AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number						Candidate number					

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE
MATHEMATICS

H

Higher Tier Paper 2 Calculator

Monday 6 November 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
28–29	
TOTAL	

IB/M/Nov17/E10

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** questions in the spaces provided

1 Circle the fraction that is equivalent to 3.875

[1 mark]

$\frac{15}{4}$ $\frac{29}{8}$ [✓ $\frac{31}{8}$] $\frac{15}{8}$

Typing 3.875 into the calculator and formatting the answer as a fraction gives 31/8

2 What is 50 as a percentage of 20?

Circle your answer.

[1 mark]

10% 40% 150% [✓ 250%]

50/20 expresses 50 as a fraction of 20. Multiplying a fraction by 100 converts it into a percentage. 50/20 x 100 = 250

3 Circle the point that does **not** lie on the curve $y = x^3$

[1 mark]

$\left(-\frac{1}{2}, -\frac{1}{8}\right)$ (5, 125) [✓ $\left(\frac{1}{3}, \frac{1}{9}\right)$] (–1, –1)

Substituting the x-coordinate into the equation finds what the y-coordinate should be. $(1/3)^3$ = 1/27 which is not 1/9 so the third option does not lie on the curve

4 Which **one** of these is a unit of density?

Circle your answer.

[1 mark]

kg/m^2 m^2/kg [✓ kg/m^3] m^3/kg

Density = mass/volume. kg is a unit of mass and m^3 is a unit of volume so the unit of density must be kg/m^3

5 Solve $4(3x - 2) = 2x - 5$

[3 marks]

$12x-8=2x-5$ ← Expanding the bracket

$10x-8=-5$ ← Subtracting 2x from both sides to get the x terms on the side with the most x

$10x=3$ ← Adding 8 to both sides to get the x term on its own

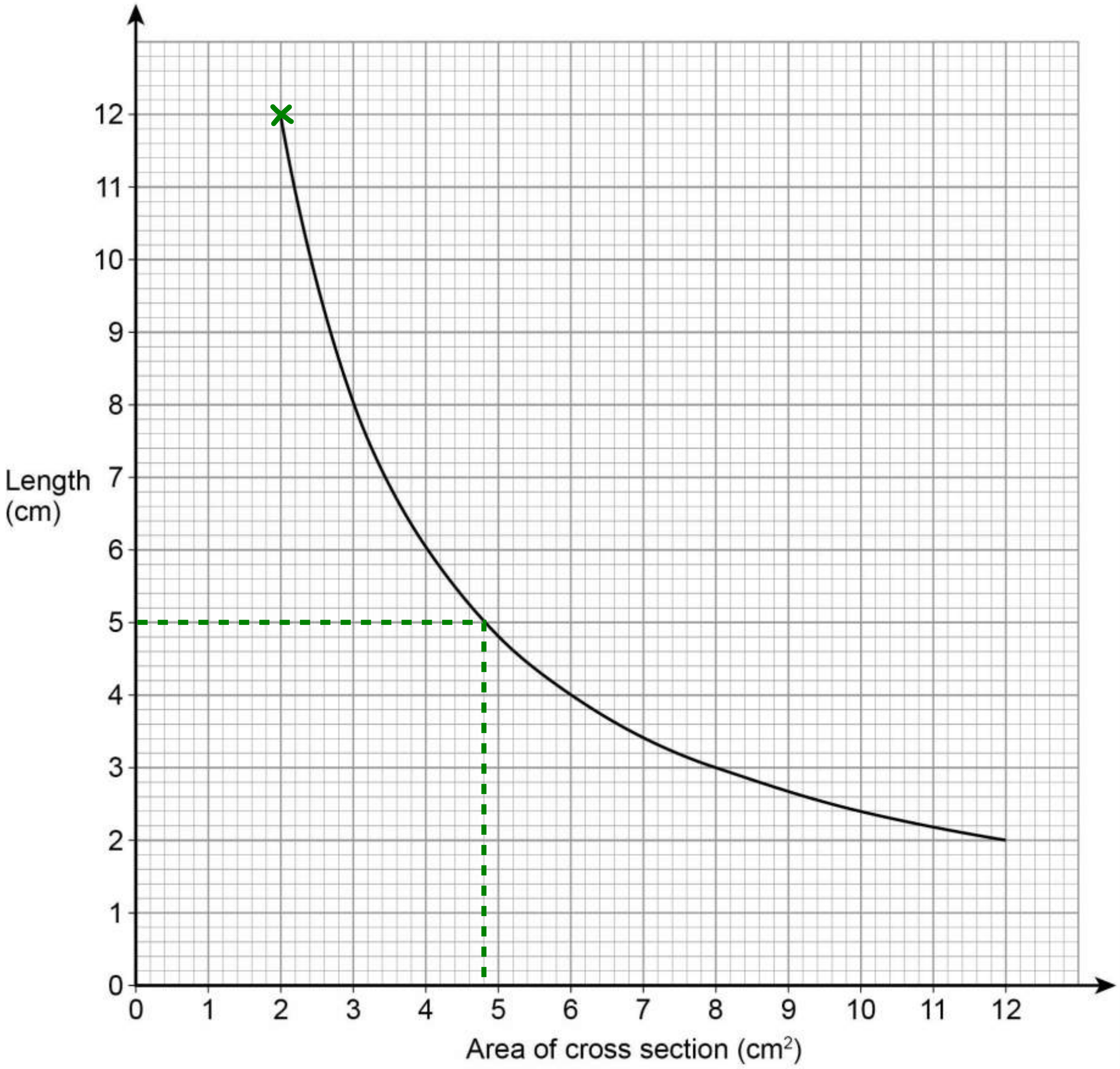
$x = \frac{3}{10}$

Dividing both sides by 10 makes x the subject and finds x

Turn over for the next question

7

6 The graph shows information about prisms with the same volume.



6 (a) Give **one** example to show the volume is 24 cm^3

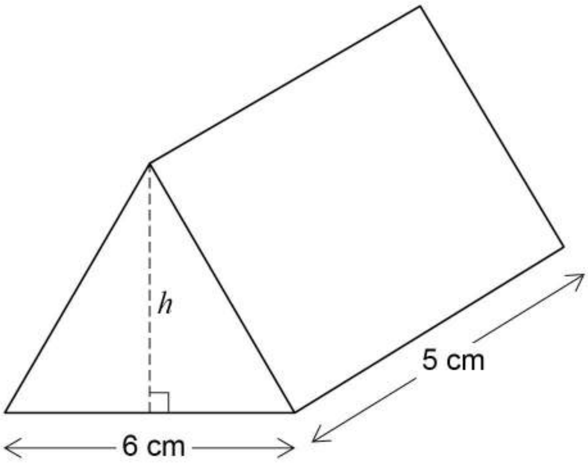
[1 mark]

$2 \times 12 = 24$

Volume of prism = area of cross section x length. At the point indicated, the area of cross section is 2 cm^2 and the length is 12 cm. Multiplying these gives a volume of 24 cm^3

6 (b) The diagram shows a prism with volume 24 cm^3

The height of the triangular cross section is h.



Work out the height, h.

[3 marks]

$4.8 = \frac{1}{2} \times 6 \times h$

On the graph, reading across from the length of 5 cm to the line then down finds that the the cross sectional area of the prism must be 4.8 cm^2. Setting this equal to the area of the triangle as this shape is the cross section. Area of triangle = 1/2 x base x height. The base is 6 cm and the height is h so substituting these in

$= 3h$

1/2 x 6 = 3, then 3 x h = 3h. So 4.8 = 3h

Answer 1.6 cm

Dividing both sides by 3 finds that h = 1.6 cm

Turn over for the next question

4

Turn over ▶

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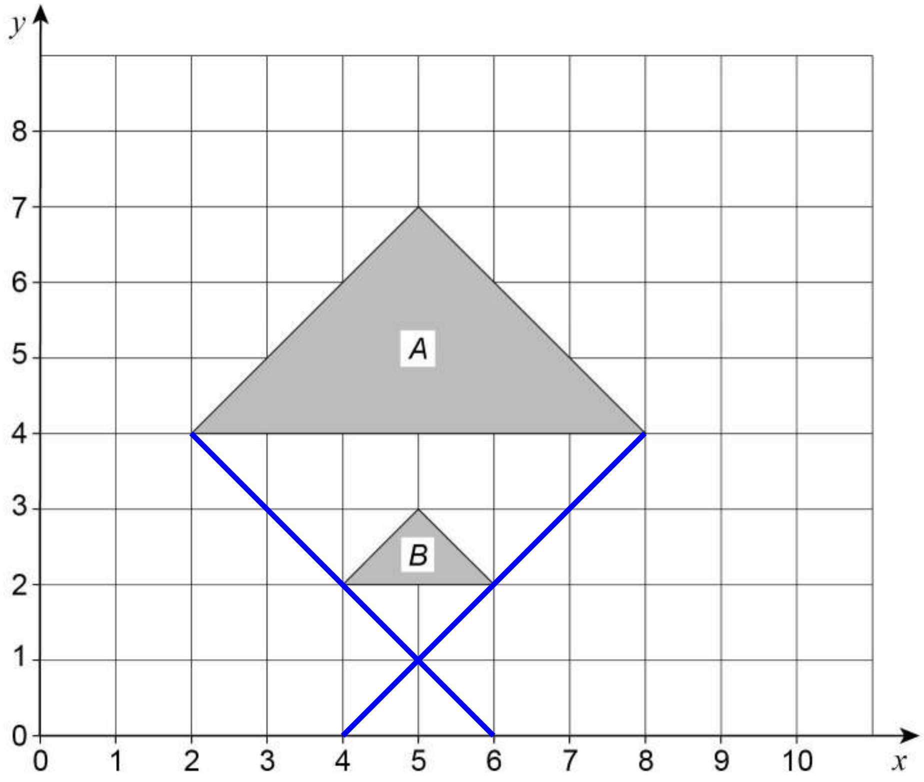
7 Describe fully the **single** transformation that maps triangle *A* to triangle *B*.



[3 marks]

Enlargement, scale factor 1/3, centre (5, 1)

It is an enlargement as it has changed size. The scale factor is 1/3 as the sides are 1/3 of the size on B and the shape is the same way up. Drawing lines through the corners of both shapes then finding where they cross works out the centre of enlargement

8 The table shows information about the distances walked by 120 students on their way to school one week.

Distance, x (miles)	**Frequency**	Midpoint	Total
$0 < x \leqslant 5$	20	2.5	50
$5 < x \leqslant 10$	48	7.5	360
$10 < x \leqslant 15$	30	12.5	375
$15 < x \leqslant 20$	22	17.5	385
	Total = 120		1170÷120

Work out an estimate for the mean distance.

[3 marks]

Highlighted in orange: working out the midpoints of each interval. Doing the mean of the upper and lower bound of each interval works out the midpoint.
For the $0 < x \leqslant 5$ interval, $0 + 5 = 5$ then $5 \div 2 = 2.5$.
For the $5 < x \leqslant 10$ interval, $5 + 10 = 15$ then $15 \div 2 = 7.5$.
For the $10 < x \leqslant 15$ interval, $10 + 15 = 25$ then $25 \div 2 = 12.5$.
For the $15 < x \leqslant 20$ interval, $15 + 20 = 35$ then $35 \div 2 = 17.5$.

Highlighted in pink: working out an estimated total for each interval. Multiplying the midpoint by the frequency for each interval works out an estimated total.
For the $0 < x \leqslant 5$ interval, 2.5 x 20 = 50.
For the $5 < x \leqslant 10$ interval, 7.5 x 48 = 360.
For the $10 < x \leqslant 15$ interval, 12.5 x 30 = 375.
For the $15 < x \leqslant 20$ interval, 17.5 x 22 = 385.

Highlighted in green: adding all of the totals gives an overall total. Dividing this by the 120 students estimates that the mean distance is 9.75 miles.

Answer 9.75 miles

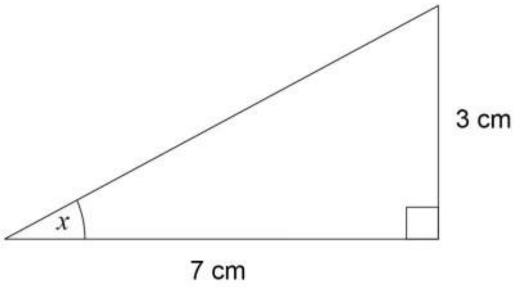
Turn over for the next question

6

Turn over ▶

9 Work out the size of angle x.

Not drawn accurately



[2 marks]

$S\acute{O}H\ C\acute{A}H\ T\acute{O}\acute{A}$

Right angled trigonometry can be used. Ticking O as we have the opposite and A as we have the adjacent. There are two ticks on TOA so this formula triangle can be used

$\tan x = \frac{3}{7}$

Covering t in the TOA formula triangle finds that tan of the angle = opposite/adjacent. The opposite is 3 cm and the adjacent is 7 cm

$x = \tan^{-1}\left(\frac{3}{7}\right)$

Doing the inverse tan of both sides gets x on its own

Answer 23.2 degrees

10 Work out the next term of this quadratic sequence.

[2 marks]

+3 +6 +9 +12

5 8 14 23 ……

23+12

The amount added increases by 3 each time

Answer 35

11 Circle the expression that is equivalent to $\frac{3x^2}{6x^2+3}$

[1 mark]

$\frac{x^2}{2x^2+3}$ $\frac{x^2}{6x^2+1}$ [✓ $\frac{x^2}{2x^2+1}$] $\frac{1}{2}+x^2$

The numerator and denominator are divided by 3 so it is equivalent as both the numerator and denominator are divided by the same amount

Turn over for the next question

5

12 The table shows information about the UK and Germany.

	Population	**Area (square miles)**
UK	64 000 000 ÷	95 000 = 674
Germany	82 000 000 ÷	140 000 = 586

Dividing the populations by the area works out the population densities

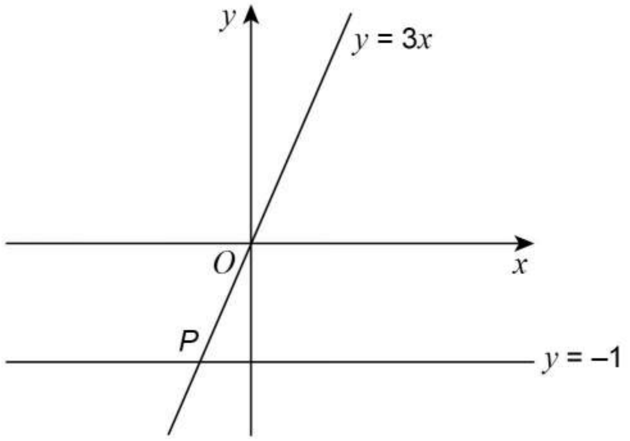
Population density = $\frac{\text{population}}{\text{area}}$

Compare the population densities of the UK and Germany.

[3 marks]

Greater for UK ← 674 is greater than 586

13 Two straight lines intersect at point *P*.



Not drawn accurately

Circle the coordinates of *P*.

[1 mark]

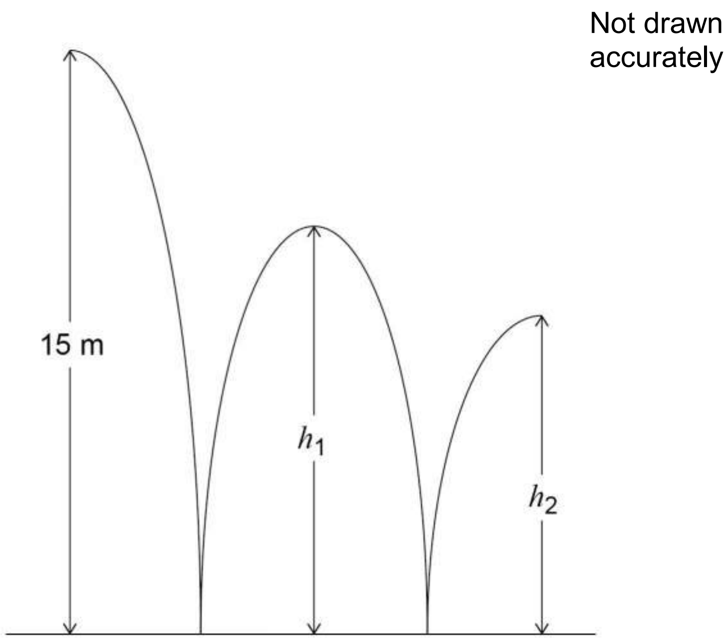
$(-3,\ -1)$ $\left(-1,\ -\frac{1}{3}\right)$ $(-1,\ -3)$ [✓ $\left(-\frac{1}{3},\ -1\right)$]

The y-coordinate must be -1 as P is on the line y = -1. Substituting -1 for y in the equation of the other line gives -1 = 3x. Dividing both sides by 3 finds that x = -1/3

Turn over for the next question

14 A ball is thrown from a height of 15 metres.

It bounces to height h_1, then to height h_2 as shown.



h_1 is three quarters of the original height.

14 (a) Jack expects h_2 to be three quarters of h_1

Work out the value of h_2 that he expects.

[2 marks]

$\frac{3}{4} \times 15$ ← Doing 3/4 of the original height works out that h_1 is 11.25 m. Of means to multiply

$\frac{3}{4} \times 11.25$ ← Doing 3/4 of h_1 works out that h_2 is 8.4... m. Of means to multiply

Answer 8.4 metres

14 (b) In fact, h_2 is two thirds of h_1

How does this affect the answer to part (a)?

Tick a box.

☐ The ball bounced higher than he expected

☑ The ball bounced lower than he expected

Show working to support your answer.

[2 marks]

$\frac{2}{3} \times 11.25 = 7.5$

Doing 2/3 of h_1 works out that h_2 is now 7.5 m, which is lower than the 8.4 m expected. Of means to multiply

Turn over for the next question

4

15 Mirek invests £6000 at a compound interest rate of 1.5% per year.

He wants to earn more than £1000 interest.

Work out the **least** time, in whole years, that this will take.

[3 marks]

$6000 + 1000$

Adding the £1000 interest to the £6000 works out that the investment needs to be worth more than £7000 when more than £1000 interest is earned

$6000 \times \left(\frac{100+1.5}{100}\right)^{x} > 7000$

Adding the 1.5% to 100% expresses the percentage it increases to each year. Putting this over 100 converts it to a fraction, which when multiplied by the £6000 increases it by 1.5%. Raising the fraction to the power of x where x is the number of years. This needs to be more than £7000

Finding the whole number value of x which gives more than £7000 by using table mode. Set f(x) as the left side of the inequality written above. Start: 1. End: 30. Step: 1. This lists out the values of the investment for each of the first 30 years. When x = 10, the investment is worth £6963.24... When x = 11, the investment is worth £7067.69... which is the first time it is worth more than £7000. As x is the number of years, the least time it will take, in whole years, is 11 years

Answer 11 years

16 (a) Factorise fully $9y^3 - 6y$

[2 marks]

3 is the highest common factor of the numbers. y is the highest common factor of the letters. So 3y is the highest common factor of both terms. Bringing 3y outside of a bracket, dividing both terms by 3y and leaving the result in the bracket

Answer $3y(3y^2-2)$

16 (b) Factorise $3x^2 - 22x + 7$

[2 marks]

$3x^2-21x-x+7$

The original expression is in the form $ax^2 + bx + c$. Multiplying a (which is 3) by c (which is 7) gives 21. Two numbers which multiply to this 21 and add to b (which is -22) are -21 and -1. Splitting the middle x term into these amounts of x

$3x(x-7)-1(x-7)$

Factorising the first two term and the last two terms separately. -1 is brought out as a factor for the last two terms as there is no higher common factor and the x term is negative

Answer $(3x-1)(x-7)$

Bringing together the 3x and -1 into a single bracket and writing this multiplied by the other bracket which repeats

Turn over for the next question

7

Turn over ►

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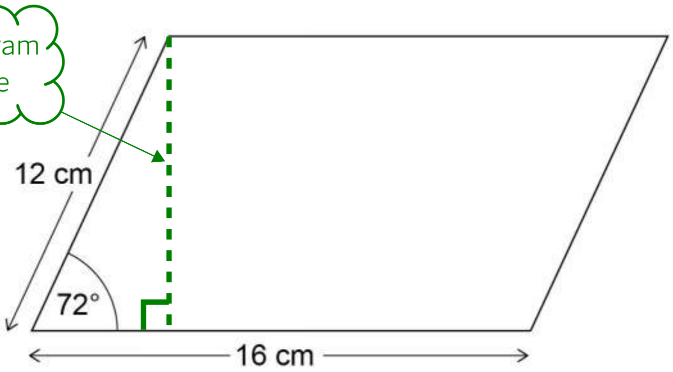
17 Work out the area of the parallelogram.

Drawing the height of the parallelogram here creates a right-angled triangle



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[3 marks]

$S\overset{\checkmark}{O}\overset{\checkmark}{H}\,C\,A\,\overset{\checkmark}{H}\,T\,\overset{\checkmark}{O}\,A$ ← Using right-angled trigonometry on the triangle to work out the height of the parallelogram. We have the hypotenuse so H is ticked and we are working out the opposite so O is ticked. There are two ticks on SOH so this formula triangle can be used

$\sin 72 \times 12$ ← Covering O in the SOH formula triangle finds that opposite = sin of the angle x hypotenuse. The angle is 72 degrees and the hypotenuse is 12 cm. So the height is 11.4... cm

$16 \times 11.4\ldots$ ← Area of parallelogram = base x height

Answer 182.6 cm^2

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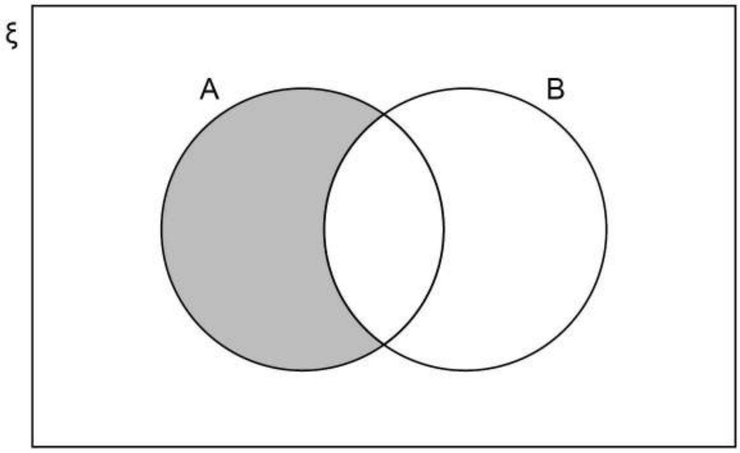
18 (a)

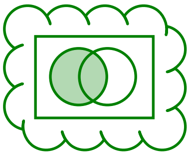


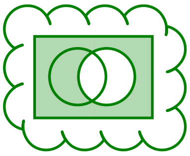
Which of these represents the shaded region?

Circle your answer.

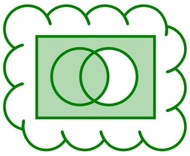
[1 mark]

A B' [✓ $A \cap B'$] $A \cup B'$

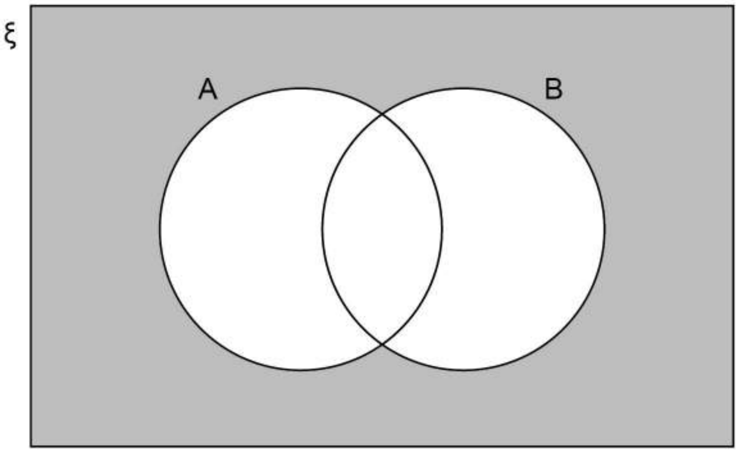




A and not B



18 (b)



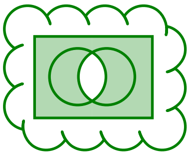
Which of these represents the shaded region?

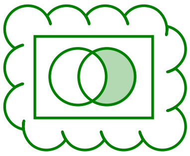
Circle your answer.

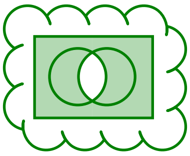
[1 mark]

[✓ $(A \cup B)'$] $(A \cap B)'$ $A' \cap B$ $A' \cup B'$

Not A or B or both







5

19 The length of a rectangle is five times the width.

The area of the rectangle is 1620 cm^2

Not drawn accurately

Work out the width of the rectangle.

[3 marks]

$5w \times w$

Area of rectangle = length x width. let w be the width. The length will be 5w

$5w^2 = 1620$

5w x w = $5w^2$, which is an expression for the area so must be equal to the area of 1620 cm^2

$w^2 = 324$

Dividing both sides by 5 to get w^2 on its own

Answer 18 cm

Square rooting both sides finds that w = 18, which is the width

20 A stone is thrown upwards with a speed of v metres per second.

The stone reaches a maximum height of h metres.

h is directly proportional to v^2

When $v = 10$, $h = 5$

Work out the maximum height reached when $v = 24$

[4 marks]

$h = kv^2$ ← $h \propto v^2$. The right side of this can be multiplied by anything and still be directly proportional. So multiplying by k (which represents a constant value) and converting it into an equation

$k = \frac{h}{v^2} = \frac{5}{10^2}$ ← Rearranging to find k by dividing both sides by v^2 then substituting 10 for v and 5 for h finds that k = 0.05

0.05×24^2 ← $h = 0.05v^2$. Substituting 24 for v in the right side works out the maximum height reached when v = 24

Answer 28.8 m

Turn over for the next question

7

Turn over ▶

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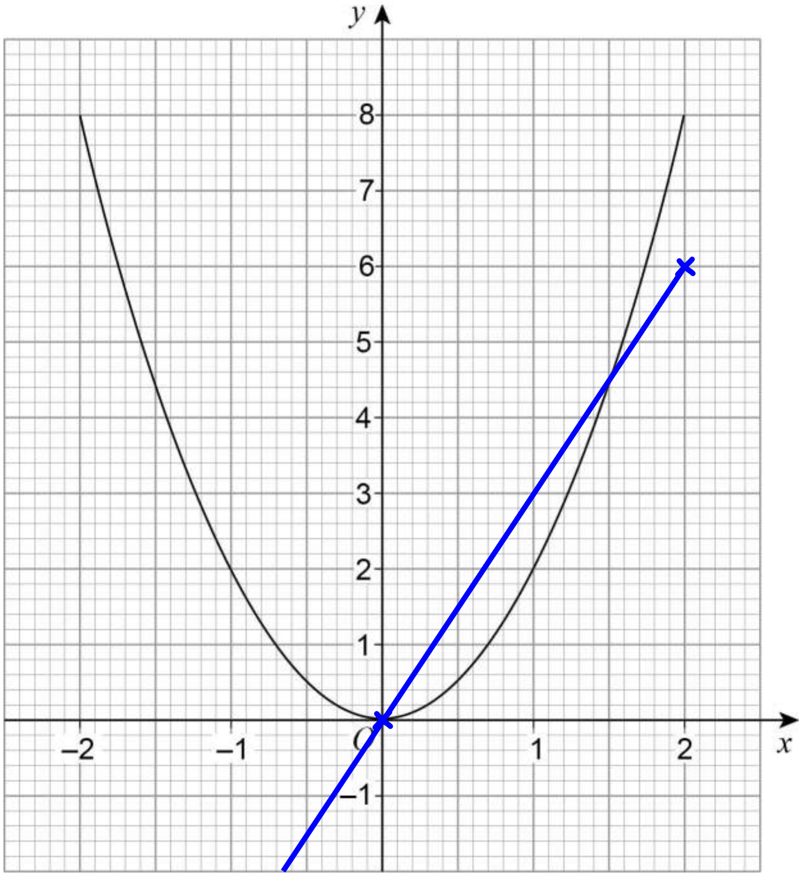
21 (a) Meera is using a **graphical** method to solve $2x^2 - 3x = 0$

She draws the graph of $y = 2x^2$ and a straight line graph on the same grid.

Here is the graph of $y = 2x^2$



Complete her method to solve $2x^2 - 3x = 0$

[2 marks]

Rearranging $2x^2 - 3x = 0$ to get the $2x^2$ on its own (as it is in the equation $y = 2x^2$) gives $2x^2 = 3x$. So drawing the graph of $y = 3x$ and working out the x-coordinates where both graphs cross works out the solutions.

When $x = 0$, $y = 3(0) = 0$. When $x = 2$, $y = 3(2) = 6$. Plotting the points (0, 0) and (2, 6) then drawing a straight line through them.

The two graphs cross when $x = 0$ and $x = 1.5$

Answer $x=0, x=1.5$

21 (b) Levi is solving $2x^2 + 5x = 0$

He uses this method.

$2x^2 + 5x = 0$ subtract $5x$ from both sides

$2x^2 = -5x$ divide both sides by x

$2x = -5$ divide both sides by 2

$x = -2.5$

Evaluate his method and his answer.

[2 marks]

Cannot divide by x as it could be 0

Dividing by 0 is undefined. Instead should rearrange into the quadratic form then solve by factorising or using the quadratic formula

Turn over for the next question

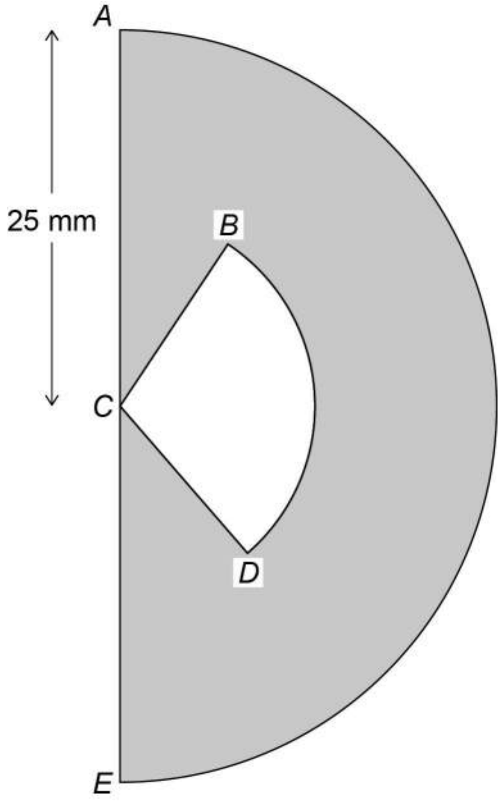
4

Using column subtraction, making sure that all the digits are in the right places values. Adding a 0 onto the end of 1.3 does not change its value

22 The cross section of an earring is a semicircle, centre *C*, radius 25 mm

The earring is black and white.

The shaded area is black.



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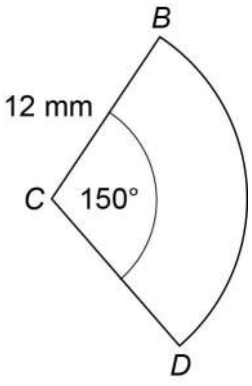
Sector *BCD* is white and has radius 12 mm



Not drawn accurately

Is more than 20% of the semicircle white?

You **must** show your working.

[5 marks]

$\frac{1}{2} \times \pi \times 25^2 = 981.7...$

Area of circle = π x radius². The radius of the semicircle is 25 mm. Doing 1/2 of this area as the semicircle is half of the full circle. Storing the exact value of the area as A on the calculator

$\frac{150}{360} \times \pi \times 12^2$

Area of circle = π x radius². The radius of the white sector is 12 mm. Doing 150/360 of this area as the white sector 150 degrees out of the full 360 degrees of the circle

$\frac{188.4...}{981.7...} \times 100$

Putting the exact value of the area of the white sector over the exact stored value of the area of the semicircle expresses the fraction of the semicircle which is white. Multiplying this by 100 converts it to a percentage

19.2%

19.2% of the semicircle is white

Answer No

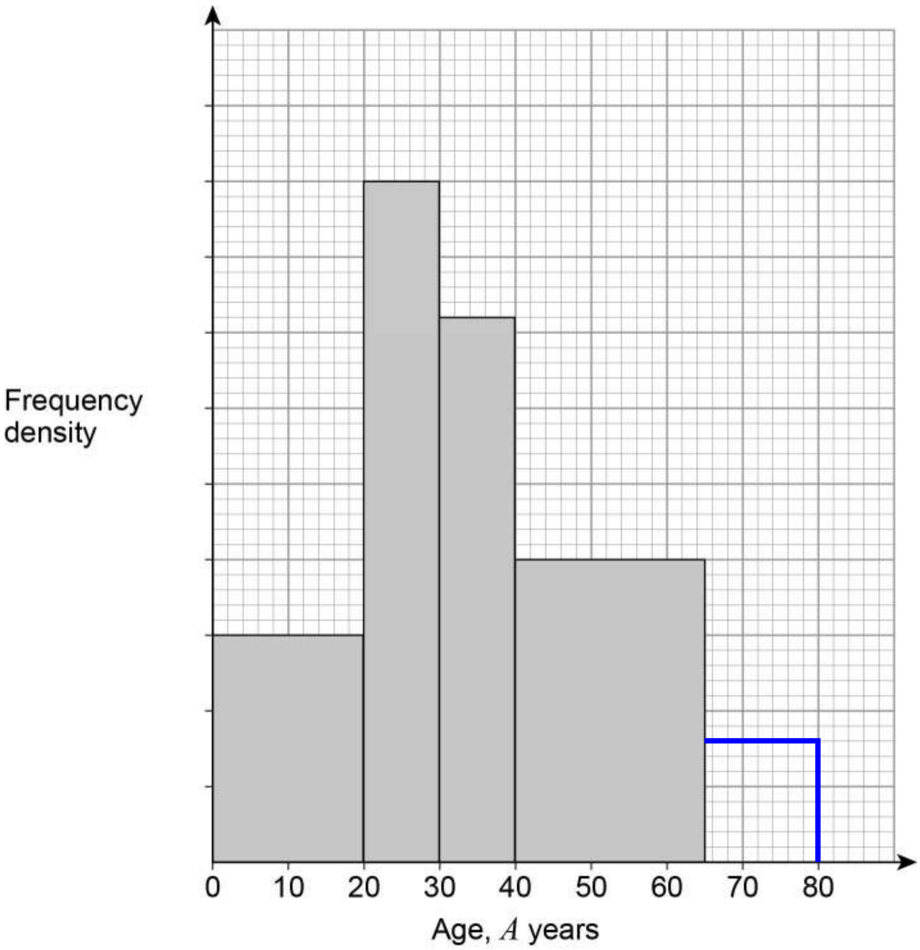
Less than 20% of the semicircle is white

Turn over for the next question

5

23 Here is some information about a tennis club.

Members of a tennis club



There are 30 members with $A < 20$

There are 12 members with $65 \leqslant A < 80$

There are no members with $A \geqslant 80$

23 (a) Complete the histogram.

[3 marks]

c F d

The area of each box represents the frequency on a histogram. Frequency = class width x frequency density. Making a formula triangle out of this

$\frac{30}{20} = 1.5$

From the formula triangle, frequency density = frequency/class width. For the first bar the frequency is 30 and the class width is 20 (as it goes from 0 to 20). So the frequency density of the first bar is 1.5

$\frac{12}{80-65} = 0.8$

From the formula triangle, frequency density = frequency/class width. For the last bar the frequency is 12 and the class width is 80 - 65 (as it goes form 65 to 80). So the frequency density of the last bar is 0.8

The first bar is 15 small boxes tall and has frequency density of 1.5. 1.5 ÷ 15 = 0.1 so each small box is worth 0.1

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23 (b) Work out the total number of members of the club.

[2 marks]

$(30-20)\times 4.5=45$

$(40-30)\times 3.6=36$

$(65-40)\times 2=50$

Multiplying the class width by the frequency density for the second, third and fourth bars works out their frequencies

$30+45+36+50+12$

Adding the frequencies of all the bars

Answer 173

Turn over for the next question

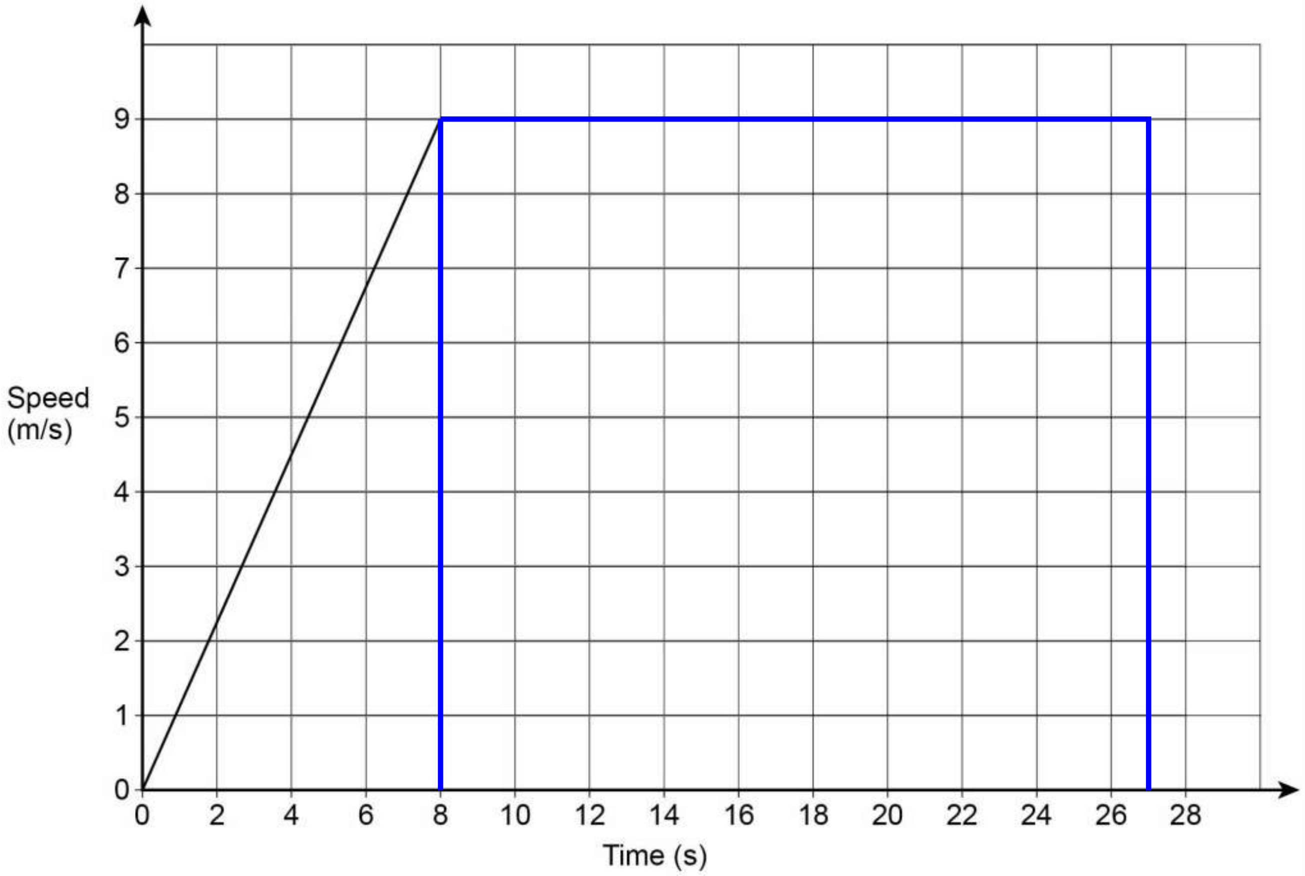
5

24 Beth ran a 200 metre race.

Here is a graph of the first 8 seconds of her race.

She completed the race at a constant speed of 9 m/s

Speed-time graph for Beth



Amy completed the race in 27 seconds.

Did Beth finish before Amy?

You **must** show your working.

[3 marks]

Beth completed the race at a constant speed of 9 m/s and Amy completed the race in 27 seconds so drawing a horizontal line from 8 seconds to 27 seconds. The area under the line on a speed-time graph is the distance so dividing it into a triangle and a rectangle to help work out the area

See next page for the working

Answer Yes

$\frac{1}{2} \times 8 \times 9 = 36$ ← Area of triangle = 1/2 x base x height. The base is 8 and the height is 9. So the area of the triangle is 36

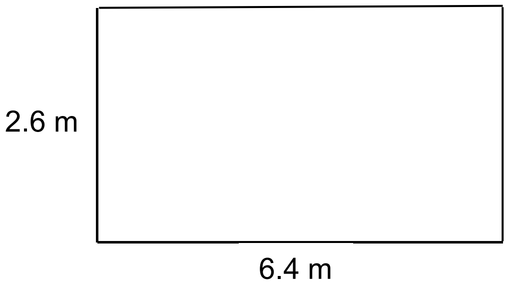
$27 - 8$ ← Subtracting the 8 seconds from the 27 seconds works out that the base of the rectangle is 19

19×9 ← Area of rectangle = base x height. The base is 19 and the height is 9. So the area of the rectangle is 171

$36 + 171 = 207$ ← Adding the area of the triangle and the rectangle works out that the area under the line is 207, which represents a distance of 207 metres

Beth would have done 207 metres in 27 seconds and this is more than the 200 metre race. So Beth must have finished before Amy

25 The dimensions of a rectangular floor are to the nearest 0.1 metres.



Not drawn accurately

A force of 345 Newtons is applied to the floor.

The force is to the nearest 5 Newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the upper bound of the pressure.

Give your answer to 4 significant figures.

You **must** show your working.

[5 marks]

$345+\frac{5}{2}=347.5$

The resolution of the force is 5 Newtons. Adding half of this to the 345 Newtons works out that the upper bound of the force is 347.5 Newtons

$6.4-\frac{0.1}{2}=6.35$

The resolution of the length is 0.1 m. Subtracting half of this from the 6.4 m works out that the lower bound of the length is 6.35 m

$2.6-\frac{0.1}{2}$

The resolution of the width is 0.1 m. Subtracting half of this from the 2.6 m works out that the lower bound of the width is 2.55 m

2.55×6.35

Area of rectangle = length x width. Multiplying the lower bound of the length by the lower bound of the width works out that the lower bound of the area is 16.1... m^2

$\frac{347.5}{16.1...}$

Pressure = force/area. Dividing the upper bound of the force by the lower bound of the area works out the upper bound of the pressure

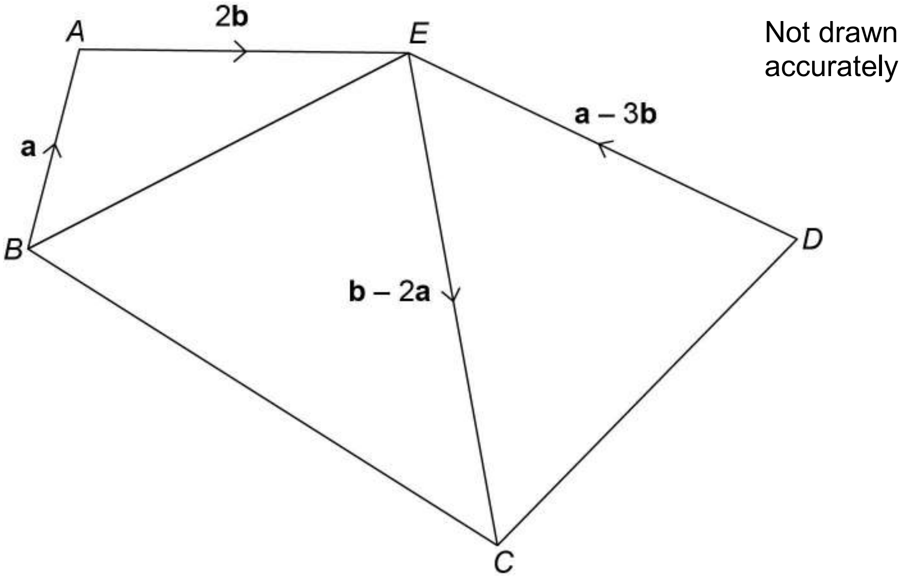
Answer 21.46 N/m^2

Rounding 21.460... to 4 significant figures

8

Turn over ►

26 *ABCDE* is a pentagon.



Not drawn accurately

Show that *BCDE* is a parallelogram.

[3 marks]

$\overrightarrow{CB} = -b + 2a - 2b - a$

$= a - 3b$

$\overrightarrow{CB} = \overrightarrow{CE} + \overrightarrow{EA} + \overrightarrow{AB}$. $\overrightarrow{CE} = -\overrightarrow{EC}$. $\overrightarrow{EA} = -\overrightarrow{AE}$. $\overrightarrow{AB} = -\overrightarrow{BA}$

Simplified by collecting like terms

CB is equal and parallel to DE

Therefore it must be a parallelogram as opposite sides are equal in length and parallel

27 Solve $\frac{x}{4} - \frac{2x}{x+2} = 1$

Give your solutions to 2 decimal places.

You **must** show your working.

[6 marks]

$x - \frac{8x}{x+2} = 4$ ← Multiplying all terms on both sides by 4 to eliminate it as a denominator

$x^2 + 2x - 8x = 4x + 8$ ← Multiplying all terms on both sides by x + 2 to eliminate it as a denominator. $x(x + 2) = x^2 + 2x$. $4(x + 2) = 4x + 8$

$x^2 - 10x - 8 = 0$ ← Bringing it into the quadratic form $ax^2 + bx + c = 0$ by subtracting 4x and 8 from both sides and collecting like terms

$\frac{--10 \pm \sqrt{(-10)^2 - 4\times1\times-8}}{2\times1}$ ← $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← Solving using the quadratic formula. a = 1. b = -10. c = -8

Answer 10.74, -0.74

↑ Rounding 10.744... and -0.744... to 2 decimal places

END OF QUESTIONS

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