AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number | | | | | |

Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE MATHEMATICS

H

Higher Tier Paper 2 Calculator

Thursday 8 November 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Do not write outside the box

Answer **all** questions in the spaces provided

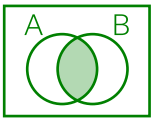
1 What does $(A \cap B)$ represent in $P(A \cap B)$?

Circle your answer.

[1 mark]

A or B or both | A but not B

not A and not B | A and B



The intersection of A and B

2 P is (4, 9) and Q is (–2, 1)

Circle the midpoint of PQ.

[1 mark]

(1, 5) (3, 4) (3, 5) (6, 8)

Working out the mean of the x-coordinates works the x-coordinate of the midpoint.
4 - 2 = 2, then 2 ÷ 2 = 1. There is only one option with an x-coordinate of 1

3 Which of these is a geometric progression?

Multiplies by the same amount between each term

Circle your answer.

[1 mark]

1 3 5 7 9 | 1 3 6 10 15

1 4 9 16 25 | 1 3 9 27 81

Each term is multiplied by 3 to get the next term

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4 The bearing of *A* from *B* is 310°

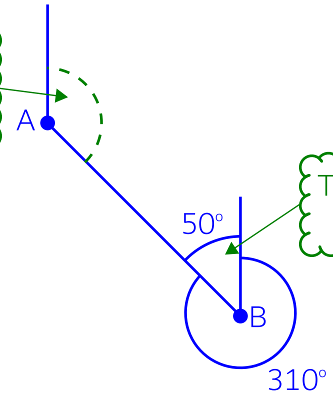
Circle the bearing of *B* from *A*.

A bearing is the angle turned clockwise from north

[1 mark]

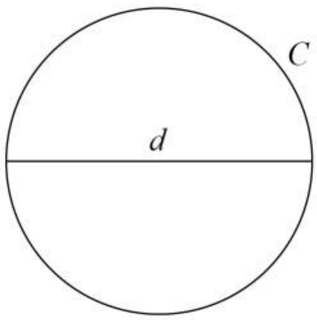
050° 110° 130° 220°

This is the angle we are trying to find. It is co-interior to the 50° so is 130° as co-interior angles add up to 180



This angle is 50° as there are 360 degrees around a point and 360 - 310 = 50

5 A circle has circumference C and diameter d.



$C = kd$

What **value** does the constant k represent?

[1 mark]

Answer π

Circumference = π x diameter

5

Turn over ▶

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6 Here is some information about 20 trains leaving a station.

Number of minutes late, t	**Number of trains**	**Midpoint**	
$0 \leqslant t < 5$	12	2.5	30
$5 \leqslant t < 10$	7	7.5 ← A	52.5 ← B
$10 \leqslant t < 15$	1	12.5	12.5
$t \geqslant 15$	0		

95÷20 ← C

6 (a) Work out an estimate of the mean number of minutes late.

[3 marks]

Answer 4.75 minutes

A: Working out the midpoints of each interval by doing the mean of the upper and lower bounds of each interval. For $0 \leqslant t < 5$, 0 + 5 = 5 then 5 ÷ 2 gives the midpoint of 2.5. For $5 \leqslant t < 10$, 5 + 10 = 15 then 15 ÷ 2 gives the midpoint of 7.5. For $10 \leqslant t < 15$, 10 + 15 = 25 then 25 ÷ 2 gives the midpoint of 12.5. There is no midpoint for $t \geqslant 15$ but this does not matter as there were no trains in this interval. The midpoints are the best estimate we can use for the minutes late for each train in each interval.

B: Estimating the total number of minutes late for each interval by multiplying the number of trains by the midpoints. 12 x 2.5 = 30. 7 x 7.5 = 52.5. 1 x 12.5 = 12.5.

C: Adding these totals estimates that the total minutes late of all the trains combined is 95 minutes. Dividing this estimated total by the 20 trains estimates the mean number of minutes late

Do not write outside the box

6 (b) The station manager looks at the information in more detail.

Number of minutes late, t	**Number of trains**
$0 \leqslant t < 2$	12
$2 \leqslant t < 4$	0
$4 \leqslant t < 6$	7
$6 \leqslant t < 8$	0
$8 \leqslant t < 10$	0
$10 \leqslant t < 12$	1

He works out an estimate of the mean using this information.

How does his estimate compare with the answer to part (a)?

Tick **one** box.

[1 mark]

☐ Higher than part (a)

☐ Same as part (a)

☑ Lower than part (a)

☐ Not possible to tell

The upper and lower bound of the intervals the 12, 7 and 1 trains are in are lower so the midpoints will be lower. This will make the estimated totals lower and so the mean will be lower

Turn over for the next question

4

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7 Work out the values of a and b in the identity

$$5(7x + 8) + 3(2x + b) \equiv ax + 13$$

[4 marks]

$35x + 40 + 6x + 3b$ ← Expanding the brackets on the left

$41x + 40 + 3b = ax + 13$ ← Simplifying by collecting like terms then putting it equivalent to the right side

There is 41x on the left so there must be 41x on the right. So a must be 41

$40 + 3b = 13$ ← 40 + 3b are the constants on the right so these must be equal to the constant 13 on the right

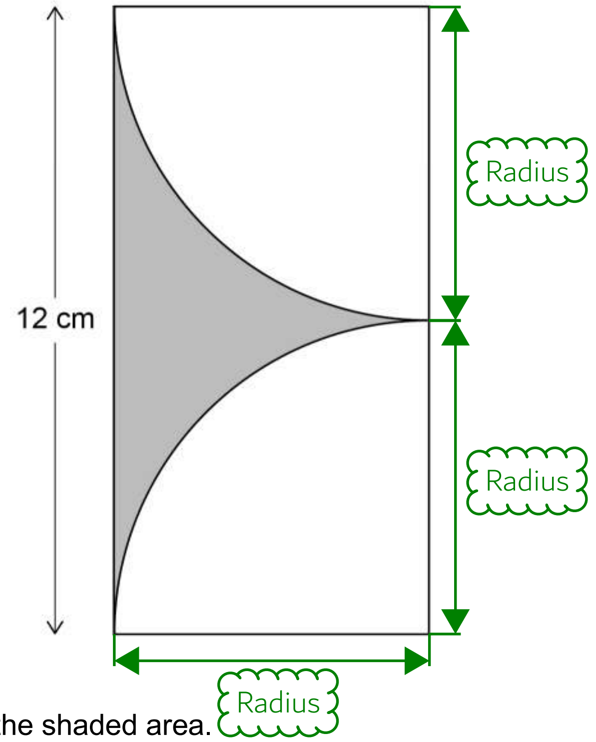
$3b = -27$ ← Subtracting 40 from both sides to get the b term on its own

Dividing both sides by 3 gets b on its own and finds that b = -9

$a =$ 41 $b =$ -9

Do not write outside the box

8 Two identical quarter circles are cut from a rectangle as shown.



Not drawn accurately

Work out the shaded area.

[4 marks]

$12 \div 2$ — 2 lots of the radius is the same as the 12 cm length as opposite sides on a rectangle are equal. So dividing the 12 cm by 2 works out that the radius is 6 cm

$12 \times 6 = 72$ — Area of rectangle = length x width. The length is 12 cm and the width is 6 cm as it is a radius. So the area of the whole rectangle is 72 cm^2

$\pi \times 6^2$ — Area of circle = π x radius^2. So the area of the whole of one of the circles is 36π cm^2

$36\pi \div 4$ — Dividing the area of one of the whole circles by 4 works out that the area of one of the quarter circles is 9π cm^2

$9\pi \times 2$ — There are 2 identical quarter circles so multiplying the area of one of them by 2 works out that the area of both of them combined is 18π cm^2

$72 - 18\pi$ — Subtracting the area of both of the quarter circles combined from the area of the rectangle gives the shaded area

Answer 15.5 cm^2

8

Do not write outside the box

9 The diagrams show the position of a tap when off and fully on.

The tap is fully on when the angle of turn is 180°

When fully on, water flows out of the tap at 14 litres per minute.

The rate at which water flows out is in direct proportion to the angle of turn.

The tap is turned 135°

The water flows into a tank with a capacity of 79.8 litres.

Will it take **less than** $7\frac{1}{2}$ minutes to fill the tank?

You **must** show your working.

[4 marks]

$\frac{135}{180} \times 14$

The tap is turned 135 degrees out of the 180 degrees, which is 135/180. Doing this fraction of the 14 litres per minute works out that the water flows at 10.5 litres per minute

$79.8 \div 10.5 = 7.6$

Dividing the 79.8 litres by the 10.5 litres per minute works out that it will take 7.6 minutes to fill the tank

No

7.6 minutes is not less than $7^1/_2$ minutes, which is 7.5 minutes as a decimal

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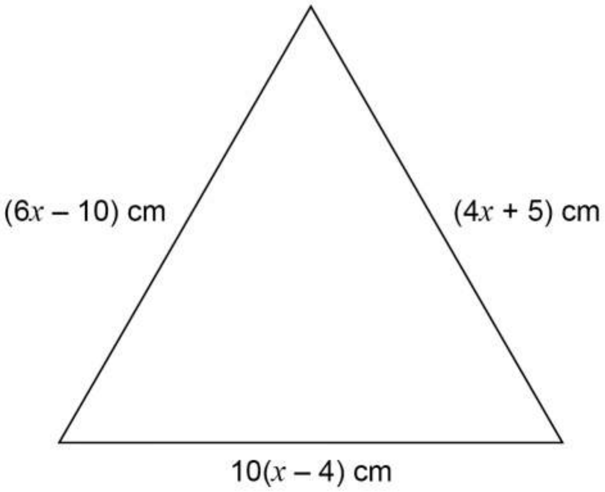
10 This triangle is equilateral.



Not drawn accurately

Is the perimeter of the triangle greater than one metre?

You **must** show your working.

[5 marks]

$6x-10=4x+5$ ← It is an equilateral triangle so all the sides are equal. Setting the (6x - 10) cm equal to the (4x + 5) cm

$2x-10=5$ ← Subtracting 4x from both sides to get all the x on the same side

$2x=15$ ← Adding 10 to both sides to get the x term on its own

$x=7.5$ ← Dividing both sides by 2 to find x

$6\times7.5-10$ ← Substituting 7.5 for x in (6x - 10) cm works out that each side is 35 cm long

$35\times3=105$ ← Multiplying the length of one side by the 3 sides of the triangle works out that the perimeter is 105 cm

Yes ← One metre is 100 cm. 105 cm is greater than this

9

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11 An approximation for the value of π is given by

$$4\left(1 - \frac{22}{57} + \frac{22}{85} - \frac{22}{105} + \frac{22}{117} - \frac{22}{242}\right)$$

Use your calculator to show that this approximation is within 0.1 of 3.14

[2 marks]

$3.14 - 3.041... = 0.09...$

Typing the approximation into the calculator gives 3.041839619. Subtracting this from 3.14 works out that the difference is 0.09816038107. As this is less than 0.1, it is within 0.1 of 3.14

12 Work out $\dfrac{9.12 \times 10^{10}}{3.2 \times 10^{4}}$

Give your answer in standard form.

[2 marks]

2850000

Typing it into the calculator exactly as it is above gives this

Answer 2.85×10^{6}

Formatting the answer into ENG notation puts it into standard form in this case

Do not write outside the box

13 Ashraf is going to put boxes into a crate.

The crate is a cuboid measuring 2.5 m by 2 m by 1.2 m

Each box is a cube of length 50 cm

He does these calculations.

volume of crate	=	$2.5 \times 2 \times 1.2$
	=	$6\ m^3$
volume of one box	=	$0.5 \times 0.5 \times 0.5$
	=	$0.125\ m^3$
number of boxes	=	$6 \div 0.125$
	=	48

He claims,

"I can put 48 boxes in the crate."

Evaluate Ashraf's method **and** claim.

[2 marks]

They are wrong as 50 cm doesn't fit into 1.2 m a whole number of times.

So the volume will not be completely filled. There will be a gap on the 1.2m length. He will not be able to put as many as 48 boxes in the crate

14 The cross section of a prism has n sides.

Circle the expression for the number of edges of the prism.

[1 mark]

$2n$ $3n$ $n + 2$ $2n + 3$

Consider a cuboid (which is a type of prism): the cross section (the rectangle at one end) has 4 sides and the cuboid has 12 sides. 3n is the only one which works as 3 x 4 = 12

7

Turn over ▶

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15 The volume of a medal is 45 cm^3

The medal is made from copper and tin.

volume of copper : volume of tin = 22 : 3

The density of copper is 8.96 g/cm^3

The density of tin is 7.31 g/cm^3

Work out the mass of the medal.

[4 marks]

$d\,{}^{m}\,v$ ← Writing the formula triangle for density, mass, volume

$22+3$ ← Adding the 22 and 3 in the ratio works out that there are 25 parts in total in the ratio which represent the total volume of the medal

$45 \div 25$ ← Dividing the total volume of the medal by the 25 parts of the ratio which represent it works out that 1 part of the ratio is worth 1.8 cm^3

$1.8 \times 22 = 39.6$ ← Multiplying the value of 1 part of the ratio by the 22 parts which represent the volume of the copper works out that the volume of the copper is 39.6 cm^3

$1.8 \times 3 = 5.4$ ← Multiplying the value of 1 part of the ratio by the 3 parts which represent the volume of the tin works out that the volume of the tin is 5.4 cm^3

$8.96 \times 39.6 = 354.816$ ← Covering m in the formula triangle finds that mass = density x volume. Multiplying the density of the copper by the volume of the copper works out that the mass of the copper is 354.816 g

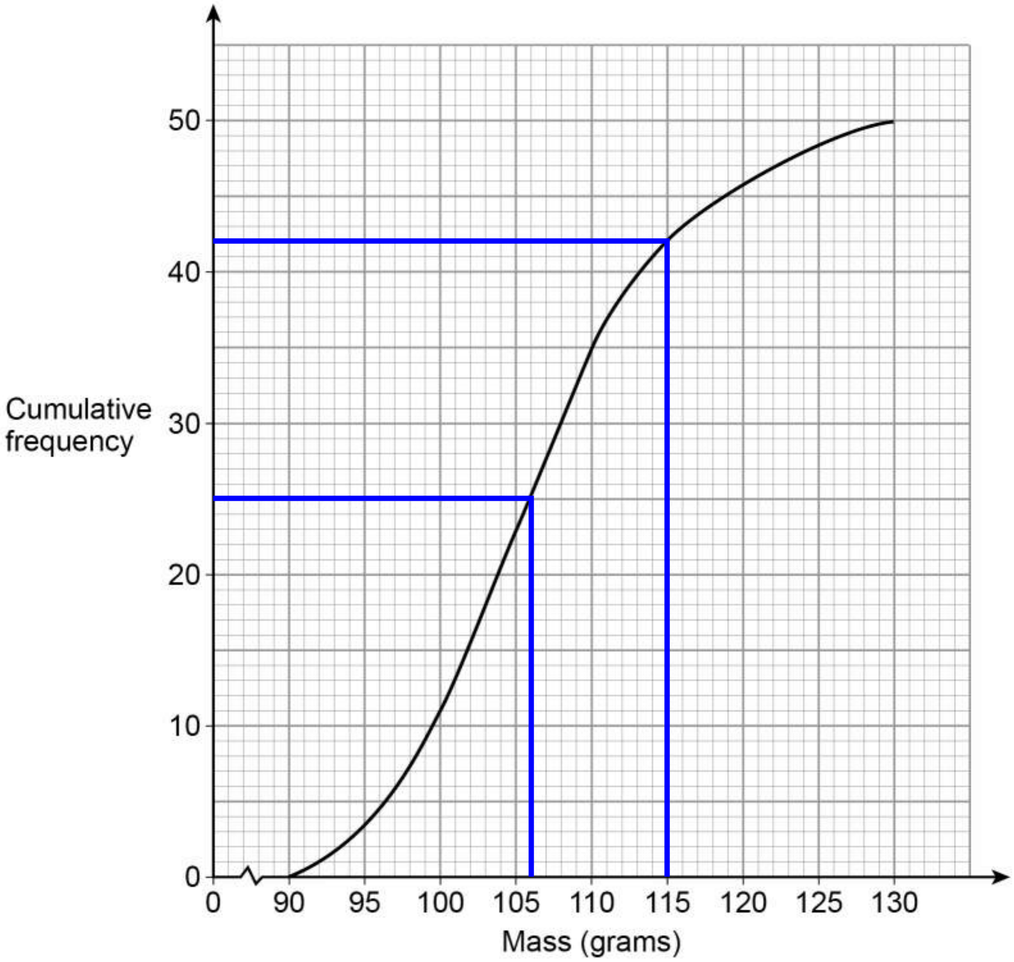
$7.31 \times 5.4 = 39.474$ ← Covering m in the formula triangle finds that mass = density x volume. Multiplying the density of the tin by the volume of the tin works out that the mass of the tin is 39.474 g

$354.816 + 39.474$ ← Adding the mass of the copper and the mass of the tin works out that the mass of the medal is 394.29 g

Answer 394.29 grams

Do not write outside the box

16 The cumulative frequency graph shows information about the masses of 50 apples.



16 (a) Use the graph to estimate the median mass of the apples.

[1 mark]

The median is roughly halfway through the data. 50 ÷ 2 = 25. So drawing a line across from 25 to the line then down gives an estimate for the median

Answer 106 grams

16 (b) Estimate the proportion of the apples that have a mass greater than 115 grams.

[2 marks]

Drawing a line up from 115 to the line then across estimates that there are 42 apples which are 115 g or less. 50 - 42 = 8 so there would be 8 out of the 50 apples which are more than 115 g

Answer $\frac{8}{50}$

7

Turn over ►

Do not write outside the box

17 a is a prime number.

b is an even number.

$N = a^2 + ab$

Circle the correct statement about N.

[1 mark]

could be even or odd	always even
always prime	always odd

A prime number could be odd or even (as 2 is prime and even and all the other primes are odd). Odd x odd = odd so a^2 could be odd but even x even = even so a^2 could be even. Even x even = even and odd x even = even so ab will be even. Even + even = even and odd + even = odd so N could be even or odd

18 A bag contains 20 discs.

10 are red, 7 are blue and 3 are green.

18 (a) Marnie takes a disc at random before putting it back in the bag.

Nick then takes a disc at random before putting it back in the bag.

Olly then takes a disc at random.

Work out the probability that they all take a red disc.

[2 marks]

$\frac{10}{20} \times \frac{10}{20} \times \frac{10}{20}$

There are 10 red disks out of a total of 20 disks so the probability of getting red is 10/20. As the disk is put back in the bag, the probability is the same for Marnie, Nick and Olly. Red AND red AND red. AND means to multiply the probabilities

Answer $\frac{1}{8}$

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18 (b) All 20 discs are in the bag.

Reggie takes three discs at random, one after the other.

After he takes a disc he does **not** put it back in the bag.

Reggie's first disc is blue.

Work out the probability that all three discs are different colours.

[3 marks]

$\frac{10}{19} \times \frac{3}{18} + \frac{3}{19} \times \frac{10}{18}$

It is given the first is blue. The next two must be different and can't be blue. They could be red AND green OR green AND red. AND means to multiply the probabilities of each event. OR means to add the probabilities. As there is 1 fewer disk each time (he does not put it back in the bag), the denominator decreases to 19 then to 18

Answer $\frac{10}{57}$

6

Turn over ►

Do not write outside the box

19

Lunch
Choose one starter and one main course

There are four starters and ten main courses to choose from.

Two of the starters and three of the main courses are suitable for vegans.

What percentage of the possible lunches have **both** courses suitable for vegans?

[3 marks]

$4 \times 10 = 40$ Using the product rule for counting. Multiplying the 4 starters by the 10 main courses works out that there are 40 possible lunches

$2 \times 3 = 6$ Using the product rule for counting. Multiplying the 2 starters by the 3 main courses suitable for vegans works out that there are 6 possible lunches with both courses suitable for vegans

$\frac{6}{40} \times 100$ 6 out of the 40 possible lunches are suitable for vegans. Expressing this as a fraction then multiplying it by 100 to convert it into a percentage

Answer 15 %

20 n is a positive integer.

Prove algebraically that $2n^2\left(\frac{3}{n} + n\right) + 6n\left(n^2 - 1\right)$ is a cube number.

[3 marks]

$6n + 2n^3 + 6n^3 - 6n$ Expanding the brackets

$8n^3$ Collecting like terms

$(2n)^3$ n is a positive integer. Multiplying it by 2 will still be a positive integer. Cubing a positive integer gives a cube number

Do not write outside the box

21 y is inversely proportional to $\sqrt{x}$

$y = 4$ when $x = 9$

21 (a) Work out an equation connecting y and x.

[3 marks]

$y \propto \frac{1}{\sqrt{x}}$ ← Inversely proportional means proportional to 1 over

$y = \frac{k}{\sqrt{x}}$ ← $1/\sqrt{x}$ can be multiplied by anything and it will still be proportional to y. So multiplying it by k, which represents a value which can be found, converts it into an equation

$4\sqrt{9} = k$ ← Multiplying both sides by $\sqrt{x}$ and substituting 4 for y and 9 for x to find k. So k = 12

Answer $y = \frac{12}{\sqrt{x}}$

Substituting 12 for k in the original equation

21 (b) Work out the value of y when $x = 25$

[2 marks]

$\frac{12}{\sqrt{25}}$ ← Substituting 25 for x into the right side of the equation found in part (a)

Answer 2.4

Turn over for the next question

11

Do not write outside the box

22 Simplify fully $\dfrac{x^5 - 4x^3}{3x - 6}$

[3 marks]

$x^3(x^2-4)$

Factorising the numerator by bringing out x^3 as a factor

$\dfrac{x^3(x+2)(x-2)}{3(x-2)}$

Factorising the numerator further using difference of two squares. $A^2 - B^2 = (A + B)(A - B)$.
Factorising the denominator by bringing out 3 as a factor

Answer $\dfrac{x^3(x+2)}{3}$

Simplifying the fraction by cancelling out $(x - 2)$, which is a common factor to the numerator and denominator

23 *PQR* is a straight line.

PQ : *QR* = 3 : 1

$\overrightarrow{PQ} = \mathbf{a}$

Not drawn accurately

Circle the vector $\overrightarrow{RQ}$

[1 mark]

$\frac{1}{3}\mathbf{a}$ $\frac{1}{4}\mathbf{a}$ $-\frac{1}{3}\mathbf{a}$ $-\frac{1}{4}\mathbf{a}$

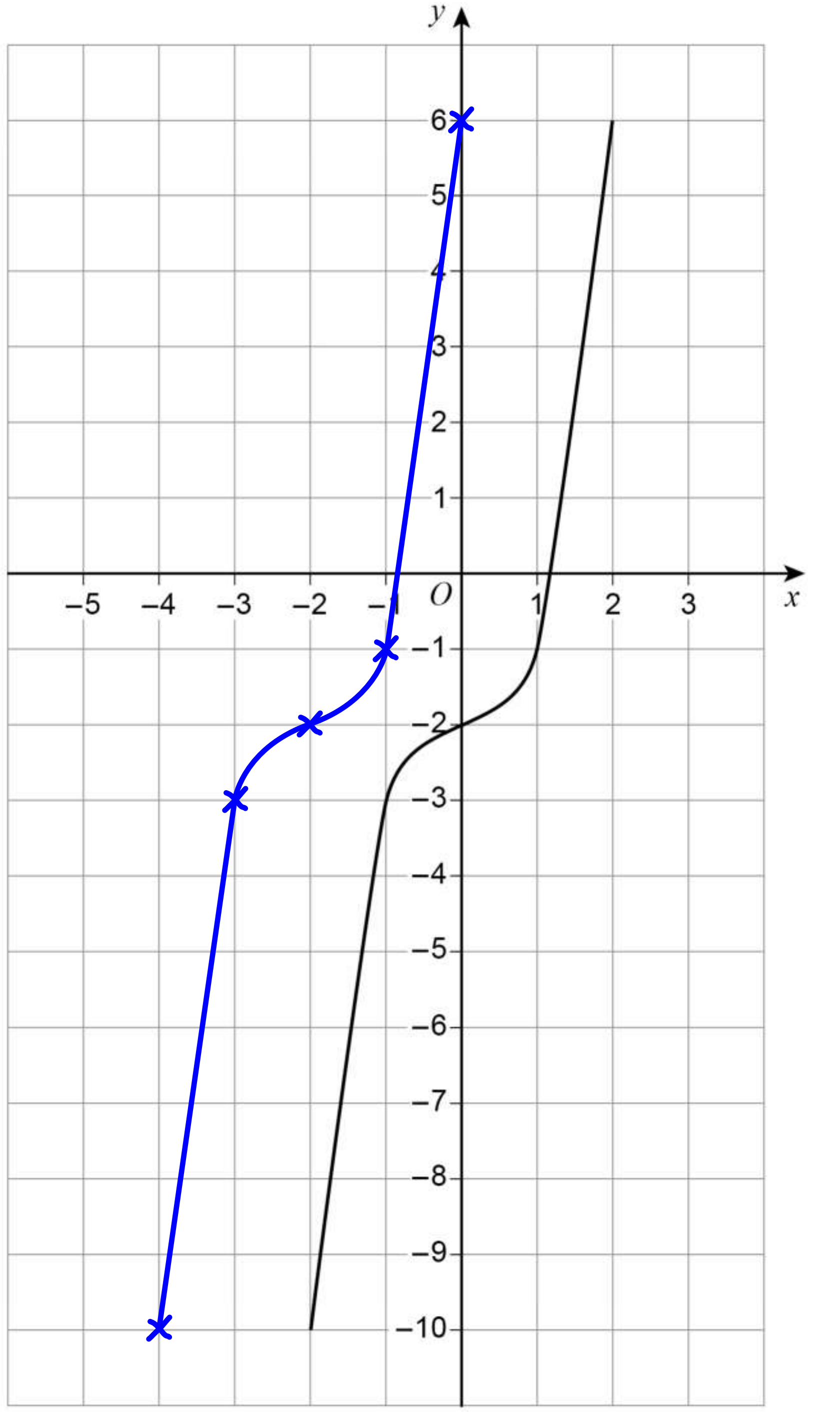
$\overrightarrow{PQ}$ is a and is represented by 3 parts of the ratio. $\overrightarrow{QR}$ is represented by 1 part so is 1/3 a. $\overrightarrow{RQ}$ is the same size but opposite direction to $\overrightarrow{QR}$ so is -1/3 a

Do not write outside the box

24 Here is a sketch of $y = f(x)$

The curve passes through the points

$(-2, -10)$ $(-1, -3)$ $(0, -2)$ $(1, -1)$ $(2, 6)$



On the grid, sketch the curve $y = f(x + 2)$

[2 marks]

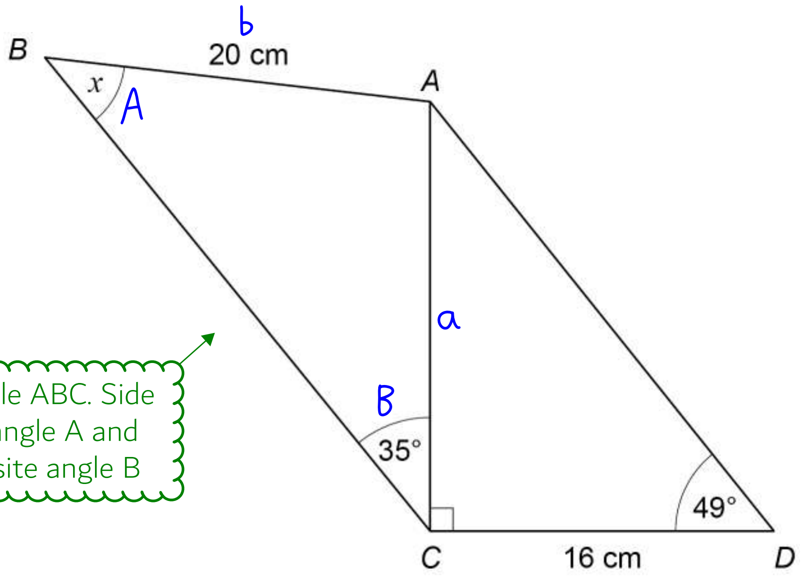
Adding 2 to x translates the graph 2 to the left

6

Turn over ▶

Do not write outside the box

25 ABC and ACD are triangles.



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Labelling triangle ABC. Side a is opposite angle A and side b is opposite angle B

Work out the size of angle x.

[5 marks]

S O H C A H T O A

AC can be found by using right-angled trigonometry in triangle ACD. Ticking A as we have the adjacent and O as we are trying to find the opposite. There are two ticks on TOA so that formula triangle can be used

$\tan 49 \times 16$

Covering O in the TOA formula triangle finds that opposite = tan of the angle x adjacent. So AC = 18.4... cm

$$\frac{\sin x}{18.4...} = \frac{\sin 35}{20}$$

The sine rule can be used to find angle x as there are two opposite pairs of sides and angles in triangle ABC. (sinA)/a = (sinB)/b. Using the sine rule with the angles as numerators to make it easier to rearrange to find x. Substituting x for A, 18.4... for a, 35 for B and 20 for b

$\sin x = 0.5...$

Multiplying both sides by 18.4...

Answer 31.9 degrees

Doing the inverse sin of both sides to get x on its own. x = $\sin^{-1}$(0.5...) = 31.86...

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26 $\quad f(x) = \dfrac{x}{x+2} \qquad g(x) = x^2 - 2$

Work out $\quad fg(x)$

Give your answer in the form $\quad a + bx^n \quad$ where a, b and n are integers.

[3 marks]

$\dfrac{x^2-2}{x^2}$

Putting g(x) into f(x) by substituting x^2 - 2 for x in f(x). The -2 and +2 cancel out leaving x^2 as the denominator

Answer $1-2x^{-2}$

Dividing the terms on the numerator by x^2 separately. x^2/x^2 = 1 and $2/x^2 = 2x^{-2}$

27 $\quad$ The point $\left(3, \dfrac{1}{64}\right)$ lies on the curve $\quad y = k^x \quad$ where k is a constant.

Show that the point $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ lies on the curve.

[3 marks]

$\dfrac{1}{64} = k^3$

The point (3 , 1/64) lies on the curve and therefore satisfies the equation. Substituting 1/64 for y (as this is the y-coordinate of the point) and 3 for x (as this is the x-coordinate of the point) in the equation

$k = \sqrt[3]{\dfrac{1}{64}} = \dfrac{1}{4}$

Rearranging to make k the subject by cube rooting both sides. So k = 1/4

$\left(\dfrac{1}{4}\right)^{\frac{1}{2}} = \dfrac{1}{2}$

Substituting in 1/2 for x (as this is the x-coordinate of the second point) and 1/4 for k finds that the y value will be 1/2. Therefore the point (1/2, 1/2) lies on the curve

11

Do not write outside the box

28 Izzy runs an 80-metre race in 14 seconds.

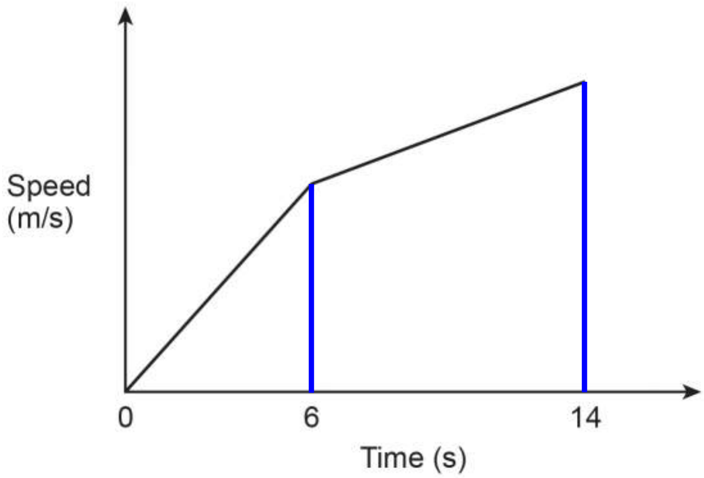
During the first 6 seconds her speed increases at a constant rate.

During the last 8 seconds her speed increases at a different constant rate.

Her speed at 14 seconds is 2 m/s more than her speed at 6 seconds.

Here is a sketch of her speed-time graph.

Not drawn accurately



28 (a) Work out her acceleration during the last 8 seconds.

State the units of your answer.

[2 marks]

Answer $\frac{2}{8}$ m/s²

Acceleration = (change in speed)/(change in time). The last 8 seconds is from 6 seconds to 14 seconds, in which her speed goes up by 2 m/s. The units of speed and time are divided (m/s ÷ s = m/s^2)

Do not write outside the box

28 (b) When Izzy finishes the 80-metre race, her speed is v m/s

Work out the value of v.

[4 marks]

$\frac{1}{2} \times 6 \times (v-2) + \frac{1}{2}(v-2+v) \times 8$

The distance is equal to the area under the graph, which can be split into a triangle and trapezium. Area of triangle = 1/2 x base x height. Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them. v is the final speed at 14 seconds so the speed at 6 seconds is v - 2

$3(v-2)+4(2v-2)$ Simplifying

$3v-6+8v-8$ Expanding the brackets

$11v-14=80$ Collecting like terms then setting the simplified expression of the distance in terms of v equal to the actual distance of 80 m

$11v=94$ Adding 14 to both sides to get the v term on its own

Answer $8.\dot{5}\dot{4}$

Dividing both sides by 11 to get v on its own

END OF QUESTIONS

6