

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

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Forename(s)

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Candidate signature

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I declare this is my own work.

# GCSE MATHEMATICS

# H

Higher Tier

Paper 2 Calculator

Friday 10 November 2023

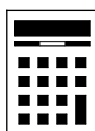
Morning

Time allowed: 1 hour 30 minutes

## Materials

For this paper you must have:

- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).



## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
<b>TOTAL</b>	

## Advice

In all calculations, show clearly how you work out your answer.



N 0 V 2 3 8 3 0 0 2 H 0 1

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

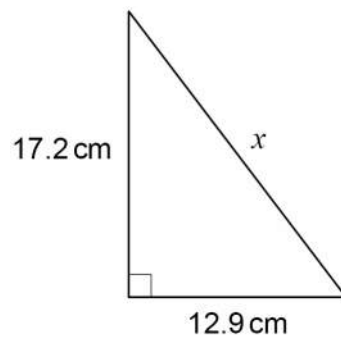
Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)



3 Use Pythagoras' theorem to work out the value of  $x$ .

[3 marks]



Not drawn  
accurately

$$12.9^2 + 17.2^2 = x^2$$

Pythagoras' Theorem is  $a^2 + b^2 = c^2$ , where  $a$  and  $b$  are the shorter sides and  $c$  is the longest side. Substituting 12.9 for  $a$ , 17.2 for  $b$  and  $x$  for  $c$

$$\sqrt{12.9^2 + 17.2^2} = x$$

Square rooting both sides to get rid of the square on  $x$

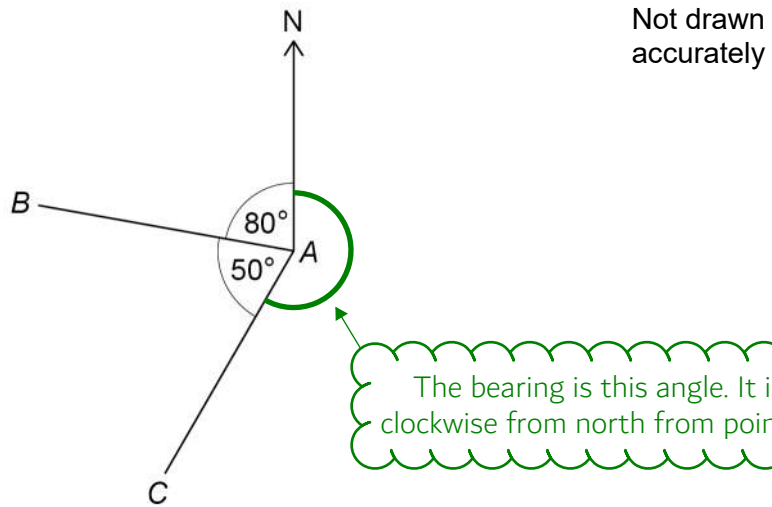
$$x = \underline{\quad 21.5 \quad} \text{ cm}$$

Turn over for the next question

Turn over ►



4 A, B and C are three points.



Work out the bearing of C from A.

[1 mark]

360-80-50

There are  $360^\circ$  in total around a point. Subtracting the other angles leaves the angle in green

Answer 230 °




5 Three shops sell the same type and size of lip balm stick.

**Shop A**



£2.39 each

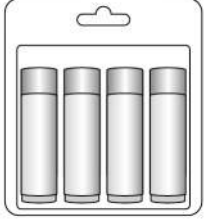
**Shop B**



£3.08 each

Buy one,  
get one half price

**Shop C**



Pack of 4

Was £11.40

Now  $\frac{1}{6}$  off

Which shop is the best value for 8 sticks and what is the total cost in that shop?

Show working to support your answer.

[5 marks]

$$2.39 \times 8 = 19.12$$

Multiplying the £2.39 by 8 works out that the total cost for 8 sticks from Shop A is £19.12

$$3.08 \div 2$$

This works out that the half price stick from Shop B is £1.54

$$3.08 + 1.54$$

This works out that buying one and getting one half price costs £4.62 from Shop B

$$4.62 \times 4 = 18.48$$

Buying one and getting one half price gets 2 sticks. So the offer should be used 4 times in order to get 8 sticks. This works out that the total cost for 8 sticks from Shop B is £18.48

$$11.40 \times \frac{5}{6}$$

$\frac{1}{6}$  off the price reduces it to  $\frac{5}{6}$  of the original as  $1 - \frac{1}{6} = \frac{5}{6}$ . Doing  $\frac{5}{6}$  of the £11.40 works out that a pack of 4 from Shop C costs £9.50

$$9.50 \times 2 = 19$$

2 packs of 4 are needed from Shop C to get 8 sticks.  $£9.50 \times 2$  works out that the total cost for 8 sticks from Shop C is £19

Shop B Total cost £ 18.48

Shop A costs £19.12. Shop B costs £18.48. Shop C costs £19.  
So Shop B is the best value as £18.48 is the lowest cost



6 Round 1 of voting for Head Student is taking place in a school.

6 (a) To reach round 2, a student must receive **at least**  $\frac{4}{15}$  of the votes.

What is the largest possible number of students that can reach round 2?

Circle your answer.

[1 mark]

15

11

3

4

$1 \div \frac{4}{15}$

See the next page for an explanation

6 (b) There are 900 votes in round 1

Sean receives 180 votes.

Amy draws a pie chart to represent the results.

Here is her method to work out the angle needed for Sean.

$$180 \div 900 \times 100 = 20$$

The angle should be  $20^\circ$

Is Amy's method correct?

Tick a box.

Yes

No

Give a reason for your answer.

[1 mark]

Should be  $180 \div 900 \times 360$

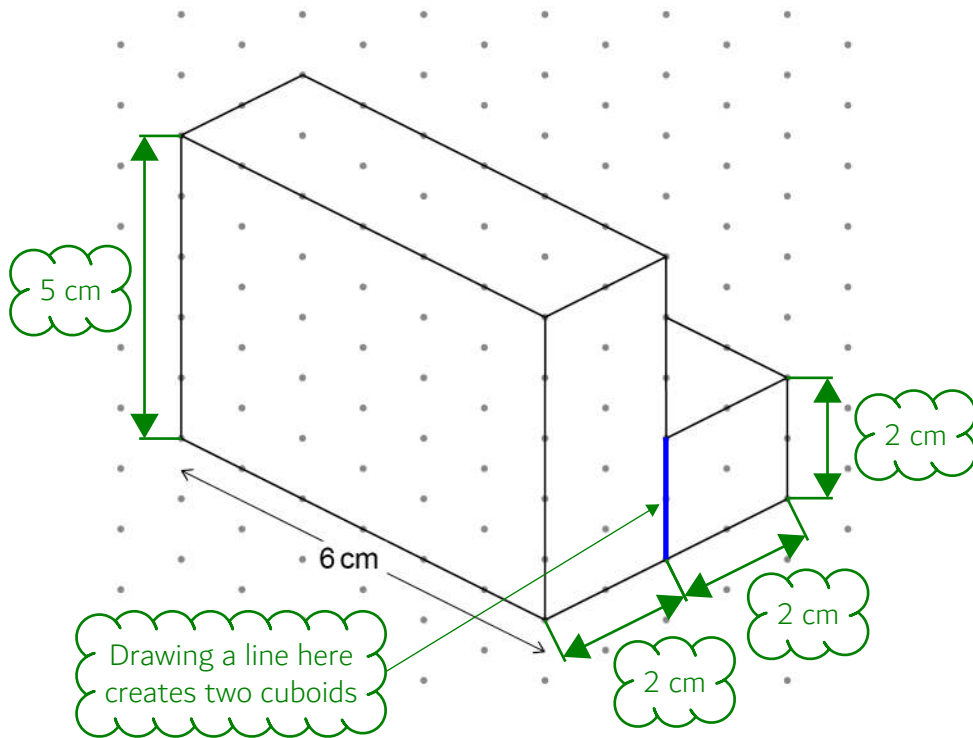
Dividing the 180 by the 900 expresses the fraction of the votes which Sean gets.  
Doing this fraction of the  $360^\circ$  in a pie chart works out that the angle should be  $72^\circ$



Assuming that each successful student received the minimum number of votes, each one would get  $\frac{4}{15}$  of the votes. Dividing 1 lot of votes by  $\frac{4}{15}$  of the votes works out that there can be 3.75 lots of the  $\frac{4}{15}$  and therefore 3.75 students reaching round 2. However this needs to be a whole number and is rounded down to 3 as 4 would be too many

7

Here is a prism drawn on an isometric grid.



Work out the volume of the prism.

**[3 marks]**

$$6 \times 2 \times 5 + 6 \times 2 \times 2$$

Adding the volume of the two cuboids works out the volume of the prism

This expresses the volume of the cuboid at the back. Volume of cuboid = length x width x height. The length is 6 cm, the width is 2 cm and the height is 2 cm

This expresses the volume of the cuboid at the front. Volume of cuboid = length x width x height. The length is 6 cm, the width is 2 cm and the height is 5 cm

Answer 84 cm<sup>3</sup>

5

Turn over ►



8

Tasha drove 198 miles.

Her average speed for the first 3 hours was 45 miles per hour.

Her average speed for the rest of the journey was 31.5 miles per hour.

Work out her average speed for the whole journey.

**[4 marks]** $s^d_t$ 

Writing the formula triangle for distance, speed, time

 $45 \times 3$ Covering d in the formula triangle finds that distance = speed  $\times$  time. Multiplying the speed of 45 miles per hour by the time of 3 hours works out that 135 miles was travelled in the first 3 hours $198 - 135$ 

Subtracting the 135 miles from the 198 miles works out that the rest of the journey was 63 miles

 $63 \div 31.5$ Covering t in the formula triangle finds that time = distance  $\div$  speed. Dividing the distance of 63 miles by the speed of 31.5 miles per hour works out that the rest of the journey took 2 hours $3 + 2$ 

Adding the 3 hours and the 2 hours works out that the whole journey took 5 hours

 $198 \div 5$ Covering s in the formula triangle finds that speed = distance  $\div$  time. Dividing the distance of 198 miles by the time of 5 hours works out that the average speed for the whole journey was 39.6 miles per hourAnswer 39.6 miles per hour

9 Here is the term-to-term rule for a sequence.

Double the previous term and add 3

The first three terms of the sequence are  $a + 1$   $2a + 5$   $4a + 13$

Show that the sum of the first **four** terms is a multiple of 3

[3 marks]

$8a + 26$  ← Doubling the third term

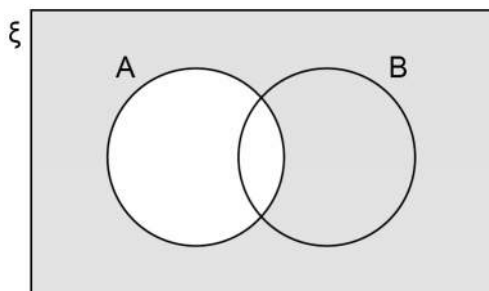
$8a + 29$  ← Then adding 3 works out that the fourth term is  $8a + 29$

$a + 1 + 2a + 5 + 4a + 13 + 8a + 29$  ← Expressing the sum of the first four terms by adding them together

$15a + 48$  ← Simplifying by collecting like terms

$3(5a + 16)$  ← Bringing out 3 as a factor shows that it is something multiplied by 3 so therefore must be a multiple of 3

10



Which of these represents the shaded region?

Circle your answer.

[1 mark]

B

$A' \cup B$

$A \cap B$

**A'**

Everything is shaded apart from A. The dash after the A means that it is not A

8

Turn over ►



- 11 A fair coin is thrown a number of times.  
The probability that **every** throw results in Heads is  $\frac{1}{64}$   
How many times is the coin thrown?

[1 mark]

The probability of getting one heads on a fair coin is  $\frac{1}{2}$ . Heads AND heads AND heads... 'AND' means to multiply the probabilities. On the calculator, starting with  $\frac{1}{2}$  and keep multiplying by  $\frac{1}{2}$  until  $\frac{1}{64}$  is reached.  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ ,  $\frac{1}{32}$ ,  $\frac{1}{64}$ .  $\frac{1}{64}$  is sixth in the list so is  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ , meaning there must be 6 heads for the probability of all heads being  $\frac{1}{64}$ . So the coin was thrown 6 times

Answer 6

- 12 Here is some information about the members of a basketball club.

	Number of members	Mean height of members
Junior	30	1.6 m
Senior	20	2.05 m

Work out the mean height of all 50 members of the club.

Give your answer as a decimal.

[3 marks]

$$\begin{matrix} t \\ m \\ n \end{matrix}$$

Mean = total/number, where total is the total height of the members and number is the number of members. Writing this as a formula triangle

$$30+20=50$$

Adding the 30 Junior members and the 20 Senior members works out that there are 50 members in total

$$1.6 \times 30 + 2.05 \times 20$$

Total = mean x number, so multiplying the mean of 1.6 by the 30 Juniors expresses the total height of the Juniors and multiplying the mean of 2.05 by the 20 Seniors expresses the total height of the Seniors. Adding these works out that the total height of all the members is 89 m

$$89 \div 50$$
Answer 1.78 m

Mean = total/number. The total height of all the members is 89 m and the number of members is 50



13 A straight line passes through (3, 14) and (12, 32)

Work out the equation of the line.

Give your answer in the form  $y = mx + c$

[3 marks]

The general equation of a straight line is  $y = mx + c$ , where  $m$  is the gradient and  $c$  is the  $y$ -intercept

$$\frac{32-14}{12-3}$$

Gradient = (change in  $y$ )/(change in  $x$ ). Change in  $y$  is expressed by subtracting the  $y$ -coordinate of the first point from the  $y$ -coordinate of the second point. Change in  $x$  is expressed by subtracting the  $x$ -coordinate of the first point from the  $x$ -coordinate of the second point. So the gradient is 2

$$14 = 2 \times 3 + c$$

Substituting the  $x$  and  $y$ -coordinates from the first point and 2 for  $m$  in the equation  $y = mx + c$

$$8 = c$$

Rearranging to find  $c$  by subtracting  $2 \times 3$  from both sides

Answer \_\_\_\_\_

$$y = 2x + 8$$

$m$  is 2 and  $c$  is 8

Turn over for the next question

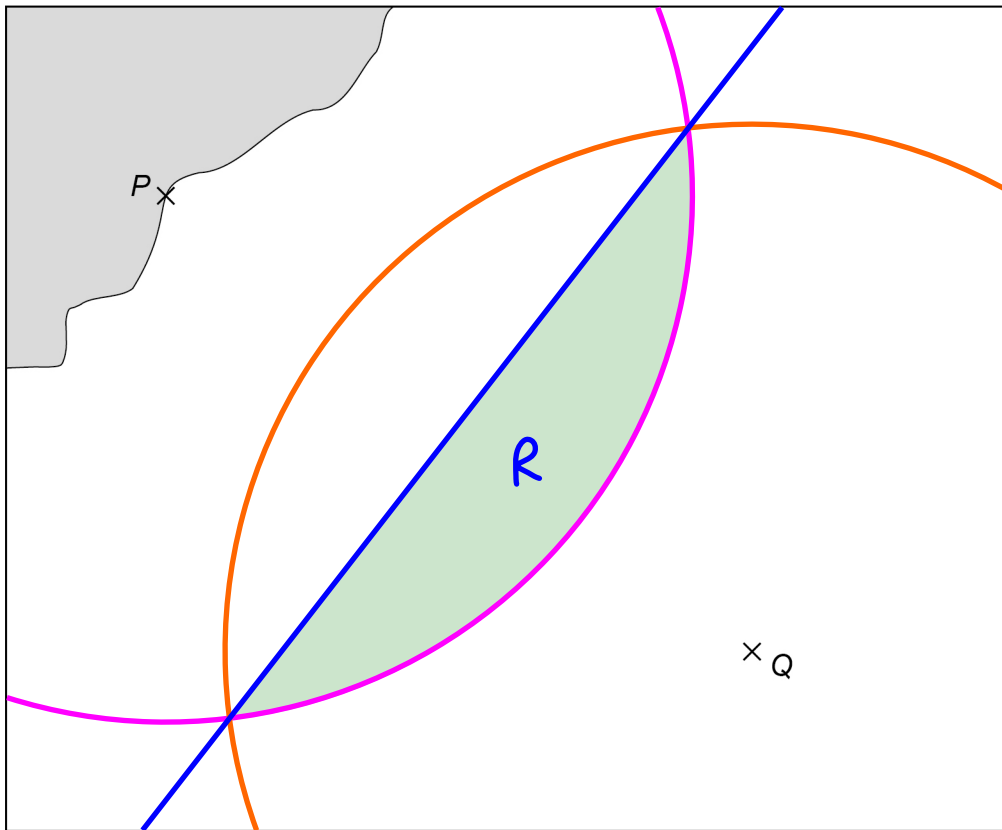
Turn over ►



14 Use a ruler and compasses in this question.

The scale diagram shows port  $P$  and lighthouse  $Q$ .

Scale: 1 cm represents 2 km



A ship is  
less than 14 km from  $P$   
and  
closer to  $Q$  than to  $P$ .

Label the region,  $R$ , where the ship could be.

Show all your construction lines.

[4 marks]

$$14 \div 2 = 7$$

Every 2 km is represented by 1 cm. So dividing the 14 km by 2 works out that it is represented by 7 cm

Scribing an arc around  $P$  using a compass with radius of 7 cm (shown in pink) gives all points which are exactly 14 km away from  $P$ . Scribing an arc around  $Q$  using a compass with radius of 7 cm (shown in orange) to construct a perpendicular bisector of  $PQ$  (shown in blue) by drawing a straight line through where both of the arcs meet. The region is highlighted in green but does not need to be highlighted in the exam



- 15 A bag contains discs.

<b>Trial</b>
A disc is chosen at random from the bag. The colour of the disc is noted. The disc is put back into the bag.

The trial is carried out 100 times.

The table shows the relative frequency of a blue disc after every 25 trials.

<b>Total number of trials</b>	25	50	75	100
<b>Relative frequency of a blue disc</b>	0.4	0.36	0.4	0.32

- 15 (a) For the trials from the 26th to the 50th, how many times was a blue disc chosen?

[2 marks]

$$0.4 \times 25 = 10$$

Multiplying the relative frequency for the 25 trials by 25 works out that the frequency of blue was 10 out of the 25 trials

$$0.36 \times 50 = 18$$

Multiplying the relative frequency for the 50 trials by 50 works out that the frequency of blue was 18 out of the 50 trials

$$18 - 10 = 8$$

Subtracting the 10 in the first 25 trials from the 18 in the first 50 trials leaves 8 blue discs from the 26th to the 50th trial

Answer 8

- 15 (b) There is a total of 1000 discs in the bag.

Work out the **best** estimate of the number of blue discs in the bag.

[1 mark]

$$0.32 \times 1000 = 320$$

The relative frequencies can be used as estimates for the probabilities of getting a blue disc. 0.32 is the best estimate for the probability as it was based in the most trials. Multiplying this probability by the 1000 discs works out an estimate of the number of blue discs in the bag

Answer 320



16  $a > 0$  and  $b < 0$

Circle the correct statement.

[1 mark]

$a - b < 0$

$-\frac{b}{a} < 0$

$\frac{1}{a} < 0$

$b^3 < 0$

Let  $a = 1$  and  $b = -1$ .  $1 - -1 = 2$ , which is not less than 0 so the first statement is not correct.  $-1/1 = 1$ , which is not less than 0 so the second statement is not correct.  $1/1 = 1$ , which is not less than 0 so the third statement is not correct.  $(-1)^3 = -1$ , which is less than 0 so the third statement works. The third statement is correct as cubing a negative number gives a negative result, which is less than 0

- 17 190 people were asked how much they spent on takeaways one month.  
The table shows information about the results.

Amount, $x$ (£)	Cumulative frequency
$0 < x \leq 10$	16
$0 < x \leq 20$	64
$0 < x \leq 30$	140
$0 < x \leq 40$	184
$0 < x \leq 50$	190

- 17 (a) How many people spent
- more**
- than £20 ?

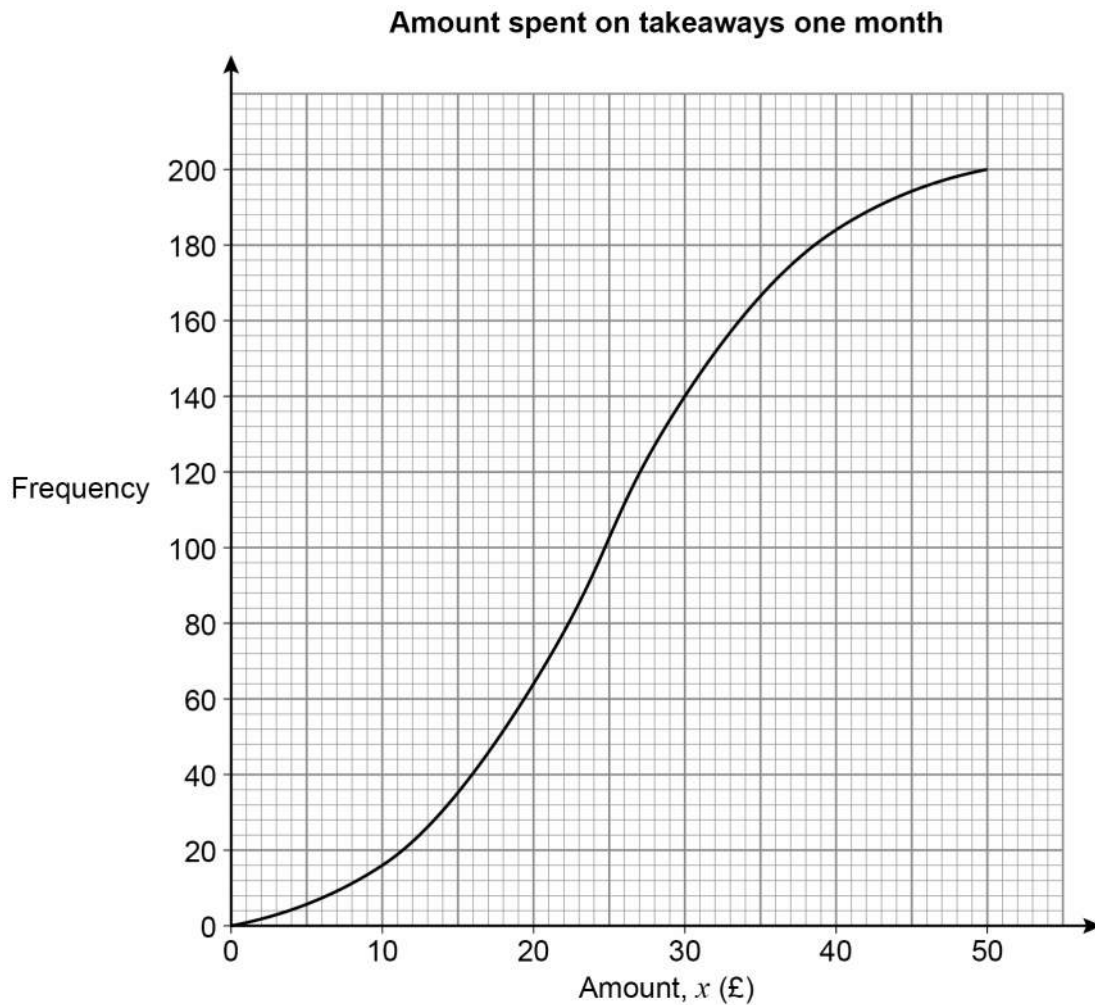
[2 marks]

$190 - 64$

Subtracting the 64 people who spent more than £0 and less than or equal to £20 from the total 190 people leaves 126 people who spent more than £20

Answer 126

- 17 (b) Farah draws this cumulative frequency curve to represent the results.



Give **two** criticisms of her graph.

[2 marks]

Criticism 1

The vertical axis should be labelled cumulative frequency

It is currently labelled as frequency

Criticism 2

The curve goes up to 200

The curve should go up to 190 as this is the highest cumulative frequency



18 By completing the square, prove that  $x^2 + 6x + 13$  is always positive.

[3 marks]

$$(x+3)^2 + 13 - 9$$

Completing the square by halving the coefficient of  $x$  (which is 6), putting this in a bracket with  $x$  and squaring the bracket. Subtracting the  $3^2$  from the end

$$(x+3)^2 + 4$$

Simplifying

The minimum value is 4 so it is always positive

The minimum value a squared bracket can have is 0. Adding 4 to this gives the minimum value of 4



19

$A$  is directly proportional to  $B^4$

The value of  $B$  is doubled.

Pete thinks that the value of  $A$  will be 8 times bigger because  $4 \times 2$  is 8

Is he correct?

Tick a box.

Yes

No

Give a reason for your answer.

**[1 mark]**

The value of  $A$  will be 16 times bigger

$A \propto B^4$ . So  $A = kB^4$ , where  $k$  is a constant. Doubling  $B$  gives  
 $A = k(2B)^4 = 16kB^4$ . This  $16kB^4$  is 16 times bigger than  $kB^4$

Turn over for the next question

Turn over ►



20 Rearrange  $p = \frac{2m+1}{1-m}$  to make  $m$  the subject.

[4 marks]

$$p(1-m)$$

Multiplying both sides by the denominator cancels out the denominator on the right.  $p(1-m)$  is now on the left

$$p-pm=2m+1$$

Expanding the bracket on the left and writing the right side with the denominator cancelled out

$$p-1=2m+pm$$

Adding  $pm$  to both sides and subtracting 1 from both sides to get all the terms involving  $m$  on one side and all the terms not involving  $m$  on the other side

$$=m(2+p)$$

Bringing out  $m$  as a factor on the right side

Answer \_\_\_\_\_

$$m = \frac{p-1}{2+p}$$

Dividing both sides by  $(2+p)$  makes  $m$  the subject



21

Jack is loading a van.

The van can safely carry 1375 kg of furniture.

Jack has already loaded 1200 kg of furniture to the nearest 50 kg

A table has mass 140 kg to the nearest 10 kg

Can the table safely be added to the furniture in the van?

You **must** show your working.**[3 marks]**

Adding half of the resolution of the measurements works out the upper bound of the masses

$$1200 + \frac{50}{2} = 1225$$

The 1200 kg is to the nearest 50 kg so adding half of the 50 to the 1200 works out that the upper bound is 1225 kg

$$140 + \frac{10}{2} = 145$$

The 140 kg is to the nearest 10 kg so adding half of the 10 to the 140 works out that the upper bound is 145 kg

$$1225 + 145 = 1370$$

Adding the two upper bounds of the masses works out that the upper bound of the total mass is 1370 kg

Yes

The total mass must be less than the upper bound so will be less than 1375 kg

22

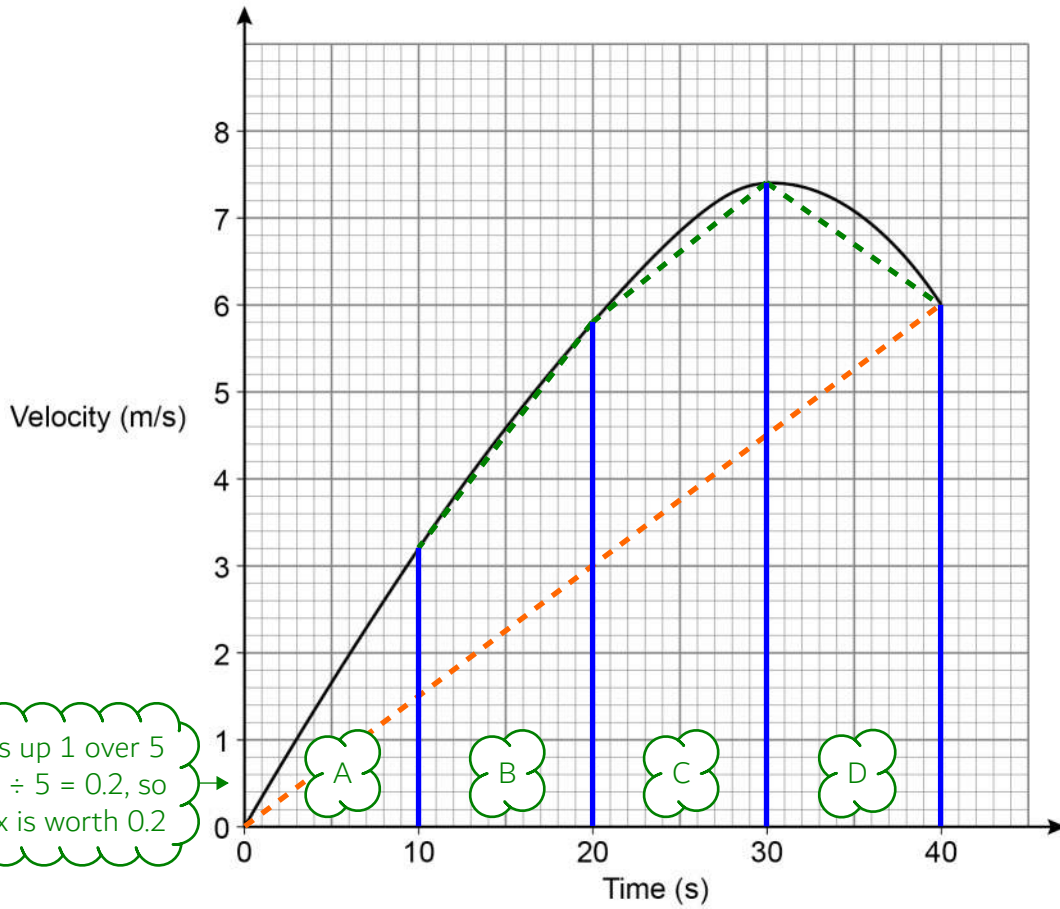
Factorise  $25a^2 - b^2$ **[1 mark]**

Answer  $(5a+b)(5a-b)$

Factorised using difference of two squares.  $x^2 - y^2 = (x + y)(x - y)$ .  $x^2 = 25a^2$ , so  $x = 5a$ .  $y^2 = b^2$ , so  $y = b$



23 Here is the velocity-time graph of a cyclist for 40 seconds.



The scale goes up 1 over 5 small boxes.  $1 \div 5 = 0.2$ , so each small box is worth 0.2

23 (a) By dividing the area under the graph into four sections of equal widths, estimate the distance travelled during the 40 seconds.

[3 marks]

$$\frac{1}{2} \times 10 \times 3.2 + \frac{1}{2} (3.2 + 5.8) \times 10 + \frac{1}{2} (5.8 + 7.4) \times 10 + \frac{1}{2} (7.4 + 6) \times 10$$

Adding the area of all the shapes gives an estimate of the total area under the curve, which is an estimate of the distance travelled

Area of triangle A. Area of triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$

Area of trapezium B, C and D. Area of trapezium =  $\frac{1}{2} (a + b) \times h$ , where a and b are the parallel sides and h is the distance between them

Answer 194 m



- 23 (b) Work out the average acceleration of the cyclist during the 40 seconds.  
State the units of your answer.

[2 marks]

$$\frac{6-0}{40-0}$$

Acceleration is the gradient on a velocity-time graph. Working out that the gradient of the orange dashed line is 0.15. Gradient = (change in y)/(change in x). The change in y from (0, 0) to (6, 40) is 6 - 0 and the change in x is 40 - 0

Answer 0.15m/s<sup>2</sup>

The velocity (m/s) was divided by time (s) and  $m/s \div s = m/s^2$

- 24 Simplify fully  $\frac{8x^2 + 4}{5x} \times \frac{3x}{14x^2 + 7}$

You **must** show your working.

[3 marks]

$$\frac{4(2x^2+1) \times 3x}{5x \times 7(2x^2+1)}$$

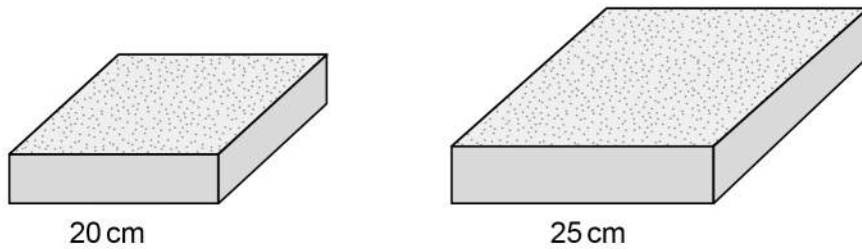
Factorising the numerators and denominators and multiplying the fractions. To multiply fractions: multiply the numerators and multiply the denominators

Answer  $\frac{12}{35}$

To simplify a fraction: cancel out common factors from the numerator and denominator. The  $(2x^2 + 1)$  is a common factor so is cancelled out. The x is a common factor so is cancelled out. This leaves  $(4 \times 3)/(5 \times 7)$



- 25 Here are two square-based paving stones.  
The stones are similar solids.



The price per  $\text{cm}^3$  is the same for both stones.  
The price of the **larger** stone is £17.50

Work out the price of the smaller stone.

[4 marks]

$$25 \div 20$$

Dividing the length of the larger stones by the length of the smaller stone works out that the length scale factor is 1.25

$$17.50 \div 1.25^3$$

The unit of length could be cm and the unit of volume could be  $\text{cm}^3$ . So cubing the length scale factor gives the volume scale factor. Dividing the price of the larger stone by this gives the price of the smaller stone. If it doesn't make sense why this works, consider that if the volume of the stone was halved, the price would be halved as the price per  $\text{cm}^3$  is the same for both stones

Answer £ 8.96



- 26 Rick borrows £1500 from a bank.  
He pays back £100 each month.

This iterative formula works out the amount he still owes at the end of each month.

$$A_{n+1} = 1.02 \times A_n - 100$$

$$A_0 = 1500$$

Work out the amount he still owes at the end of the 2nd month.

[3 marks]

$$A_1 = 1.02 \times 1500 - 100$$

$$A_2 = 1.02 \times 1430 - 100$$

On the calculator, entering 1500 then pressing =/exe. Entering 1.02 x Ans - 100. Pressing =/exe gives  $A_1$ , which is the amount he still owes at the end of the 1st month. Pressing =/exe again gives  $A_2$ , which is the amount he still owes at the end of the 2nd month

Answer £ 1358.60

Turn over for the next question



27  $g(x) = a \times b^x$  where  $a$  and  $b$  are constants.

$$g(0) = 8 \quad \text{and} \quad g(3) = 343$$

Work out the value of  $g(1)$

[4 marks]

$$A \times B^0 = A = 8$$

Substituting 0 for  $x$  in  $g(x)$  expresses  $g(0)$ . Anything to the power of 0 is 1 so  $B^0 = 1$ .  $A \times 1 = A$ . This must equal to 8 as  $g(0) = 8$

$$8 \times B^3 = 343$$

Substituting 3 for  $x$  in  $g(x)$  expresses  $g(3)$ . Substituting 8 for  $A$ . This must be equal to 343 as  $g(3) = 343$

$$B^3 = 42.875$$

Dividing both sides by 8 to get the  $B^3$  on its own

$$B = 3.5$$

Cube rooting both sides finds that  $B$  is 3.5

$$8 \times 3.5$$

Substituting 1 for  $x$  in  $g(x)$  expresses  $g(1)$ . Substituting 8 for  $A$  and 3.5 for  $B$

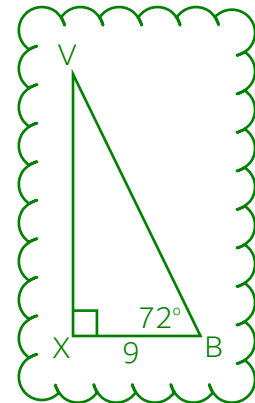
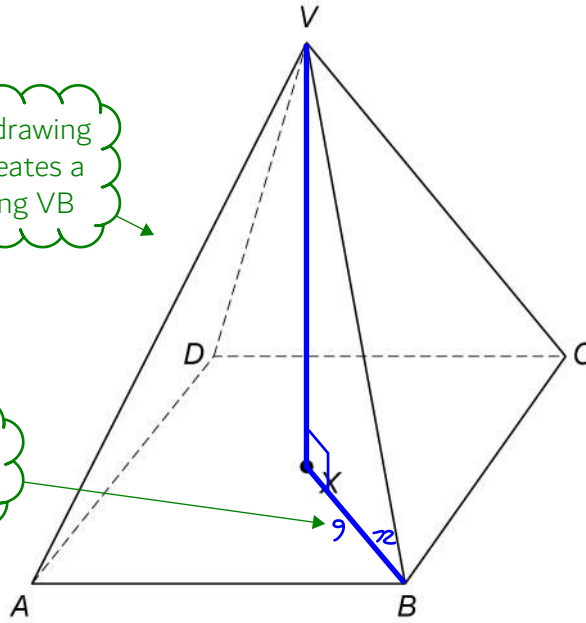
Answer 28



- 28  $VABCD$  is a pyramid with a horizontal square base.  
 $X$  is the centre of the base.  
 $V$  is vertically above  $X$ .  
 $BD = 18$  cm  
 Angle  $VBX = 72^\circ$

Drawing on angle  $VBX$  then drawing a vertical line from  $X$  to  $V$  creates a right-angled triangle involving  $VB$

$XB$  must be 9 as this is half of  $BD$ .  $X$  is the midpoint



Work out the length of  $VB$ .

[3 marks]

SOH CAH TOA

Writing SOH CAH TOA as formula triangles. Ticking A as we have the adjacent and H as we are looking for the hypotenuse

$$\frac{9}{\cos 72}$$

There are two ticks on the CAH formula triangle so this one can be used. From the formula triangle, hypotenuse = adjacent / (cos of the angle)

Answer 29.1 cm



- 29 A code is three letters, each of which is in the word LOGIC  
 Vinny assumes that letters in the code may be used more than once.  
 He works out how many possible codes there are.  
 In fact, the first two letters are the same and the third is different.  
 How many of Vinny's codes are **not** possible? [2 marks]

$$5 \times 5 \times 5 = 125$$

There are 5 different letters in LOGIC. So there are 5 possible letters for each letter of the code as they can be used more than once. Using the product rule for counting works out that Vinny works out 125 codes

$$5 \times 1 \times 4 = 20$$

There are 5 possible letters for the first letter of the code. There is only 1 possibility for the second letter of the code given that the first letter is already chosen and that the first two letters are the same. There are 4 possibilities for the third letter given that 1 of the 5 letters has already been used. Using the product rule for counting works out that there are actually 20 possible codes

$$125 - 20$$

Answer 105

Subtracting the 20 possible codes from Vinny's 125 codes works out that 105 of Vinny's codes are not possible

END OF QUESTIONS

