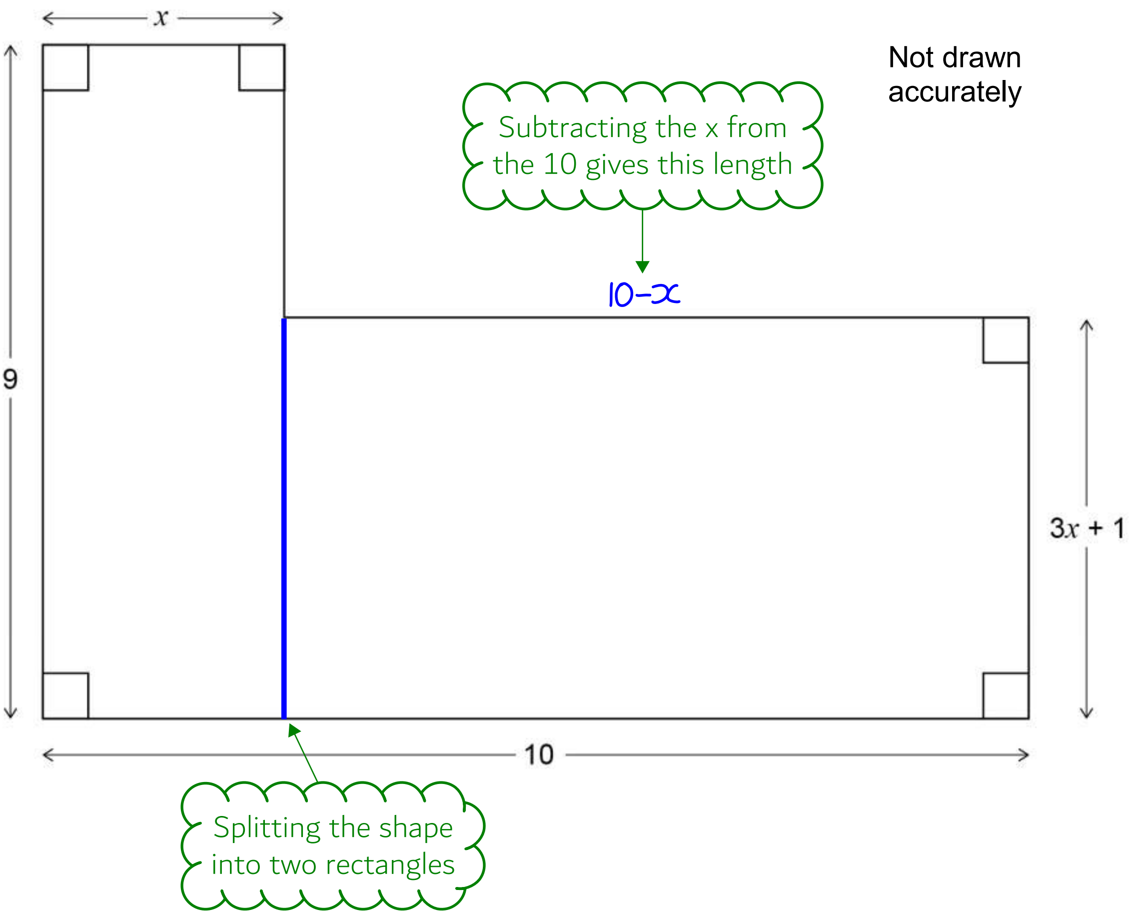
Answer 25.8 degrees

Turn over for the next question

6

26 Here is an L-shape.

All dimensions are in centimetres.



The area of the L-shape is 65 cm^2

Work out the value of x.

[6 marks]

$(10-x)(3x+1)$

Area of rectangle = base x height. x - 10 is the base and 3x + 1 is the height of the rectangle on the right

$30x+10-3x^2-x+9x=65$

Expanding the brackets and adding the area of the rectangle on the left must equal to the area of the L-shape. Area of rectangle = base x height. x is the base and 9 is the height of the rectangle on the left so its area is 9x

$-3x^2+38x-55=0$

Rearranging into the quadratic form $ax^2 + bx + c = 0$ by collecting like terms and subtracting 65 from both sides

$\frac{-38\pm 38^2-4\times-3\times-55}{2\times-3}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solving using the quadratic formula. a = -3, b = 38 and c = -55

The other solution is 11 however this is ignored as 10 - x would be 10 - 11 which would give a negative length, which is not possible

Answer $\frac{5}{3}$

Turn over for the next question

6

27 Prove that $x^2 + x + 1$ is always positive.

[3 marks]

$(x+0.5)^2+1-0.5^2$ ← Completing the square by halving the coefficient of x (which was 1), putting this in a bracket with x, squaring the bracket and subtracting the 0.5^2

$(x+0.5)^2+0.75$ ← $1 - 0.5^2 = 0.75$

Minimum value is 0.75 ← As the minimum value the squared bracket can have is 0 and adding the 0.75 to this minimum gives 0.75

END OF QUESTIONS

3