OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Monday 13 November 2023 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number						Candidate number				

First name(s) ____________________

Last name ____________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

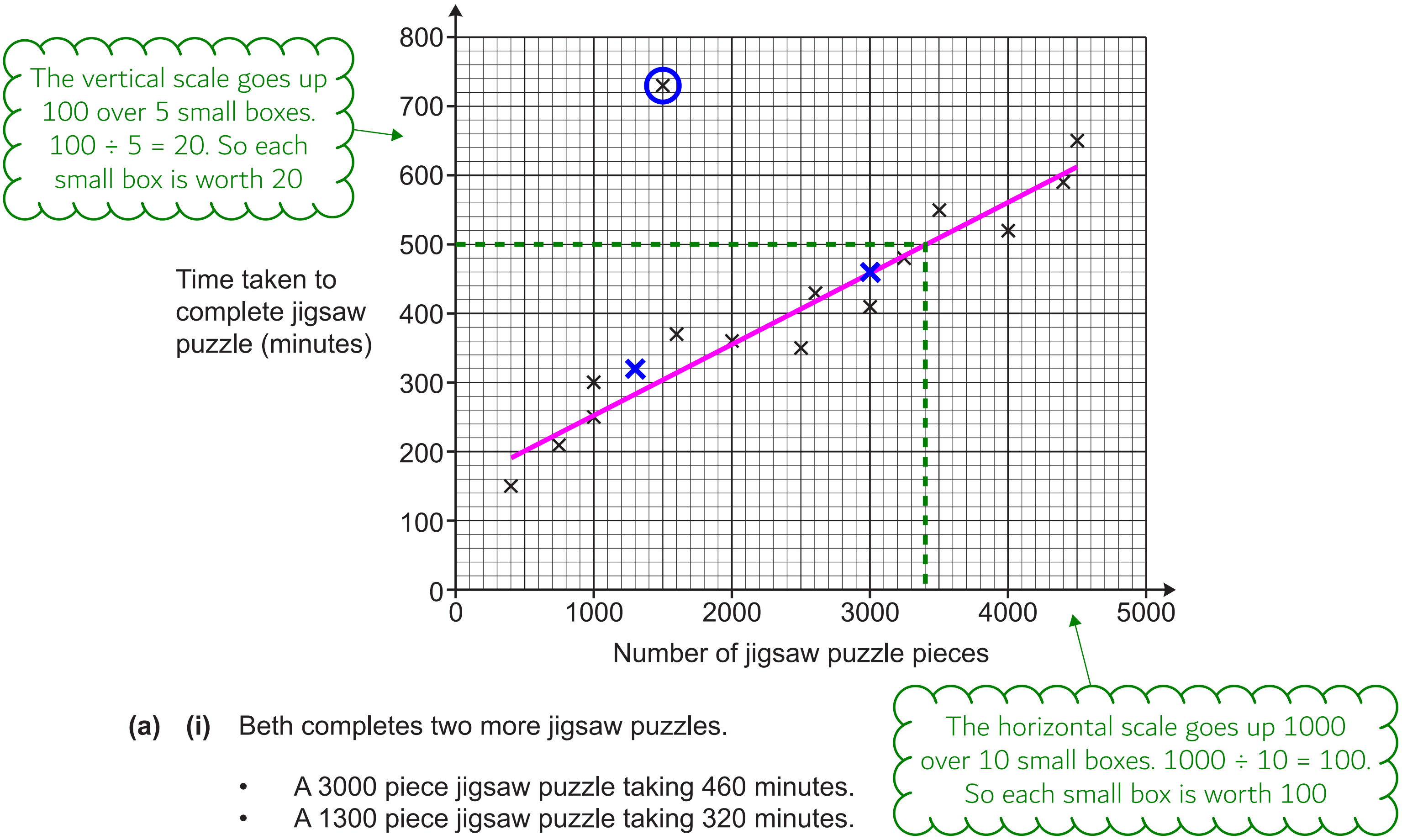
Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Beth completes some jigsaw puzzles and records the following information.

- The number of pieces in the jigsaw puzzle.
- The time taken to complete the jigsaw puzzle, in minutes.

Beth shows this information in a scatter diagram.



(a) **(i)** Beth completes two more jigsaw puzzles.

- A 3000 piece jigsaw puzzle taking 460 minutes.
- A 1300 piece jigsaw puzzle taking 320 minutes.

Show this information on the scatter diagram. **[1]**

(ii) Describe the type of correlation shown on the scatter diagram.

As the number of jigsaw puzzle pieces increases, the time taken to complete the jigsaw puzzle generally increases. This is positive correlation

(a)(ii) Positive .... **[1]**

(b) One of Beth's jigsaw puzzles was described as "the most difficult jigsaw puzzle you will ever try".

(i) Circle the most likely jigsaw puzzle on the scatter diagram. **[1]**

(ii) Give a reason why you chose this jigsaw puzzle.

It took the most time even with low pieces

....

.... **[1]**

(c) **(i)** Draw a line of best fit on the scatter diagram. Shown in pink **[1]**

(ii) Use your line of best fit to estimate how many pieces are in a jigsaw puzzle that takes Beth 500 minutes to complete.

Reading across from 500 minutes to the line of best fit then down to the number of jigsaw pieces

(c)(ii) ....3400................ pieces **[1]**

(d) Explain why Beth should **not** use her scatter diagram to estimate how long it will take to complete a jigsaw puzzle containing 8000 pieces.

It is outside the range of the data

Data is only given up to 4500 pieces. The trend might not continue beyond this point **[1]**

2 A restaurant menu has 4 main courses and 3 side dishes.
For their meal, each customer chooses 1 main course and 1 side dish.

Main course	
Beef burger	£6
Lasagna	£7
Veggie burger	£5
Turkey stew	£6

Side dish	
Salad	£2
Chips	£3
Garlic bread	£1

Work out the percentage of possible meals that cost less than £8.

	2	3	1
6	×	×	✓
7	×	×	×
5	✓	×	✓
6	×	×	✓

Writing the prices of the main courses against the prices of the side dishes. Adding them to work out all the possible prices and putting a tick if it is less than £8 and a cross if it is not less than £8

$\frac{4}{12} \times 100$

4 out of the 12 possible meal costs are less than £8. Putting this as a fraction then multiplying by 100 to convert it into a percentage

$33.\dot{3}$ % **[4]**

3 Write these numbers in order of size, starting with the smallest.

0.36% $\qquad \frac{1}{333}$ $\qquad 0.03$ $\qquad 3.1 \times 10^{-3}$

0.36 0.30 3 0.31

Expressing them all as percentages to 2 decimal places as percentages are easier to compare. Converting the fraction, decimal and standard form to percentages by multiplying them by 100

0.30, 0.31, 0.36, 3 is the order of the percentages starting with the smallest. Writing the original numbers

$\frac{1}{333}$, 3.1×10^{-3} , 0.36% , 0.03 **[4]**

smallest

4 Casey's mobile phone gives a weekly report showing the amount of time they use their phone.

This week the report says that the phone was used

- for 217 minutes
- 24% more than last week.

Calculate Casey's phone usage last week.

100+24 ← Let 100% be the full amount of Casey's phone usage last week. This has been increased by 24% so adding the 24% to the 100% works out that it has increased to 124%

217÷124 ← 124% of the full amount of Casey's phone usage last week is 217 minutes. Dividing the 217 minutes by the 124 works out that 1% is 1.75 minutes

1.75×100 ← Multiplying the value of 1% by 100 gives the full 100%

175 minutes **[3]**

5 A sheet of A4 card weighs 1.19×10^{-2} kg.

(a) Work out the weight of 500 sheets of the A4 card.

$1.19 \times 10^{-2} \times 500$ ← Multiplying the weight of one sheet of A4 card by 500 works out the weight of 500 sheets of the A4 card

(a) ....5.95.......... kg **[2]**

(b) Card is classified using W, the weight in grams per square metre (gsm).

$$W = \frac{\text{weight in grams}}{\text{area in square metres}}$$

A sheet of A4 card is a rectangle that is 21 cm by 29.7 cm.

Calculate W for this A4 card.

$1.19 \times 10^{-2} \times 1000 = 11.9$ ← There are 1000 g in 1 kg so multiplying the weight of one sheet of A4 card by 1000 converts the weight into grams

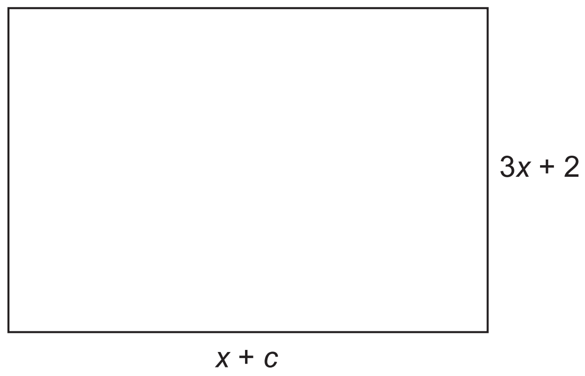
$21 \div 100 = 0.21$
$29.7 \div 100 = 0.297$ ← There are 100 cm in 1 m so dividing the lengths in cm by 100 converts them into metres

0.21×0.297 ← Area of rectangle = length x width. Multiplying the lengths in metres gives the area of the A4 sheet of card in square metres

$\frac{11.9}{0.06237}$ ← Putting the weight in grams over the area in square metres gives W

(b) ....190.8.......... gsm **[4]**

6 The area of this rectangle can be written as $ax^2 + bx - 10$.



Not to scale

Find the values of *a*, *b* and *c*.
You must show your working.

$(x+c)(3x+2)$ ← Area of rectangle = length x width

$3x^2+2x+3cx+2c$ ← Expanding the brackets

$3x^2+(2+3c)x+2c \equiv ax^2+bx-10$ ← Collecting like terms and setting equivalent to the expression for the area

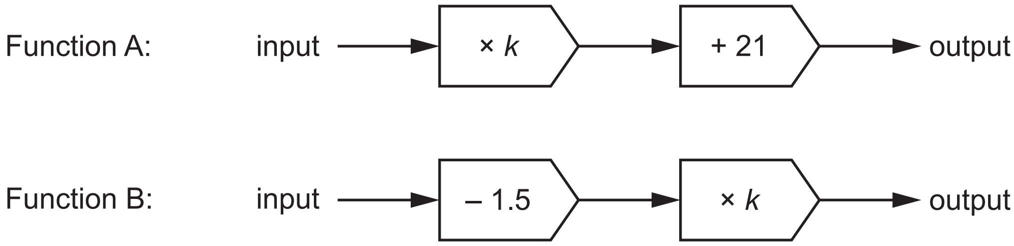
a must be 3 as there is the same number of x^2 on both sides and there is $3x^2$ on the left

$-10 \div 2$ ← 2c is the constant on the left and -10 is the constant on the right so 2c = -10. Dividing both sides by 2 finds that c = -5

$2+3\times-5$ ← Substituting -5 for c finds that there is -13x on the left so there must be -13x on the right. So b = -13

a = 3 , *b* = -13 and *c* = -5 **[5]**

7 Here are two functions.



5 is input into Function A.
5 is also input into Function B.
The output of Function A is 10 times the output of Function B.

Work out the value of *k*.
You must show your working.

$5k + 21 = 10 \times (5 - 1.5) \times k$

Expressing the output of function A by multiplying the 5 by k then adding 21. Setting this equal to 10 times the output of function B, which is expressed by subtracting 1.5 from the 5 then multiplying by k

$= 35k$

Simplifying the right side of the equation. 10 x (5 - 1.5) = 35

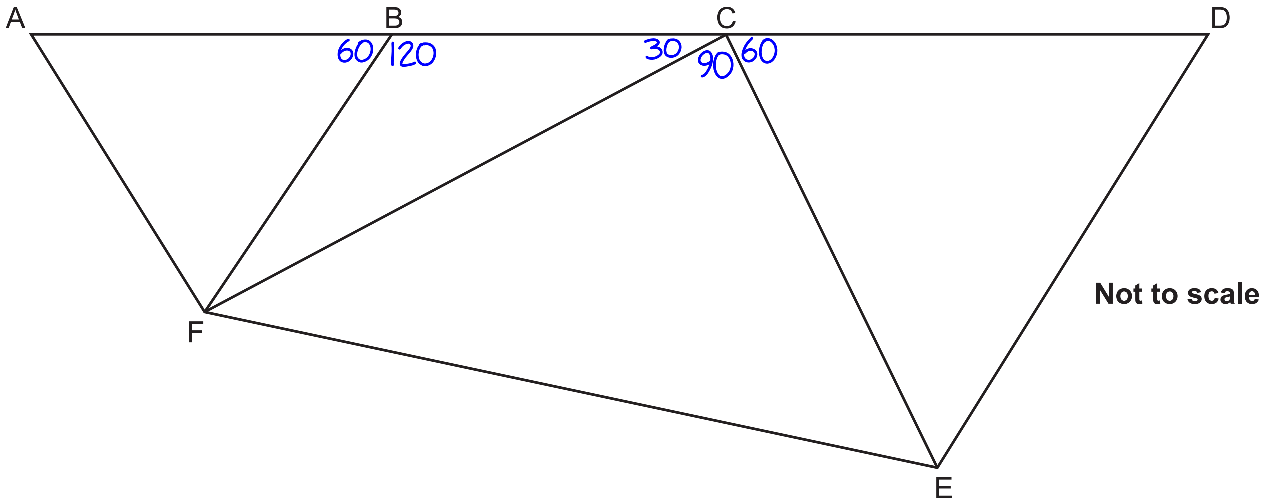
$21 = 30k$

Subtracting 5k from both sides to get the k terms all on the same side

Dividing both sides by 30 gets k on its own

$k =$ ......0.7...... **[5]**

8 The diagram shows four triangles that are joined together.



- Points A, B, C and D lie on a straight line.
- Triangles ABF and CDE are equilateral triangles.
- Triangle BCF is an isosceles triangle with BF = BC.

Show that triangle CEF is a right-angled triangle.
Give a reason for each stage of your working.
Use the template below to help present your work. You may not need all of the lines.

Angle ABF = 60° because angles in a triangle add up to 180° and all of the angles in an equilateral triangle are equal. 180 ÷ 3 = 60

Angle DCE = 60° because angles in a triangle add up to 180° and all of the angles in an equilateral triangle are equal. 180 ÷ 3 = 60

Angle FBC = 120° because angles around a point on a straight line add up to 180°.
180 - 60 = 120 ← Subtracting angle ABF from 180 leaves angle FBC

Angle BCF = 30° because angles in a triangle add up to 180° and the base angles of an isosceles triangle are equal. (180 - 120) ÷ 2 = 30 ← Subtracting angle FBC from 180 leaves the total of the base angles in triangle BCF. As these are equal, this total can be divided by 2 to work out each base angle

Angle FCE = 90° because angles around a point on a straight line add up to 180°.
180 - 30 - 60 = 90 ← Subtracting angles BCF and DCE from 180 leaves angle FCE

Angle .... = ...° because ....

Angle .... = ...° because ....

[5]

9 A large box of chocolates contains dark, milk and white chocolates.
When Riley opens the box, the ratio of dark to milk to white chocolates is 3 : 2 : 4. ← First ratio
Riley's family eat 6 of the dark chocolates, none of the milk chocolates and all of the white chocolates.
The ratio of dark to milk chocolates is now 9 : 8. ← Second ratio

How many **white** chocolates did Riley's family eat?

12:8:16 ← There are the same number of milk chocolates represented by the first and second ratio. So multiplying all the parts in the first ratio by 4 so that there are 8 parts for milk chocolates and this is the same as the second ratio. This is the third ratio

12−9 ← Subtracting the 9 parts representing the dark chocolates in the second ratio from the 12 parts representing the dark chocolates in the third ratio works out that 3 parts of the second and third ratio is worth 6 chocolates as this is how many dark chocolates were eaten

6÷3 ← Dividing the 6 chocolates by the 3 parts which represent them works out that each part of the second and third ratio is worth 2 chocolates

16×2 ← Multiplying the 16 parts which represent the number of white chocolates in the second ratio by the value of each part works out that there were originally 32 white chocolates

..........32.......... white chocolates **[4]**

10 **(a)** x and y are related by the equation $xy = 36$.

Tick the correct statement.

- [ ] y is directly proportional to x
- [x] y is inversely proportional to x
- [ ] y is not proportional to x

As multiplying x must divide y by the same factor

[1]

(b) y is inversely proportional to x^4.
$y = 2.5$ when $x = 2$.

Find a formula linking x and y.

$y \propto \frac{1}{x^4}$ ← Writing the proportion

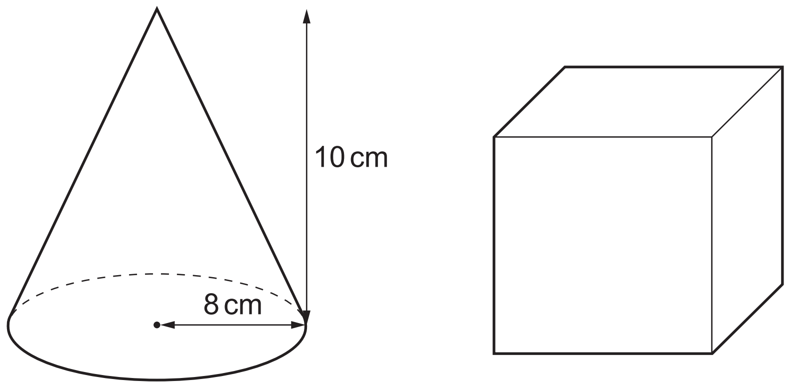
$y = \frac{k}{x^4}$ ← Multiplying the right side of the proportion by k, which represents an unknown value. It can now be written as an equation

$k = 2.5 \times 2^4$ ← Multiplying both sides of the equation by x^4 to make k the subject and substituting 2.5 for y and 2 for x. This finds that k = 40

Substituting the value of k back in the equation →

(b) $y = \frac{40}{x^4}$ **[3]**

11 The diagram shows a cone and a cube.
The cone has radius 8 cm and height 10 cm.



The volume of the cone is equal to the volume of the cube.

Work out the length of one side of the cube.

[The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

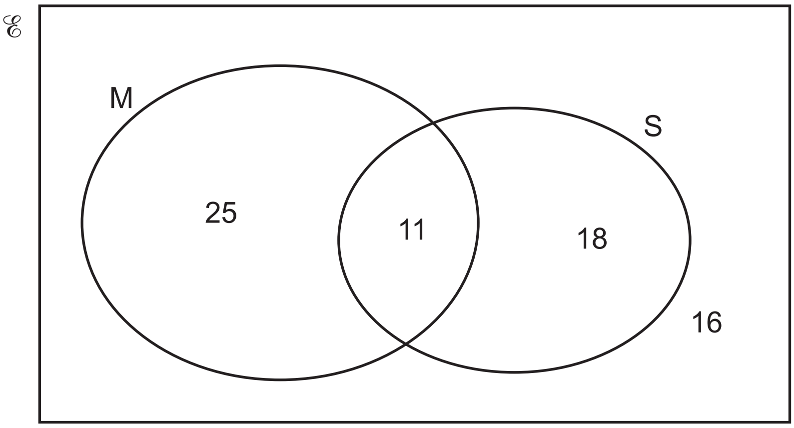
$\sqrt[3]{\frac{1}{3}\pi \times 8^2 \times 10}$

Substituting 8 for r and 10 for h in the right side of the formula for the volume of the cone expresses the volume of the cone. This is also the volume of the cube. Volume of cube = length^3, so cube rooting the volume gives the side length

8.8 cm **[4]**

12 A cafe owner recorded information about customer orders for coffee.
They recorded whether the customer asked for milk (M) and whether the customer asked for sugar (S).

The results are shown in this Venn diagram.



(a) One of the customers is chosen at random.

Find the probability that the customer asked for sugar.

11+18
29+25+16

Adding the two numbers in the S ring works out that 29 customers asked for sugar. Then adding the other numbers in the Venn diagram to this works out that there were 70 customers in total

(a) .... $\frac{29}{70}$ .... **[2]**

29 out of the 70 customers asked for sugar

(b) One of the customers is chosen at random.

Find the probability that the customer asked for sugar given that they asked for milk.

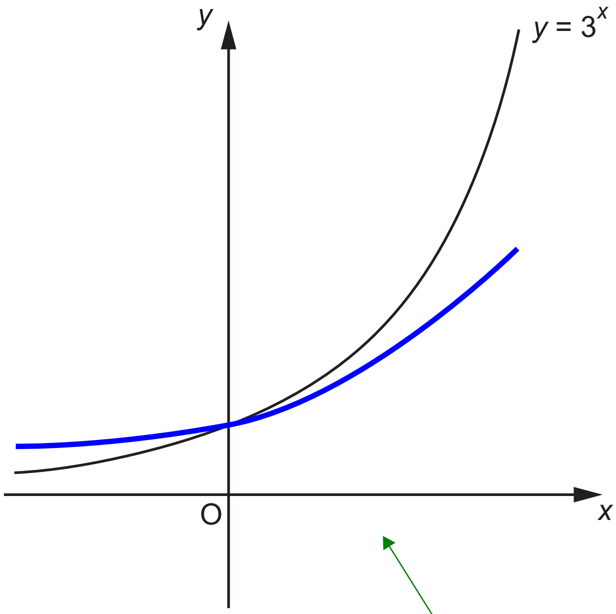
25+11

Adding the two numbers in the M ring works out that 36 customers asked for milk

(b) .... $\frac{11}{36}$ .... **[2]**

11 out of the 36 customer who asked for milk also asked for sugar

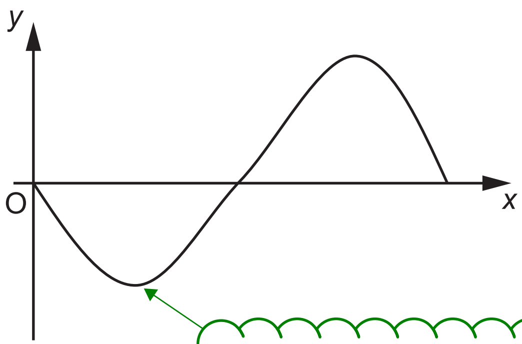
13 **(a)** The graph of $y = 3^x$ is sketched below.



On the same axes, sketch the graph of $y = 2^x$. **[3]**

Using table mode on the calculator and setting f(x) = 3^x and g(x) = 2^x. Start: -5, end: 5, step: 1. This gives a table of values for both equations. Comparing both finds that they are similar curves which both cross the y-axis at 1 but 2^x is greater when x is negative and less when x is positive

(b) Charlie sketches this graph.



x should be 90 at this point and it is giving a negative value

Charlie says

The equation of my graph is y = sinx.

(i) Explain how you know that Charlie is **not** correct.

The graph is negative when x = 90

sin90 = 1

[1]

(ii) Write down a possible equation for Charlie's graph.

It could be a reflection of y = sinx in the x-axis. This flips the sign on the right side of the equation

(b)(ii) $y = -\sin x$ **[1]**

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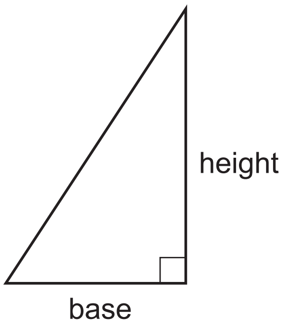
14 Here is a right-angled triangle.



Not to scale

The area of the triangle is $100\,cm^2$, correct to the nearest $10\,cm^2$.
The length of the base of the triangle is 8 cm, correct to the nearest cm.

Calculate the largest possible height of the triangle.

$\frac{1}{2}bh = A$ ← 1/2 x base x height = area of triangle

$bh = 2A$ ← Multiplying both sides by 2 eliminates the 1/2 on the left

$h = \frac{2A}{b}$ ← Dividing both sides by b makes h the subject

$= \frac{2\left(100 + \frac{10}{2}\right)}{8 - \frac{1}{2}}$ ← Substituting in the upper bound of the area and the lower bound of the base (as dividing by less makes it more) gives the upper bound of the height. The upper bound of the area is expresses by adding half of the resolution (which is 10 as it is to the nearest $10\ cm^2$) to the 100. The lower bound of the base is expressed by subtracting half of the resolution (which is 1 as it is to the nearest 1 cm) from the 8

.......................28....................... cm **[4]**

15 (a) Sasha and Taylor are asked to find how many solutions the equation $5(x+2)^2 = 45$ has.

Here is **Sasha**'s answer.

$5(x+2)^2 = 45$

$(x+2)^2 = 9$

$x + 2 = 3$

$x = 1$

There is <u>one solution</u>.

Here is **Taylor**'s answer.

$5(x+2)^2 = 45$

$(x+2)^2 = 9$ ← Divided both sides by 5

$x + 2 = 3$ or $x + 2 = -3$ ← Square rooted both sides

$x = 1$ or $x = -5$ ← Subtracted 2 from both sides

There are <u>two solutions</u>.

Decide who is correct, Sasha or Taylor, and give the reason for your decision.

Taylor is correct because the square root of 9 is 3 and -3

[1]

(b) Solve this equation algebraically.
Give your answers correct to **2** decimal places.
You must show your working.

$x^2 - 5x + 3 = 0$

Using the quadratic formula as it is in the quadratic form $ax^2 + bx + c = 0$ and the fact that the answers need to be to 2 decimal places means that it is probably not possible to factorise. a = 1, b = -5, c = 3

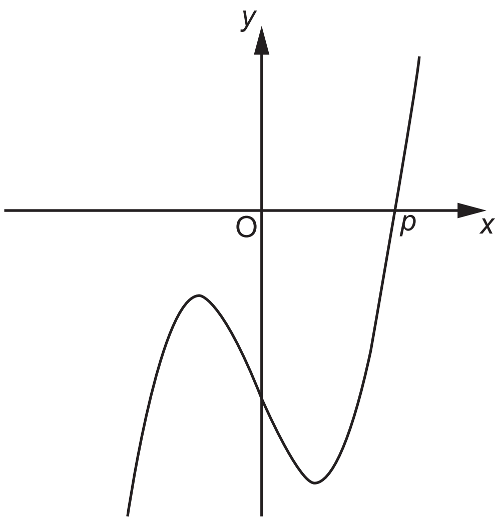
→ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

→ $\dfrac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 3}}{2 \times 1}$

Rounding 4.302... to 2 decimal places and rounding 0.697... to 2 decimal places

(b) $x =$ 4.30 or $x =$ 0.70 **[4]**

16 The graph of $y = x^5 - 70x - 150$ is sketched below.
The root of the equation $x^5 - 70x - 150 = 0$ is p.



(a) Show that $3 < p < 4$. **[3]**

$3^5 - 70 \times 3 - 150 = -117$
$4^5 - 70 \times 4 - 150 = 594$
Sign change so solution between x = 3 and x = 4

Substituting 3 for x gives a negative result and substituting 4 for x gives a positive result. As it is a continuous function, it must be 0 at some point between x = 3 and x = 4

(b) Find a smaller interval that contains the value of p.
You must show calculations to support your answer.

$3.2^5 - 70 \times 3.2 - 150 = -38.4...$

Using table mode, enter $f(x) = x^5 - 70x - 150$. Start: 3, end: 4, step: 0.1. This substitutes in all the values between 3 and 4 to 1 decimal place. 3.2 still gives a negative result so there will be a change in sign between x = 3.2 and x = 4

(b) ......3.2...... $< p <$ ......4...... **[3]**

17 Hiro invests £2500 for 2 years in a bank account paying r% per year compound interest.
At the end of 2 years, the amount in the bank account is £2704.

Calculate r.

$2500\left(\frac{100+r}{100}\right)^2 = 2704$ ← Adding r% to 100% expresses the percentage it increases to each year. Putting this over 100 converts it into a fraction, which increases the £2500 by r% when it is multiplied. Raising the fraction to the power of 2 as it is increased by r% 2 times. Increasing £2500 by r% 2 times must give the £2704

$\left(\frac{100+r}{100}\right)^2 = 1.0816$ ← Dividing both sides by 2500

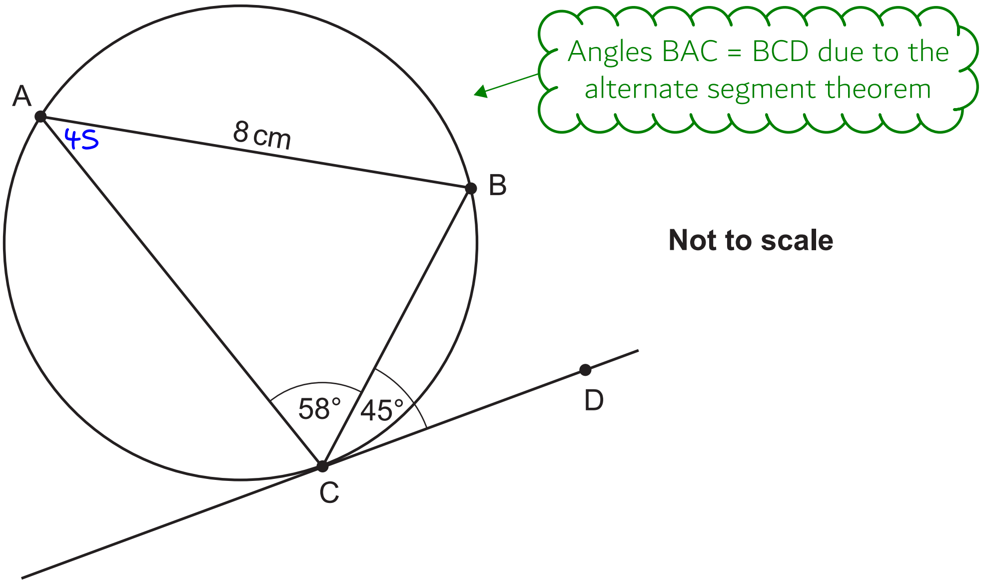
$\frac{100+r}{100} = 1.04$ ← Square rooting both sides

$100 + r = 104$ ← Multiplying both sides by 100

Subtracting 100 from both sides makes r the subject

$r =$ ....4.................... **[4]**

18 A, B and C are points on the circumference of a circle.



Not to scale

Points C and D lie on a tangent to the circle at C.
Angle BCD = 45°.
Angle ACB = 58°.
AB = 8 cm.

Find the length of BC.

$$\frac{BC}{\sin 45} = \frac{8}{\sin 58}$$

There are two opposite pairs of sides and angles so the sine rule can be used. a/sinA = b/sinB. Substituting BC for a, 45 for A, 8 for b and 58 for B

$$BC = \frac{8}{\sin 58} \times \sin 45$$

Multiplying both sides by sin45 makes BC the subject

..........6.7.......... cm **[4]**

19 A fitness centre records how long each customer spends in the gym.
This **cumulative frequency** table summarises the results.

Time (*t* minutes)	**Cumulative frequency**
$t \leqslant 10$	6
$t \leqslant 20$	24
$t \leqslant 30$	35
$t \leqslant 40$	48
$t \leqslant 50$	60
$t \leqslant 60$	74

Time (t minutes)	Frequency	Midpoints	Total
$0 \le t \le 10$	6	5	30
$10 < t \le 20$	18	15	270
$20 < t \le 30$	11	25	275
$30 < t \le 40$	13	35	455
$40 < t \le 50$	12	45	540
$50 < t \le 60$	14	55	770

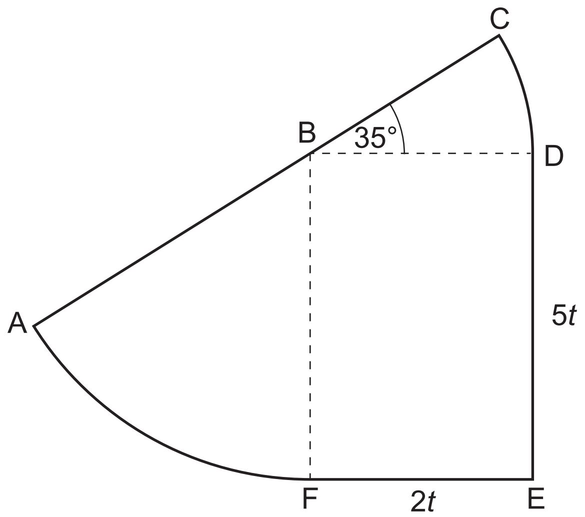
$2340 \div 74$

Calculate an estimate of the mean time the customers spend in the gym.
You must show your working.
You may use the table above to help present your work.

The lowest time spent in the gym could be 0 minutes. Writing the cumulative frequency times in intervals and working out the frequency for each one by subtracting the previous cumulative frequency. For example, the frequency for $10 < t \le 20$ is 18 as 24 - 6 = 18. Working out the midpoint of each of the intervals by doing the mean of the lower and upper bound of each. For example, for $10 < t \le 20$: 10 + 20 = 30 then 30 ÷ 2 = 15. Multiplying the frequencies by the midpoints to give an estimated total time for each interval. Adding all these totals estimates that the total time the customers spend in the gym is 2340 minutes. Dividing this by the 74 customers works out an estimated mean

.................31.6................. minutes **[5]**

20 This shape is formed from a rectangle and two sectors of circles.



Not to scale

Points A, B and C lie on a straight line.
Angle CBD = 35°.
DE = $5t$ and EF = $2t$.

(a) Explain why BC = $2t$.
Give a reason for each step of your explanation.

BD = 2t as opposite sides of a rectangle are equal

Both EF and BD are opposite sides of the rectangle

BC = 2t as radii are equal

BC is a radius and BD is a radius of the same sector. So they must be equal

[2]

(b) Show that the perimeter of the shape is $\frac{23}{12}\pi t + 14t$. **[5]**

180-35-90

ABC is a straight line and angles around a point on a straight line add up to 180°. So subtracting angle CBD and angle FBD (which must be 90° as it is the angle in a rectangle) from 180° finds that angle ABF is 55°

$\frac{35}{360} \times 2 \times \pi \times 2t + \frac{55}{360} \times 2 \times \pi \times 5t + 5t + 2t + 5t + 2t$

Adding all the outside sides expresses the perimeter

Sides AB, BC, DE and EF

This expresses the length of arc AF. Circumference = 2 x π x radius. The radius of sector BAF is 5t. Doing 55/360 of the circumference as this is the fraction of the whole circle which the sector is

This expresses the length of arc CD. Circumference = 2 x π x radius. The radius of sector BDC is 2t. Doing 35/360 of the circumference as this is the fraction of the whole circle which the sector is

$\frac{23}{12}\pi t + 14t$

Simplifying the expression of the perimeter

21 Simplify fully.

$$\frac{x^3 + 8x^2 + 15x}{x^3 - 9x}$$

To simplify fractions: divide both the numerator and denominator by a common factor. So the numerator and denominator need to be factorised to express them as factors then the ones in common can be cancelled out

$x(x^2+8x+15)$ ← Bringing x out as a factor on the numerator as this is a factor of all three terms

$x(x+5)(x+3)$ ← Factorising the $x^2 + 8x + 15$. 5 and 3 add to the 8 and multiply to the 15 so putting these in brackets with x

$x(x^2-9)$ ← Bringing x out as a factor on the denominator as this is a factor of both terms

$x(x+3)(x-3)$ ← Factorising the $x^2 - 9$ by using difference of two squares. $A^2 - B^2 = (A + B)(A - B)$

Both x and (x + 3) are common factors to the numerator and denominator so these can be cancelled out

$\frac{x+5}{x-3}$ .... **[5]**

END OF QUESTION PAPER