Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE
MATHEMATICS

Higher Tier Paper 3 Calculator

H

Tuesday 11 June 2019 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** questions in the spaces provided

1 Work out £1.50 as a fraction of 60p

Circle your answer.

[1 mark]

$\frac{2}{5}$ $\frac{1}{4}$ $\frac{4}{1}$ [✓ $\frac{5}{2}$]

£1.50 is 150p. 150/60 = 5/2

2 For a biased dice, P(6) = $\frac{3}{5}$

Circle the probability of two sixes when the dice is rolled twice.

[1 mark]

$\frac{6}{25}$ $\frac{6}{10}$ [✓ $\frac{9}{25}$] $\frac{9}{5}$

Six AND six. AND means to multiply the probabilities so 3/5 x 3/5

3 Circle the lowest common multiple (LCM) of 5, 15 and 25

[1 mark]

5 45 [✓ 75] 150

75 is the lowest of the numbers which is a multiple of 5, 15 and 25. Dividing the 75 by the 5, 15 and 25 gives whole number results which shows that it is a multiple of each of them

4 Circle the **two** roots of $(x - 5)(x + 3) = 0$

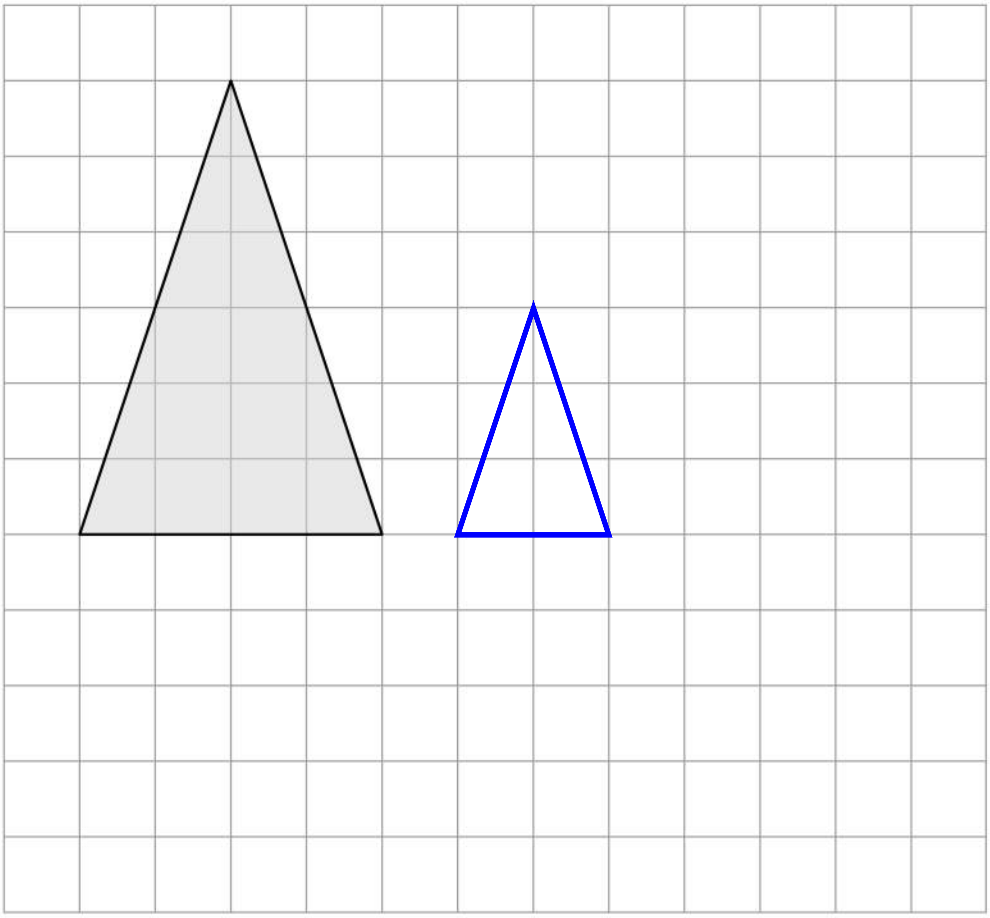
[1 mark]

–5 [✓ –3] 3 [✓ 5]

Either x - 5 = 0 or x + 3 = 0. Rearranging these to make x the subject gives x = 5 or x = -3

5 On the grid, draw an enlargement of the triangle with scale factor $\frac{1}{2}$

[2 marks]



The base was 4 cm. Multiplying this by 1/2 works out that the new base should be 2 cm.
The height was 6 cm. Multiplying this by 1/2 works out that the new height should be 3 cm

6

6 To the nearest pound, Jon has £9

To the nearest 50p, Ellie has £6.50

Work out the maximum possible total amount of money.

[3 marks]

$9+\frac{1}{2}=9.50$ ←

Adding half of the resolution (which is £1 as this is what it is to the nearest) to the £9 works out that the upper bound of the money Jon has is £9.50. However this is not possible as it rounds to £10 to the nearest pound so it needs to be a penny less. So the most Jon could have is £9.49

$6.50+\frac{0.50}{2}=6.75$ ←

Adding half of the resolution (which is £0.50 as this is what it is to the nearest) to the £6.50 works out that the upper bound of the money Ellie has is £6.75. However this is not possible as it rounds to £7 to the nearest 50p so it needs to be a penny less. So the most Ellie could have is £9.74

$9.49+6.74$ ←

Adding the most Jon could have and the most Ellie could have works out the maximum possible total amount of money

Answer £ 16.23

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7 Two solids, J and K, have the same density.

Complete the table.

Include units in your answers.

[3 marks]

	J	**K**
Mass	48 g	78 g
Volume	8 cm^3	13 cm^3 C
Density	6 g/cm^3 A	6 g/cm^3 B

$d \overset{m}{\ } v$ ← Writing the formula triangle for density, mass, volume

A: Covering d in the formula triangle finds that density = mass ÷ volume. Dividing the mass of 48 g by the volume of 8 cm^3 works out that the density of J is 6 g/cm^3. The unit is g/cm^3 as g was divided by cm^3.

B: J and K have the same density.

C: Covering v in the formula triangle finds that volume = mass ÷ density. Dividing the mass of 78 g by the density of 6 g/cm^3 works out that the volume of K is 13 cm^3

8 Rearrange $y = 3x - 2$ to make x the subject.

Circle your answer.

[1 mark]

$x = \frac{y}{3} - 2$ [✓ $x = \frac{y+2}{3}$] $x = \frac{y-2}{3}$ $x = \frac{y}{3} + 2$

Add 2 to both sides then divide both sides by 3 to make x the subject

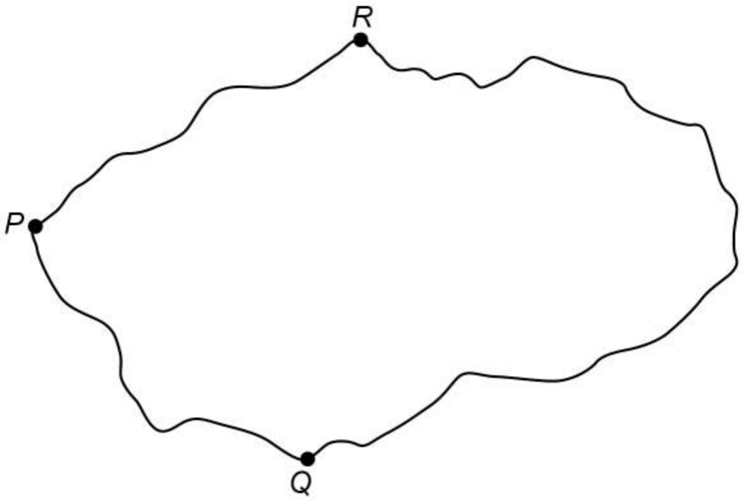
7

9 Towns P, Q and R are connected by roads PQ, PR and QR.

PR is 10 km longer than PQ.

QR is twice as long as PR.

The total length of the three roads is 170 km



Not drawn accurately

Work out the length of PQ.

[4 marks]

$x+x+10+2x+20$

Let x be PQ. PR is x + 10 and QR is 2(x + 10), which is 2x + 20. Expressing the total length of the three roads

$4x+30=170$

Collecting like terms and setting the expression equal to the actual total length of 170 km

$4x=140$

Subtracting 30 from both sides to get the x term on its own

Answer 35 km

Dividing both sides by 4 finds that x = 35, which is the length of PQ

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10 Mia wants to borrow £6000 and repay it, with interest, after two years.

She sees two offers for loans.

Offer 1	**Offer 2**
Compound interest	Compound interest
3% per year	First year 1%
	Second year 5%

Mia says,

> "I will pay back the same amount because the average of 1% and 5% is 3%"

Is she correct?

You **must** show your working.

[3 marks]

$$6000 \times \left(\frac{100+3}{100}\right)^2 = 6365.40$$

Adding 3% to 100% expresses the percentage the amount increases to each year. Putting this over 100 converts it into a fraction which increases the £6000 by 3% when multiplied. Raising the fraction to the power of 2 as it needs to be increased by 3% twice. So £6365.40 needs to be paid back with Offer 1

$$6000 \times \frac{100+1}{100} \times \frac{100+5}{100} = 6363$$

Adding 1% to 100% expresses the percentage the amount increases to each year. Putting this over 100 converts it into a fraction which increases the £6000 by 1% when multiplied. Doing the same for the 5%. So £6363 needs to be paid back with Offer 2

No

£6365.40 is not the same amount as £6363

Turn over for the next question

7

11 Here are two sets of numbers, A and B.

Set A

200	160
104	100

Set B

270	400	483
300	x	

mean of Set A : mean of Set B = 3 : 8

Work out the value of x.

[4 marks]

$\overset{t}{m \quad n}$ ← Writing the formula triangle for mean, total, number

$200+160+104+100$ ← Adding all the numbers in Set A works out that their total is 564

$564 \div 4$ ← Covering m in the formula triangle finds that mean = total ÷ number, where total is 564 and number is 4 as there are 4 numbers in Set A. So the mean of Set A is 141

$141 \div 3$ ← 3 parts of the ratio represent the mean of Set A. So dividing the mean of Set A by 3 works out that 1 part of the ratio represents 47

47×8 ← Multiplying the value of 1 part of the ratio by 8 works out that the 8 parts are worth 376 so this is the mean of Set B

376×5 ← Covering t in the formula triangle finds that total = mean x number. So multiplying the mean of Set B by the 5 numbers in Set B works out that their total is 1880

$1880-270-400-483-300$ ← Subtracting the other numbers in Set B from their total leaves x

Answer 427

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Do not write outside the box

12 A straight line

has gradient 4

and

passes through the point (5, 23)

Work out the equation of the line.

Give your answer in the form $y = mx + c$

[3 marks]

$c = 23 - 4(5)$

Rearranging y = mx + c to get c = y - mx then substituting 23 for y, 5 for x and 4 for m (the gradient). So c is 3

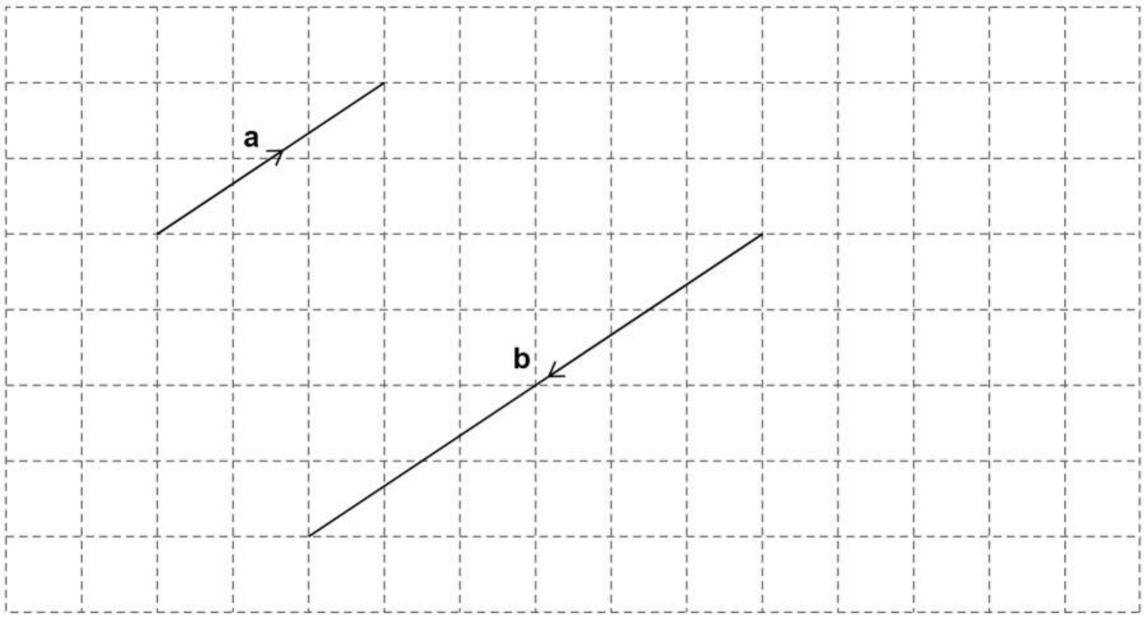
Answer $y = 4x + 3$

m is 4 (as this is the gradient) and c is 3

Turn over for the next question

7

13 (a) Vectors **a** and **b** are drawn on a grid.



Write **b** in terms of **a**.

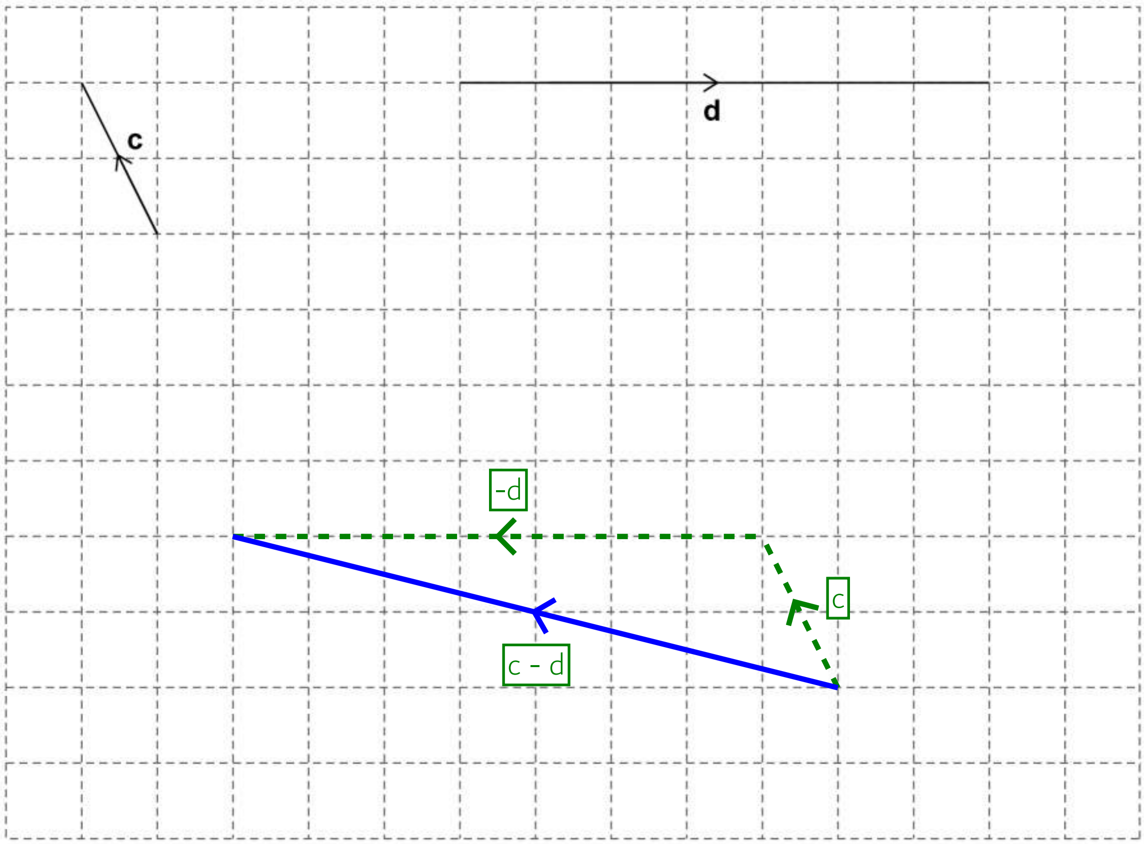
[1 mark]

b = -2a

Vector b is twice as long as a but in the opposite direction

Do not write outside the box

13 (b) Vectors **c** and **d** are drawn on a grid.



On the grid above, draw a vector representing **c – d**

[2 marks]

Combining the vectors c and -d by imagining them drawn tip to tail. -d is the same length as d but in the opposite direction

Turn over for the next question

3

14 For Class X, number of boys : number of girls = 7 : 8

For Class Y, number of boys : number of girls = 3 : 4

Which statement **must** be true?

Tick **one** box.

[1 mark]

☐ Class X has more boys than class Y

☐ Class X has twice as many girls as class Y

☑ Class X has a greater proportion of boys than class Y

☐ Class X has the same proportion of boys as class Y

The proportion of boys in Class X is 7/(7 + 8) = 7/15 = 0.46...
The proportion of boys in Class Y is 3/(3 + 4) = 3/7 = 0.42...

15 Simplify fully $\frac{a^3b^2}{cd} \times \frac{c}{ab^5}$

[3 marks]

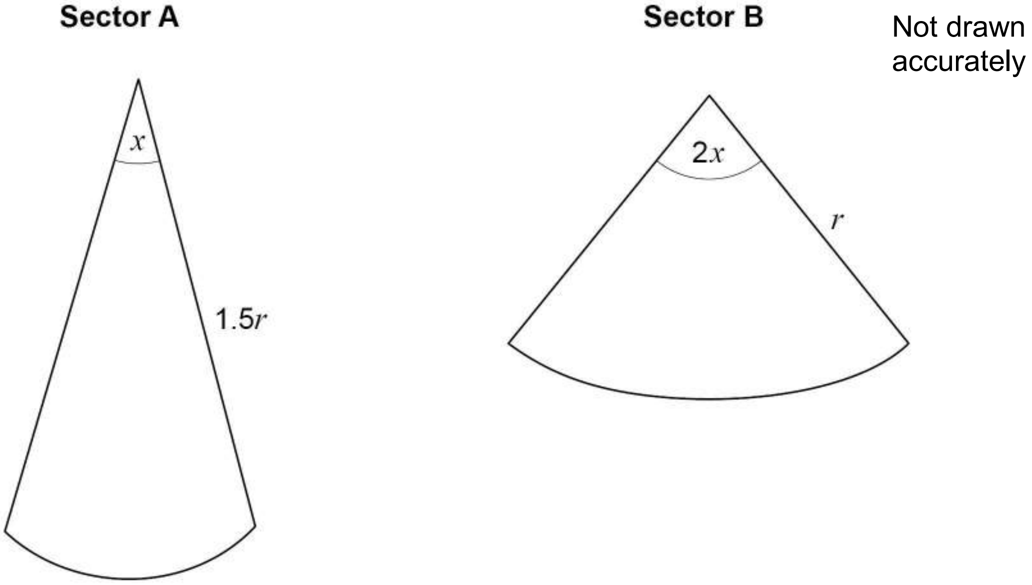
$$\frac{a^3b^2c}{ab^5cd}$$

The fractions are multiplied by multiplying the numerators and multiplying the denominators

Answer $\frac{a^2}{b^3d}$

Cancel out common factors to simplify the fraction. Dividing both the numerator and denominator by a cancels out a from the denominator and $a^3 \div a = a^2$. Dividing both the numerator and denominator by b^2 cancels out b^2 from the numerator and $b^5 \div b^2 = b^3$. Dividing both the numerator and denominator by c cancels out c from both.

16 Here are two sectors from different circles.



Which sector has the bigger area?

Tick a box.

☑ Sector A ☐ Sector B

Show working to support your answer.

[2 marks]

$\frac{x}{360} \times \pi \times (1.5r)^2 = \frac{2.25}{360} x\pi r^2$ ← Area of Sector A

$\frac{2x}{360} \times \pi \times r^2 = \frac{2}{360} x\pi r^2$ ← Area of Sector B

Area of a sector = a/360 x πr^2, where a is the angle of the sector and r is the radius. 2.25/360 is greater than 2/360 and the rest of the expressions of the areas are the same so Sector A must have the bigger area

6

Turn over ►

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17 A factory makes kettles.

Four samples of kettles are tested for faults.

Each sample has size 200

Here are the relative frequencies of faulty kettles in the samples.

Sample	P	Q	R	S
Relative frequency	0.03	0.035	0.015	0.01

Work out the range of the number of faulty kettles in the four samples.

[3 marks]

0.035 − 0.01 ← Subtracting the smallest relative frequency from the largest relative frequency works out that the range in the relative frequencies is 0.025

0.025 × 200 ← Multiplying the range of the relative frequencies by the 200 in the sample size works out the range of the number of faulty kettles in the samples

Answer 5

18 (a) Write $\overset{\frown}{x(3x} - 9) = 4$ in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

[1 mark]

Answer $3x^2 - 9x - 4 = 0$

Expanding the brackets then subtracting 4 from both sides

18 (b) Solve $x(3x - 9) = 4$

Give your answers to 2 decimal places.

[2 marks]

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(3)(-4)}}{2(3)} \leftarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftarrow$$

Using the quadratic formula on the answer to part (a) as it is in the quadratic form $ax^2 + bx + c = 0$.
$a = 3$, $b = -9$ and $c = -4$

Answer $x = 3.39$, $x = -0.39$

Rounding 3.392... and - 0.392... to 2 decimal places

Turn over for the next question

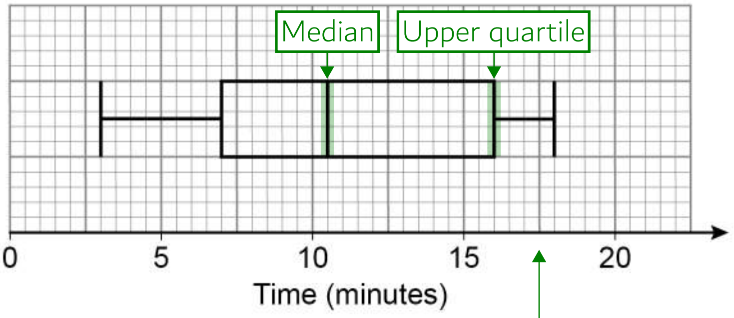
6

19 Here is some information about the times people took to complete a survey.

Fastest time	3 minutes
Slowest time	18 minutes
Median	11 minutes
Lower quartile	7 minutes
Interquartile range	8 minutes

Ben draws this box plot to show the information.

Time to complete a survey



The scale goes up 5 over 10 boxes.
5 ÷ 10 = 0.5 so every small box is worth 0.5

Make **two** criticisms of his box plot.

[2 marks]

Criticism 1 Median is at 10.5 ← Should be two boxes after 10, at 11

Criticism 2 Upper quartile should be at 15 ← Adding the interquartile range to the lower quartile works out the upper quartile. 7 + 8 = 15

20 d is directly proportional to the square of v.

$d = 6$ when $v = 20$

20 (a) Work out an equation connecting d and v.

[3 marks]

$d = kv^2$ ← Converting the proportion to an equation by multiplying the right side by k, which represents a constant

$k = \frac{6}{20^2}$ ← Dividing both sides by v^2 to get k on its own and substituting 6 for d and 20 for v. So k = 0.015

Answer $d = 0.015v^2$

Substituting 0.015 for k in the original equation

20 (b) Work out the value of d when $v = 30$

[2 marks]

0.015×30^2 ← Substituting 30 for v in the right side of the equation found in part (a)

Answer 13.5

Turn over for the next question

7

Turn over ►

21 Hanif makes green paint by mixing blue paint and yellow paint in the ratio

blue : yellow = 7 : 3

He buys blue paint in 50-litre containers, each costing £225

He buys yellow paint in 20-litre containers, each costing £80

He wants to

sell the green paint in 5-litre tins

make 40% profit on each tin.

How much should he sell each tin for?

[5 marks]

$225 \div 50 = 4.50$ ← Dividing the £225 by the 50 litres in each container of blue paint works out that blue paint costs £4.50 per litre

$80 \div 20 = 4$ ← Dividing the £80 by the 20 litres in each container of yellow paint works out that yellow paint costs £4 per litre

$7+3$ ← There is 5 litres in total in each tin of green paint. Adding the 3 and 7 parts in the ratio works out that there are 10 parts in total in the ratio which represent this 5 litres

$5 \div 10$ ← Dividing the 5 litres by the 10 parts of the ratio which represents it works out that 1 part of the ratio is worth 0.5 litres

$0.5 \times 7 = 3.5$ ← Multiplying the value of 1 part of the ratio by the 7 parts works out that the 7 parts is worth 3.5 litres, which is the amount of blue paint in each tin

$0.5 \times 3 = 1.5$ ← Multiplying the value of 1 part of the ratio by the 3 parts works out that the 3 parts is worth 1.5 litres, which is the amount of yellow paint in each tin

$3.5 \times 4.50 = 15.75$ ← Multiplying the 3.5 litres of blue paint by the £4.50 per litre of blue paint works out that the blue paint in each tin costs £15.75

1.5×4 ← Multiplying the 1.5 litres of yellow paint by the £4 per litre of yellow paint works out that the yellow paint in each tin costs £6

$15.75 + 6$ ← Adding the cost of the blue paint and yellow paint in each tin works out that each tin will cost £21.75 to make

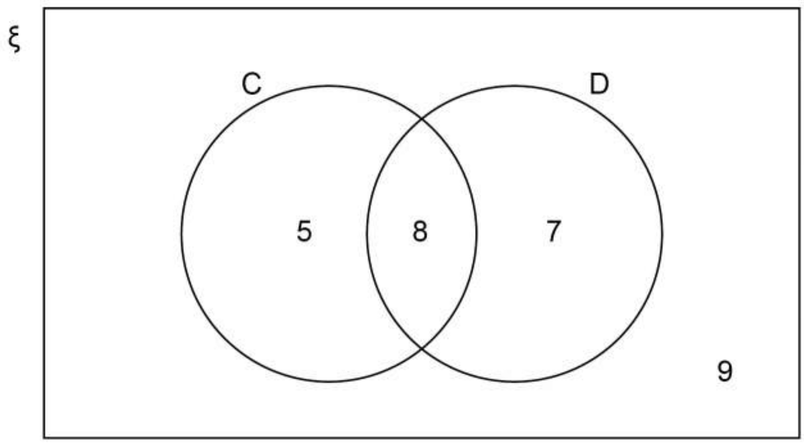
$21.75 \times \frac{100+40}{100}$ ← Adding 40% to 100% expresses the percentage the price of each tin will increase to after the profit. Putting this over 100 converts it to a fraction, which increases the £21.75 by 40% when multiplied. This works out the price he should sell each tin for

Answer £ 30.45

22 ξ = 29 students in a class

C = students who own a cat

D = students who own a dog



22 (a) A student is chosen at random.

Circle the probability that the student owns a cat or a dog but not both.

[1 mark]

[✓ $\frac{12}{29}$] $\frac{13}{29}$ $\frac{15}{29}$ $\frac{20}{29}$

5 own a cat but not a dog. 7 own a dog but not a cat. 5 + 7 = 12. There are 12 out of the 29 students who own a cat or a dog but not both

22 (b) A student who owns a dog is chosen at random.

Circle the probability that the student also owns a cat.

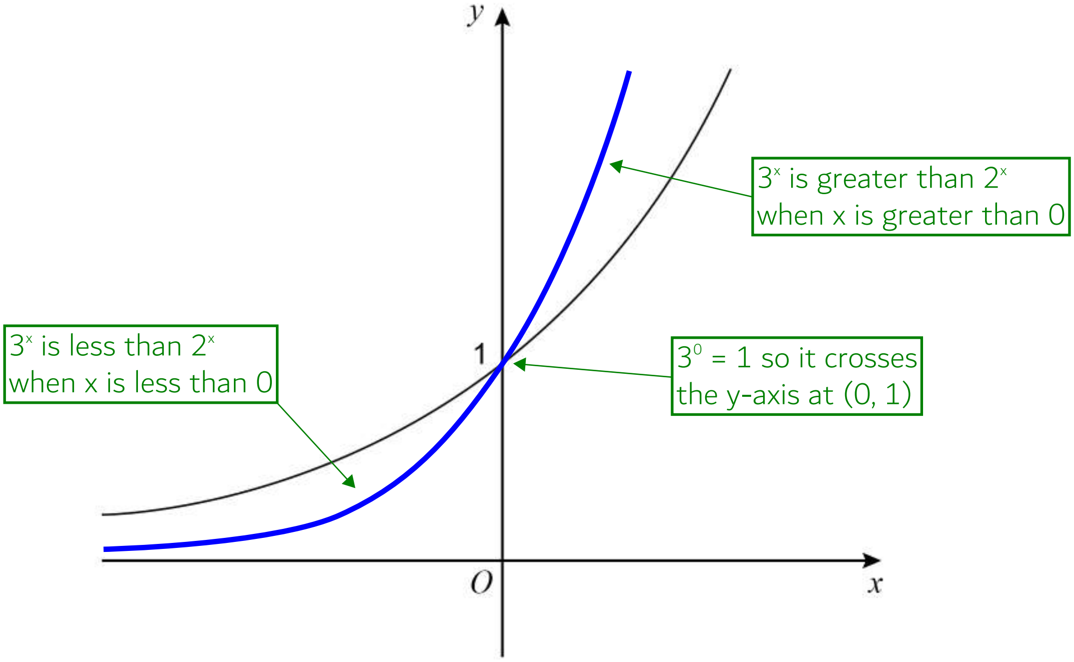
[1 mark]

$\frac{7}{15}$ [✓ $\frac{8}{15}$] $\frac{7}{29}$ $\frac{8}{29}$

Out of the 15 who own a dog, 8 own a cat

$\frac{}{7}$

23 Here is a sketch of the curve $y = 2^x$



3^x is greater than 2^x when x is greater than 0

3^x is less than 2^x when x is less than 0

$3^0 = 1$ so it crosses the y-axis at (0, 1)

On the axes above, sketch the curve $y = 3^x$

[2 marks]

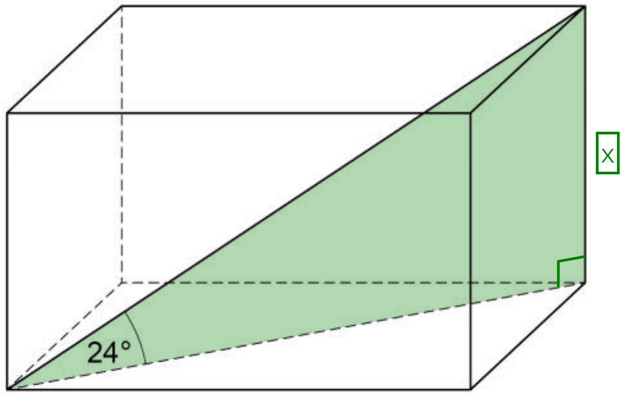
Using table mode on the calculator to create a table of values for $y = 2^x$ and $y = 3^x$ so that they can be more easily visualised and compared. Set $f(x) = 2^x$. Set $g(x) = 3^x$. Start: -5. End: 5. Step: 1

Do not write outside the box

24 The length of a diagonal of a cuboid is 20 cm

The diagonal makes an angle of 24° with the base.

The area of the base is 150 cm^2



Work out the volume of the cuboid.

[3 marks]

$S\overset{\checkmark}{O}\overset{\checkmark}{H}\;C^{A}\overset{\checkmark}{H}\;T\overset{\checkmark}{O}A$ ← Using right-angled trigonometry to find the length x in the green right-angled triangle. The diagonal is the hypotenuse so ticking H. x is the opposite so ticking O. There are two ticks on the SOH formula triangle so this can be used

$\sin 24 \times 20$ ← Covering O in the SOH formula triangle finds that opposite = sin of the angle x hypotenuse. So the opposite, x, is 8.1... cm

$150 \times 8.1...$ ← The cuboid can be treated like a prism. The base is the cross section and has an area of 150 cm^2. Multiplying this by the height of the cuboid gives the volume

Answer 1220.2 cm^3

5

Turn over ▶

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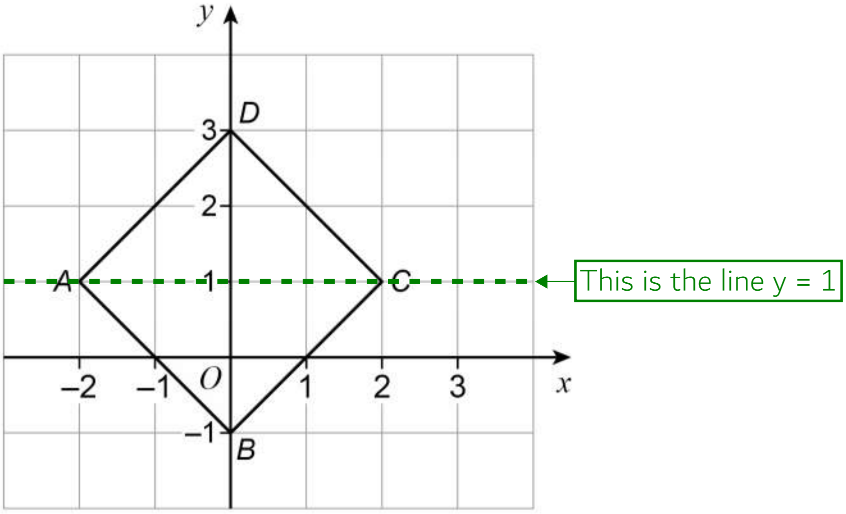
25 *ABCD* is a square.

A is (–2, 1) *B* is (0, –1) *C* is (2, 1) *D* is (0, 3)



25 (a) A **single** transformation of *ABCD* is such that

B is mapped to *D*

D is mapped to *B*

A and *C* are invariant points.

Describe fully the transformation.

[2 marks]

Reflection in the line y = 1

25 (b) A different **single** transformation of *ABCD* is such that

B is mapped to *D*
D is mapped to *B*
the only invariant point is (0, 1)

Describe fully the transformation.

[3 marks]

Rotation by 180 degrees centre (0, 1) ← The point rotating around does not change

26 $g(x) = 16 - x$ $h(x) = x^3$

Solve $gh(x) = 24$

[3 marks]

$16 - x^3 = 24$ ← Substituting h(x) for x in g(x) to get the composite function gh(x). Setting it equal to 24

$-x^3 = 8$ ← Subtracting 16 from both sides

$x^3 = -8$ ← Dividing both sides by -1

$x =$ -2

Cube rooting both sides

Turn over for the next question

8

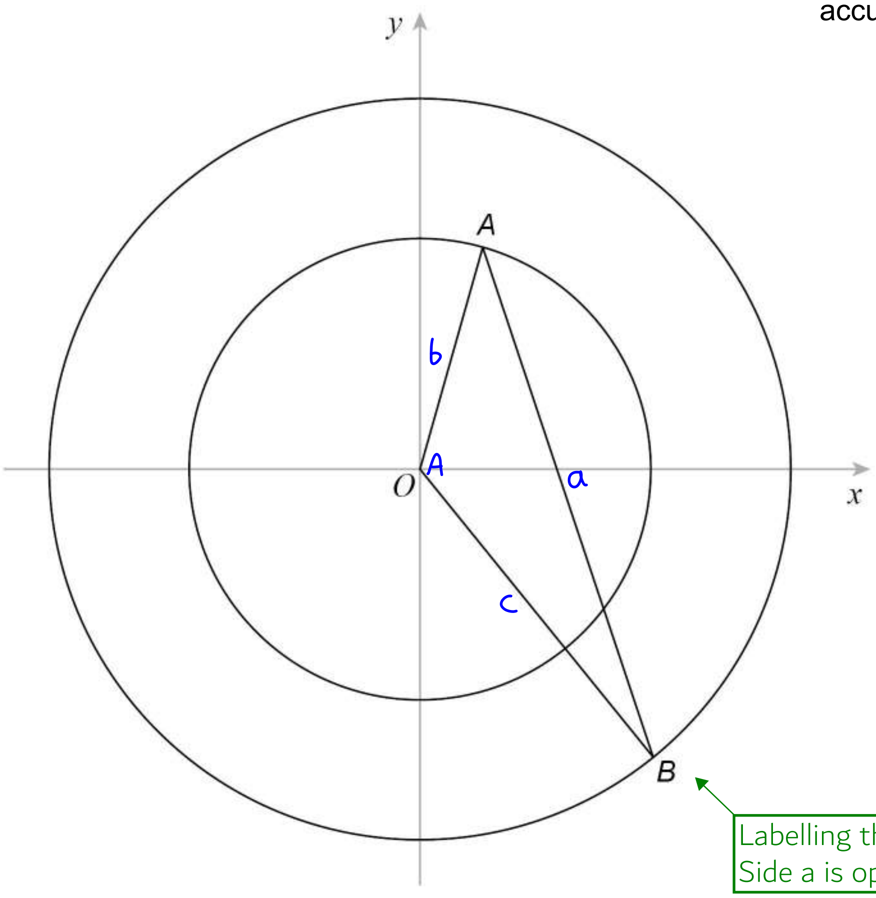
27 In this question, all lengths are in centimetres.

A is a point on a circle, centre O.

B is a point on a different circle, centre O.

$AB = 20$

Not drawn accurately



Labelling the triangle.
Side a is opposite angle A

The equation of the larger circle is $x^2 + y^2 = 144$

radius of smaller circle : radius of larger circle = 4 : 5

Work out the size of angle *AOB*.

[5 marks]

$\sqrt{144}$ ← The general equation of a circle with its centre at the origin is $x^2 + y^2 = \text{radius}^2$. So square rooting the 144 finds that the radius of the larger circle is 12 cm

$12 \div 5$ ← 5 parts of the ratio represent the radius of the larger circle. So dividing the radius of the larger circle by 5 works out that 1 part of the ratio is worth 2.4 cm

2.4×4 ← Multiplying the value of 1 part of the ratio by 4 works out that the 4 parts represent 9.6 cm, which is the radius of the smaller circle

$20^2 = 9.6^2 + 12^2 - 2 \times 9.6 \times 12 \times \cos A$ ← There aren't two opposite pairs of sides and angles so the sine rule can't be used. So the cosine rule should be used. $a^2 = b^2 + c^2 - 2bc\cos A$. Substituting the 20 cm for a, the radius of the smaller circle for b and the radius of the larger circle for c

$400 = 236.16 - 230.4\cos A$ ← $20^2 = 400$. $9.6^2 + 12^2 = 236.16$. $-2 \times 9.6 \times 12 = -230.4$

$163.84 = -230.4\cos A$ ← Subtracting 236.16 from both sides

$-0.71... = \cos A$ ← Dividing both sides by -230.4

Answer 135.3 degrees

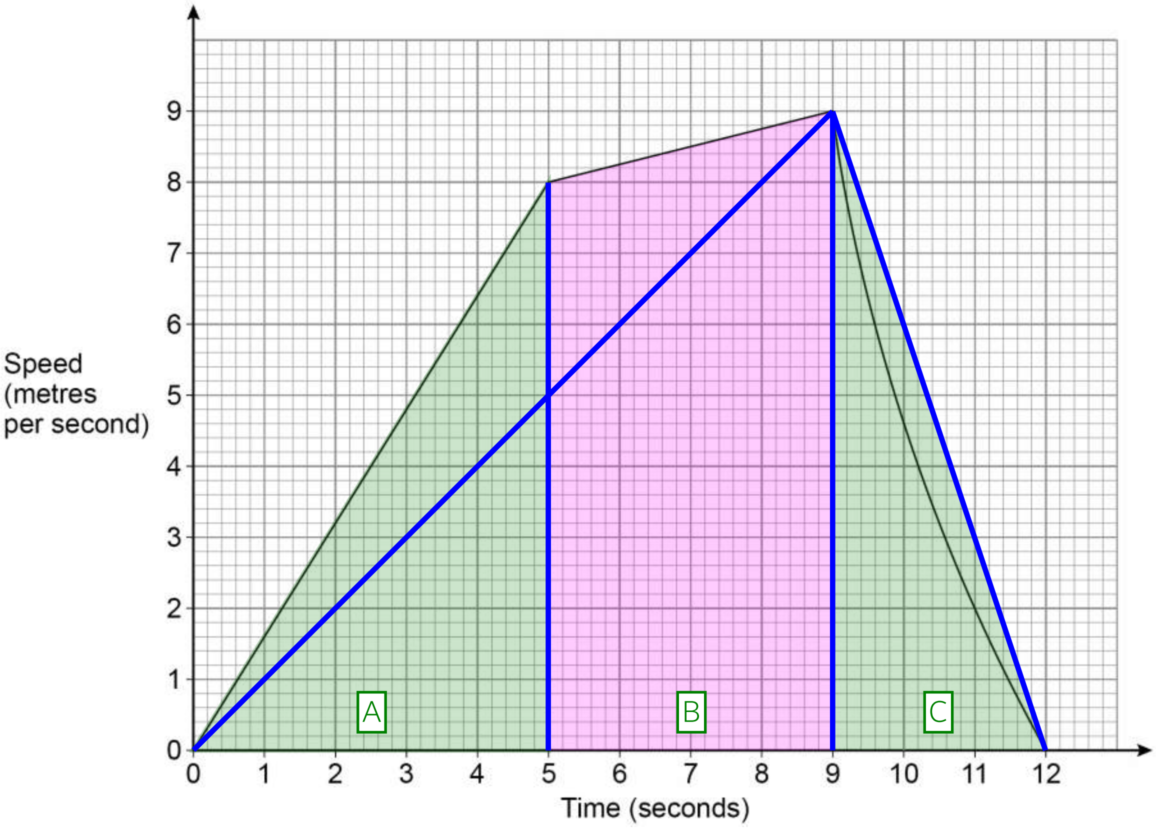
Doing the inverse cos of both sides. $\cos^{-1}(-0.71...) = 135.3...$

Turn over for the next question

5

28 Leo runs for 12 seconds.

The graph shows his speed.



28 (a) Show that the distance he runs is less than 67.5 metres.

[4 marks]

$\frac{1}{2} \times 5 \times 8 = 20$ ← Area of triangle = 1/2 x base x height. So the area of triangle A is 20

$\frac{1}{2}(8+9) \times 4 = 34$ ← Area of trapezium = 1/2 (a + b) x h, where a and b are the parallel sides and h is the distance between them. So the area of trapezium B is 34

$\frac{1}{2} \times 3 \times 9$ ← Area of triangle = 1/2 x base x height. So the area of triangle C is 13.5

$20 + 34 + 13.5$ ← Adding the areas of A, B and C gives 67.5, which is an estimate of the area under the graph, which is an estimate of the distance in metres

67.5 ← 67.5 metres is an overestimate as the curve goes below triangle C. So the distance must be less than 67.5 metres

Do not write outside the box

28 (b) Work out his average acceleration for the first 9 seconds.

State the units of your answer.

[2 marks]

$\frac{9-0}{9-0}$

Average acceleration is the gradient of the straight line drawn from (0, 0) to (9, 9). Gradient = (change in y)/(change in x). Change in y is 9 - 0 and change in x is 9 - 0. So the gradient is 1

Answer $1\,m/s^2$

Change in speed (m/s) was divided by change in time (s). m/s ÷ s = m/s^2

END OF QUESTIONS

6