OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Friday 10 November 2023 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

H

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

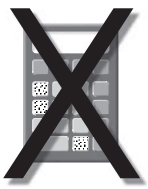
First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.



Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

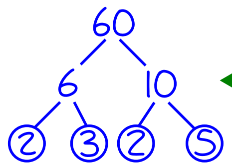
Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Write 60 as a product of its prime factors.



Doing a factor tree and ending at prime numbers by circling them

Writing the product of the circled numbers gives 60 as a product of its prime factors

$2^2 \times 3 \times 5$ .... **[2]**

2 By writing each number correct to **1** significant figure, find an estimate for this calculation.

$$\frac{486}{\sqrt{101.2}}$$

$$\frac{500}{\sqrt{100}}$$

486 is 500 to 1 significant figure. 101.2 is 100 to 1 significant figure

The square root of 100 is 10 then 500/10 = 50

50 .... **[2]**

3 **(a)** Simplify.

$3a^2 \times 4a^5$

It can all be multiplied in any order. 3 x 4 = 12. $a^2 \times a^5 = a^{2+5} = a^7$. Then $12 \times a^7 = 12a^7$

(a) $12a^7$ .... **[2]**

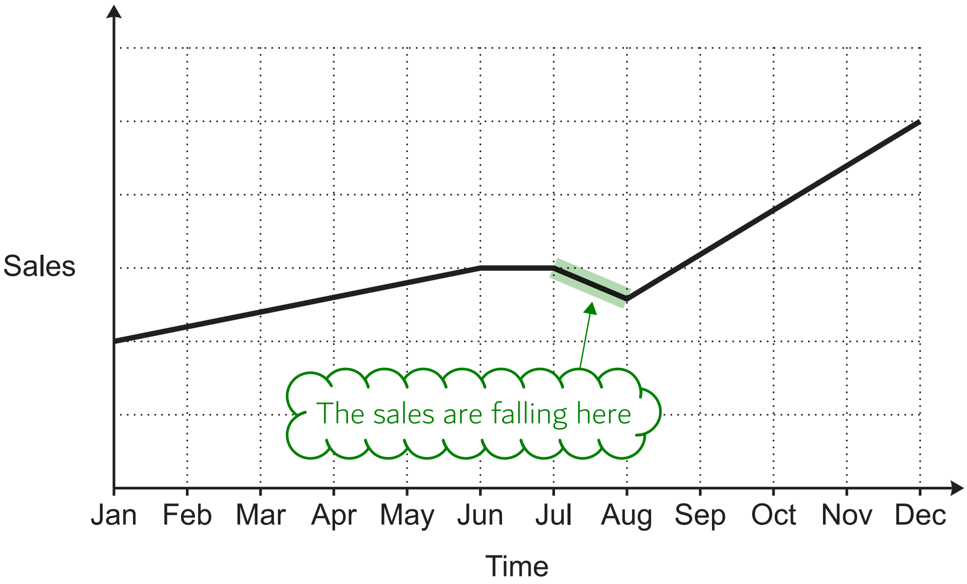
(b) Factorise fully.

$4x^2 - 12x$

4 is the highest common factor of 4 and 12. x is the highest common factor of x^2 and x. So bringing 4x out as a factor, dividing both terms by the 4x and leaving the result in a bracket

(b) $4x(x-3)$ .... **[2]**

4 A sales representative and a manager discuss this graph of sales over the last year.



(a) The sales representative says

I can tell from the graph that, over the last year, sales have risen every month.

Is the sales representative correct?
Give a reason for your answer.

......No...... because ......sales fell between July and August......

.... **[1]**

(b) The manager says

I can tell from the graph that sales are now more than double what they were at the start of the year.

Is the manager correct?
Give a reason for your answer.

......No...... because ......the scale may not start from 0......

If the vertical scale starts from 0 then the manager is correct as in December the sales are over twice as high as in January. However no numbers are given on the vertical axis and it may start from a higher number meaning that it might not be double **[1]**

5 Work out.

(a) $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Column vectors are in the form $\binom{x}{y}$
Adding the x-components: 3 + -2 = 1
Adding the y-components: -1 + 4 = 3

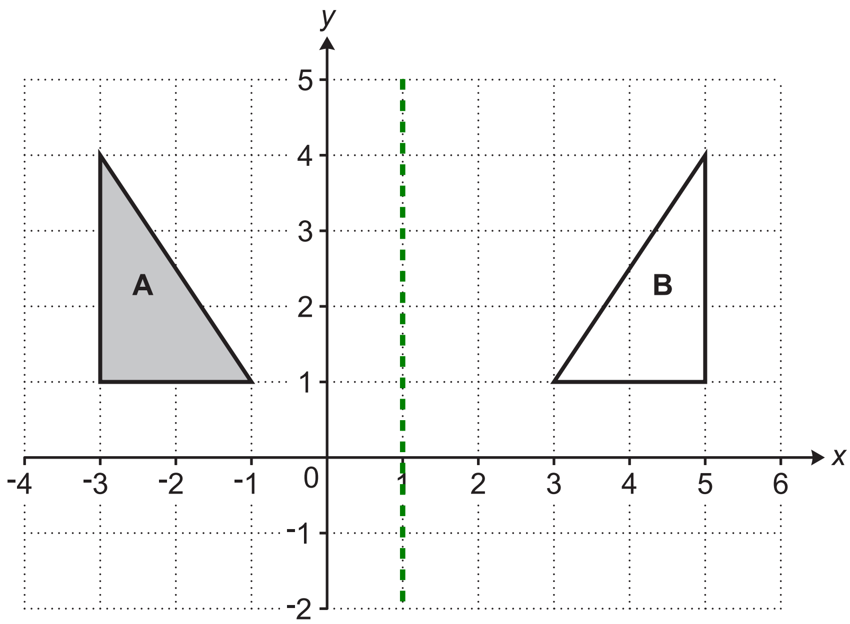
(a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ **[1]**

(b) $\begin{pmatrix} 5 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

Dealing with the x-components: -2 x 1 = -2 then 5 - 2 = 3
Dealing with the y-components: -2 x -4 = 8 then 3 + 8 = 11

(b) $\begin{pmatrix} 3 \\ 11 \end{pmatrix}$ **[2]**

6 Triangle **A** and triangle **B** are drawn on the coordinate grid.



Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

Reflection in x = 1

It is a reflection in the dashed line, which has the equation x = 1 as all the x-coordinates on the line are 1 **[2]**

7 A motorist wants to buy a new car but does not have enough money.

The price of the car is £16 000.

The motorist sees this notice for a deal on the car they want to buy.
The number of equal monthly payments is hidden.

Pay 15% of the price of the car now and then equal monthly payments of £300

Work out the number of monthly payments if the total cost of the car to the motorist is £17 400.
You must show your working.

$2\overline{)1600}$ = 0800
2400

10% of £16000 can be found by dividing £16000 by 10 to give £1600. Dividing this by 2 finds that 5% of £16000 is £800. Adding the £1600 and £800 gives £2400, which is 15% of £16000

17400
-2400
15000

Subtracting the £2400 from the £17400 works out that the total of the equal monthly payments is £15000

$300\overline{)15000}$ = 50

Dividing the £15000 by the £300 works out that £15000 is 50 lots of £300 and therefore there must be 50 monthly payments

........................50........................ **[4]**

8 The density of some concrete is 2.4 g/cm^3.
A lump of this concrete has a volume of 400 cm^3.

Work out the mass of the lump of concrete.

$d\ m\ V$ ← Density = mass/volume. Writing this as a formula triangle

$2.4 \times 400 = 960.0$ ← From the formula triangle, mass = density x volume

........960........ g **[2]**

9 A box contains only red, green and black pens.
The ratio of red pens to green pens to black pens is 1 : 4 : 11.

(a) Work out the percentage of the pens that are green.

$1+4+11$ ← Adding the 1, 4 and 11 works out that there are 16 parts in total in the ratio

$\frac{4}{16} = \frac{1}{4}$ ← 4 out of the 16 parts are for green. Simplifying this fraction by dividing both the numerator and denominator by 4 gives 1/4, which is 25%

(a) ....25........ % **[2]**

(b) There are 24 more green pens than red pens.

Work out the total number of pens in the box.

$4-1$ ← Subtracting the 1 part for red from the 4 parts for green works out that 3 parts of the ratio represent the 24 more green pens than red pens

$24 \div 3$ ← Dividing the 24 by the 3 parts of the ratio works out that 1 part of the ratio is worth 8 pens

$16 \times 8 = 128$ ← There are 16 parts in total in the ratio so multiplying the value of 1 part of of the ratio by 16 works out that there are 128 pens in total

(b) ....128........ **[4]**

10 **(a)** Find the next term of this sequence.

81
×3
243

1 (x3) 3 (x3) 9 (x3) 27 (x3) 81 (x3) ....243.......... **[1]**

(b) In the Fibonacci sequence below, the next term is found by adding the two previous terms.
The third term is 0.83 and the fourth term is 1.29.

Complete the first, second and fifth terms of the sequence.

..........0.37.......... ....0.46.......... 0.83 1.29 ....2.12.......... **[3]**

0.83 + 1.29 = 2.12 ← This works out the 5th term

1.29 − 0.83 = 0.46 ← Subtracting the 3rd term from the 4th term works out what must be added to the 3rd term to get the 4th term so works out what the 2nd term is

0.83 − 0.46 = 0.37 ← Subtracting the 2nd term from the 3rd term works out what must be added to the 2nd term to get the 3rd term so works out what the 1st term is

11 **(a)** Work out.

$0.8 \div 0.004$

$4\overline{)800}$ = 200

Division is much easier when dividing by a whole number. 0.004 must be multiplied by ten 3 times to get 4, so the 0.8 must be multiplied by ten 3 times to give 800 to make an equivalent division

(a) ....200.......... [1]

(b) A carpenter has a plank of wood of length *w* metres.

The carpenter cuts $\frac{3}{5}$ of the plank of wood into 20 equal pieces.

Each piece has length 0.06 metres.

Work out the value of *w*, the original length of the plank of wood.
You must show your working.

$0.06 \times 20 = 1.20$

Multiplying the 0.06 metres by the 20 equal pieces works out that 3/5 of the plank of wood is 1.20 metres

$3\overline{)1.20}$ = 0.40

Dividing the 1.20 metres by 3 works out that 1/5 of the plank of wood is 0.40 metres

$0.40 \times 5 = 2.00$

Multiplying the 0.40 metres by 5 undoes the 1/5 and works out that the plank of wood is 2 metres

(b) *w* = ...2.......... [4]

12 Write down an equation for the line that is parallel to $y = 3x - 5$ and passes through the point (0, 1).

The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept. The gradient of the line y = 3x - 5 is 3 so the gradient of the line parallel to this must also have a gradient of 3. The y-intercept is given as this is where the x-coordinate is 0. The y-coordinate is 1 at this point

..........$y = 3x + 1$.......... [2]

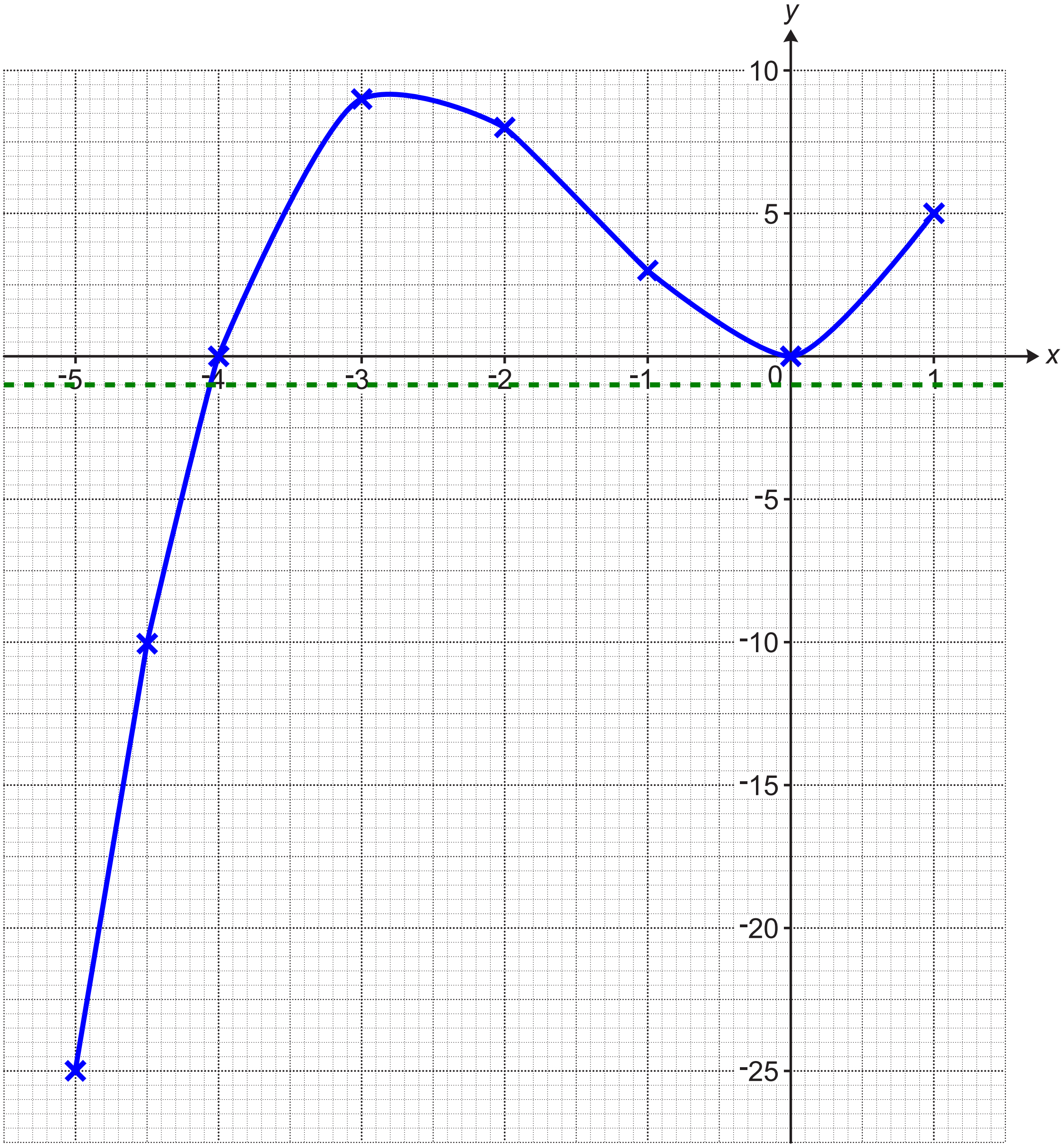
y = (-1)³ + 4(-1)² = -1 + 4 x 1 = -1 + 4 = 3

13 (a) Complete the table for $y = x^3 + 4x^2$.

x	-5	-4.5	-4	-3	-2	-1	0	1
y	-25	-10.1	0	9	8	3	0	5

[1]

(b) Draw the graph of $y = x^3 + 4x^2$ for $-5 \leqslant x \leqslant 1$.



[3]

(c) The equation $x^3 + 4x^2 = k$, where k is an integer, has exactly one solution for $-5 \leqslant x \leqslant 1$.

Find the greatest possible value of k.

y = -1 is the highest a line can be drawn horizontally which only crosses the curve once where y is equal to an integer

(c) k = ...-1........................ **[1]**

14 A teacher is planning a theme day for the 500 pupils at their school.
The teacher asks a sample of 20 pupils from year 8 which theme they would prefer.

The results are shown in the table.

Theme	**Number of pupils**
Sport	7
Art and design	3
Recipes	2
Music and movies	8

(a) Describe **two** disadvantages of the teacher's sampling method.

1. Small sample .... Asking more pupils would get more reliable data ....

....

2. Only included year 8 .... The sample should include pupils from all year groups as preferences may change with age .... **[2]**

(b) Using the results from the table the teacher estimates that 175 pupils in the school would prefer a sport theme.

Here is the teacher's method.

$\frac{7}{20} \times 500 = 175$

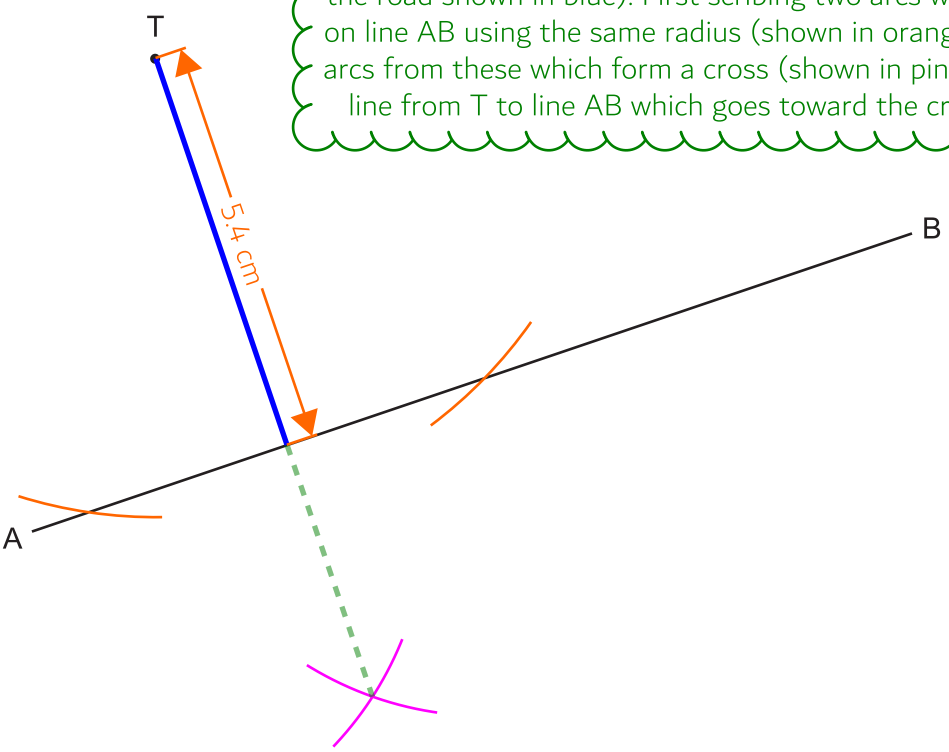
Write down **one** assumption the teacher has made when making their estimate.

The sample is representative ....

.... It is assumed that 7/20 of the 500 pupils prefer a sports theme because 7/20 of the sample prefer a sports theme. The fractions may not be the same if the sample is not representative .... **[1]**

15 The diagram shows a town T and a straight road AB.

Scale: 2 cm represents 1 km



A new straight road is built from town T to the road AB.
The road is the shortest possible distance from town T to the road AB.

(a) **Using ruler and compasses only**, construct the road from town T to the road AB. **[2]**

(b) The new road costs £200 000 per kilometre to build.
The road constructor says

The new road will cost over £600 000 to build.

Show that the road constructor is incorrect.

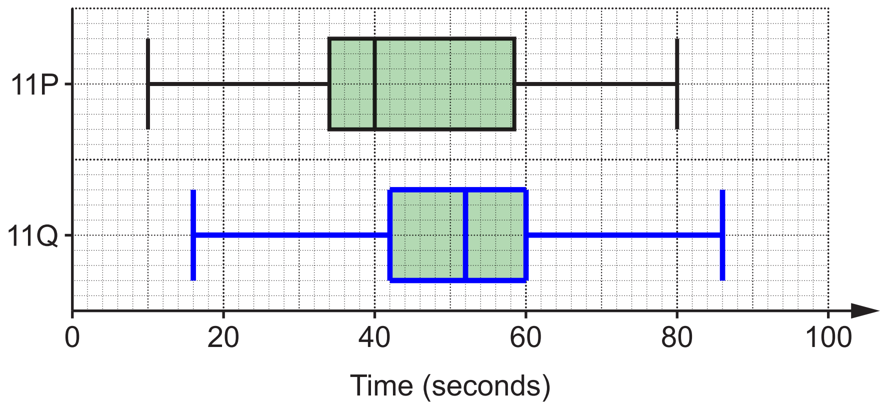
$5.4 \div 2$ ← Measuring the length of the road on the diagram finds that it is 5.4 cm. Dividing this by 2 works out that it is 2.7 lots of 2 cm

2.7×200000 ← It will therefore be 2.7 lots of 1 km so the road is 2.7 km long, Multiplying this by the £200000 per kilometre works out that the new road will cost £540000

540000 ← This is not over £600000 so the road constructor is incorrect

[3]

16 The box plot shows the distribution of the times, in seconds, taken by class 11P to complete a problem.



(a) The times, in seconds, taken by class 11Q to complete the same problem are summarised below.

- median = 52
- lower quartile = 42
- interquartile range = 18
- range = 70
- highest score = 86

The scale goes up 20 over 10 small boxes. 20 ÷ 10 = 2 so each small box is worth 2. Adding the interquartile range to the lower quartile finds that the upper quartile was 60. Subtracting the range from the highest score finds that the lowest score was 16

Show the distribution of 11Q's times as a box plot on the diagram above. **[3]**

(b) Which class has the more **consistent** times?
Give a reason for your answer.

11Q because the interquartile range was less

The interquartile range is the distance between the lower and upper quartiles. This can be seen visually by the width of the boxes which are highlighted in green **[1]**

17 y is directly proportional to the cube of x.

Complete the table.

x	1	2	5
y	7	56	875

[4]

$y \propto x^3$ ← Writing out the proportion

7×2^3 ← x was multiplied by 2 from 1 to 2. So y must be multiplied by 2^3

$875 \div 7$ ← Working out that y was multiplied by 125 from 7 to 875

$\sqrt[3]{125}$ ← So x must be multiplied by the cube root of 125. Multiplying 1 by 5 gives 5

18 Work out.

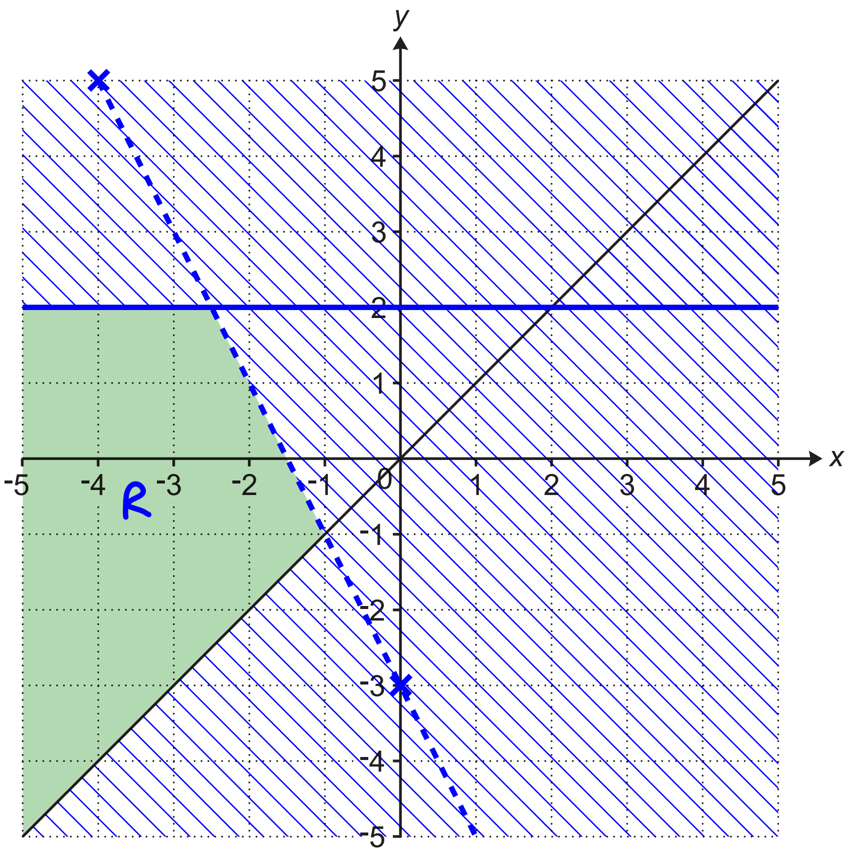
$0.\dot{2}\dot{8} + \frac{4}{9}$

Give your answer as a fraction in its simplest form.

The method is shown on the next page

$\frac{8}{11}$ .... **[4]**

19 The graph of $y = x$ is drawn on the grid.



The region **R** satisfies the following inequalities.

$y \geqslant x$ $\qquad$ $y \leqslant 2$ $\qquad$ $y < -2x - 3$

Explanation is on the next page

By drawing two more straight lines on the grid, find and label the region **R**. **[6]**

$x = 0.\dot{2}\dot{8}$ ← Letting x be the recurring decimal

$100x = 28.\dot{2}\dot{8}$ ← There are two recurring digits so multiplying by ten twice to get the recurring digits in the same decimal places

$99x = 28$ ← Subtracting x from 100x cancels out the recurring digits

$\frac{28}{99} + \frac{44}{99}$ ← Dividing both sides by 99 finds that the recurring decimal is 28/99 as a fraction

$\frac{72}{99}$ ← Adding the numerators and the denominator stays the same

Simplifying the fraction by dividing both the numerator and denominator by 9. It cannot go simpler as 8 and 11 cannot be divided by the same amount to get smaller whole numbers

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y is greater so crossing out below the line of y = x.

Plotting the line of y = 2 and crossing out everything above it as y is less.

Plotting the line of y = -2x - 3. Dashing it as y cannot be equal. Crossing out everything above it as y is less. When x = 0, y = -2(0) - 3 = -3 so plotting the point (0 , -3). When x = -4, y = -2(-4) - 3 = 8 - 3 = 5 so plotting the point (-4 , 5). Drawing a straight line through these two points

The region is highlighted in green but does not need to be shaded

20 Work out.

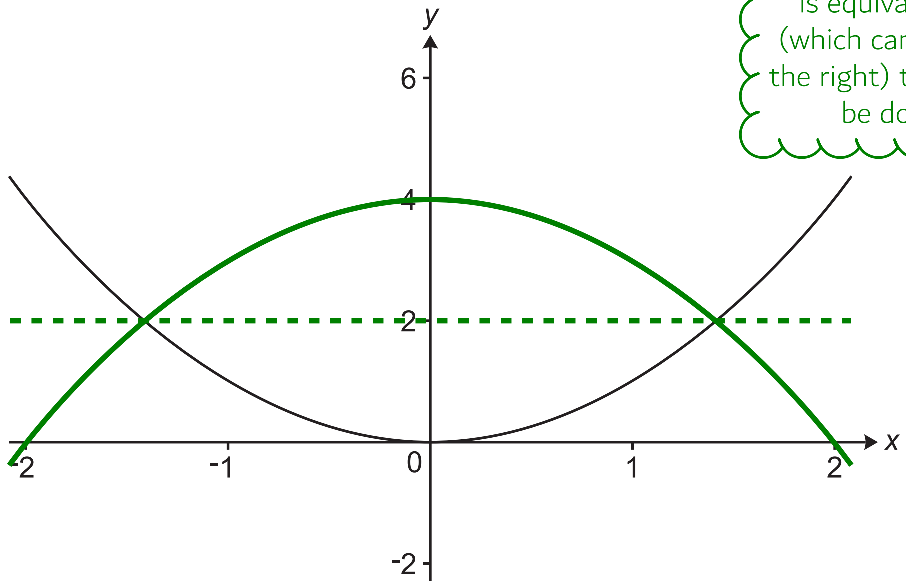
$36^{-\frac{1}{2}}$

The power of 1/2 means to do the positive square root of 36, which is 6. The negative power means to do the reciprocal (1 over)

$\frac{1}{6}$ **[2]**

21 The graph of $y = x^2$ is shown below for $-2 \leqslant x \leqslant 2$.

Reflecting on the line y = 2 (the dashed line) is equivalent to a reflection in the x-axis (which can be done by flipping the signs on the right) then a translation up 4 (which can be done by adding 4 to the right)



(a) Find the equation of the image when $y = x^2$ is reflected in the line $y = 2$.

(a) $y = -x^2 + 4$ **[2]**

(b) Describe the **single** transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 6x - 5$.

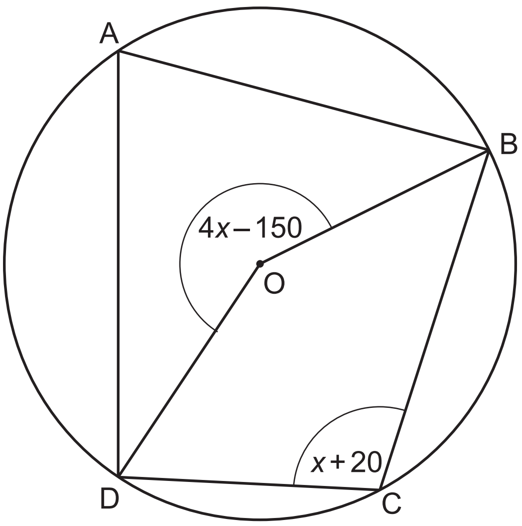
$y = (x+3)^2 - 5 - 9$ ← Completed the square on $y = x^2 + 6x - 5$ by halving the coefficient of x (which is 6 so halves to 3) and putting this in a bracket with x and squaring it. Subtracting 3^2 (which is 9) from the outside

The graph is still an x^2 graph so is the same shape but the turning point is now at (-3 , -14) instead of at (0 , 0). The turning point can be found by using the completed the square form. x = -3 for the bracket to equal to 0, which will give the minimum value when it is squared. When the bracket is equal to 0, y = -5 - 9 = -14

Translation $\begin{pmatrix} -3 \\ -14 \end{pmatrix}$ **[4]**

22 In this question all angles are given in degrees.

A, B, C and D are points on the circumference of a circle, centre O.



Not to scale

(a) Find the size of angle BCD.

$2(x+20)$
$2x+40=4x-150$

The angle at the centre is double the angle at the circumference. So doubling angle BCD must equal to the reflex angle DOB

$190=2x$

Subtracting 2x from both sides to get the x terms on the same side and adding 150 to both sides to get the x term on its own

$2\overline{)190}$ = 95, $95+20$

Dividing both sides by 2 finds that x = 95. Adding 20 to this finds angle BCD

(a) ....115..........° **[4]**

(b) Find the size of angle DAB.
Give a reason for your answer.

$180 - 115 = 65$

Angles BCD and DAB are opposite angles in the cyclic quadrilateral (as all four corners are on the circle) so they must add up to 180°. Subtracting angle BCD from 180 leaves angle DAB

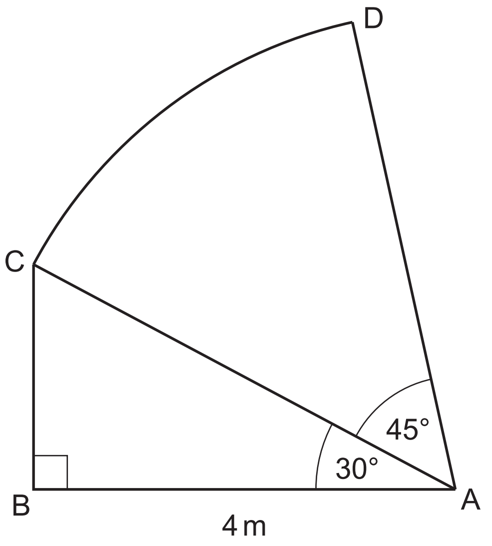
.....65..... ° because opposite angles of a cyclic quadrilateral add up to 180°

.... **[2]**

Turn over

23 In the diagram,

- ABC is a right-angled triangle
- ACD is the sector of a circle with centre A.



Not to scale

(a) Show that the area of the sector ACD is $\frac{8}{3}\pi$ m^2. **[6]**

$S^{O}_{H}\ C^{A}_{H}\ T^{O}_{A}$ ← Doing right-angled trigonometry on triangle ABC by writing SOH CAH TOA as formula triangles. Ticking H as we are looking for the hypotenuse AC and ticking A as we have the adjacent AB. There are two ticks on the CAH formula triangle so this one can be used

0	30	45	60	90
4	3	2	1	0

← Working out cos30 by listing out the angles of 0, 30, 45, 60, 90 degrees and then writing 4, 3, 2, 1, 0 under these. The 3 is under the 30. Square rooting this and putting it over 2 works out that cos30 = √3/2

$4 \div \frac{\sqrt{3}}{2}$ ← From the CAH formula triangle, hypotenuse = adjacent/(cos of the angle) = 4/cos30

$\pi \times \left(\frac{8}{\sqrt{3}}\right)^2 \times \frac{45}{360}$ ← To divide by a fraction: keep the first number, change the sign to a multiply, flip the fraction. 4 x 2/√3 = 8/√3. Area of circle = π x radius². Side AC is the radius. Multiplying the area of the whole circle by 45/360 as there are 360° around the centre of a circle and there are 45° out of these

$\pi \times \frac{64}{3} \times \frac{1}{8}$ ← Squaring the first fraction by squaring the numerator and squaring the denominator. 8² = 64 and (√3)² = 3. Simplifying 45/360 to 1/8 by dividing both the numerator and denominator by 45

$\frac{8}{3}\pi$ ← Doing 1/8 of 64 gives 8. The denominator is not effected. Putting the π at the end

(b) Work out the total area of the shape ABCD.

Give your answer in the form $\left(\frac{a\sqrt{k}}{b} + \frac{8}{3}\pi\right)$ m^2.

0 30 45 60 90
0 1 2 3 4

Working out sin30 by writing the angles of 0, 30, 45, 60, 90 degrees and then writing 0, 1, 2, 3, 4 under these. The 1 is under the 30. Square rooting this and putting it over 2 works out that sin30 = 1/2

$\frac{1}{2} \times 4 \times \frac{8}{\sqrt{3}} \times \frac{1}{2}$

Area of triangle = 1/2 absinC, where a and b are two sides and C is the angle between them. Substituting 4 for a, 8/√3 for b and 1/2 for sinC

$\frac{8}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

The multiplication can be done in any order. 1/2 x 1/2 = 1/4. Then 1/4 x 4 = 1. Then 1 x 8/√3 = 8/√3. Rationalising the denominator by multiplying both the numerator and denominator by √3

For the numerator: 8 x √3 = 8√3. For the denominator: √3 x √3 = 3. Adding the area of the sector to the area of the triangle gives the area of the shape ABCD

(b) $\frac{8\sqrt{3}}{3} + \frac{8}{3}\pi$ m^2 **[3]**

24 The x-coordinates of the intersections of the graphs of $y = x^2 + ax + 7$ and $y = 3x + b$ are the solutions to the equation $x^2 + 8x + 13 = 0$.

Find the value of a and the value of b.

$0 = x^2 + (a - 3)x + 7 - b$ ← Doing simultaneous equations on the two equations of the graphs expresses an equation in terms of x which would be solved to find the x-coordinates of the intersections of the two graphs. Subtracting the the second equation from the first equation does this. Putting the resulting equation in the same form as the $x^2 + 8x + 13 = 0$

$a - 3 = 8$ ← The equation given and the equation formed are equivalent. So the coefficients can be equated. a - 3 is the coefficient of x, which must be equal to the 8 which is also the coefficient of x. Solving this equation by adding 3 to both sides finds that a = 11

$7 - b = 13$ ← The constants of 7 - b must be equal to the constant 13. As 6 must be added to 7 to get 13, b must be -6

a = ...11......

b = ...-6...... **[3]**

25 Riley has a set of cards.
Each card has a triangle or circle drawn on it and is coloured red or blue.

The table gives some information about the number of each type of card.

	Number of cards with a triangle	**Number of cards with a circle**
Number of blue cards	3	8
Number of red cards	9	2

Riley chooses three of these cards at random without replacement.
All three of these cards have a circle drawn on them.

Riley says

The probability that two of these three cards are blue and one is red is less than $\frac{1}{2}$.

Is Riley correct?
You must show your working.

BBR, BRB, RBB ← Listing out the outcomes which would result in two of the cards being blue and one being red. Blue AND blue AND blue OR blue AND red AND blue OR red AND blue AND blue. AND means to multiply the probabilities. OR means to add the probabilities

$\frac{8}{10} \times \frac{7}{9} \times \frac{2}{8} \times 3$ ← Writing out the probability of getting blue AND blue AND red, given that they all have circles on them. There are 10 cards with a circle. Out of these, 8 are blue. So the probability of the first card being blue is 8/10. There is then one fewer blue card and one fewer card with a circle. So the probability of the second card being blue is 7/9. There is then one fewer card in total. So the probability of the third card being red is 2/8. Multiplying the probability of blue AND blue AND red by 3 as the probabilities of the other two possible outcomes is the same (they have the same numerators and denominators multiplied together)

$\frac{1}{5} \times \frac{7}{3} \times \frac{1}{1} \times 1$ ← Simplifying the multiplication by cancelling out numerators and denominators

$\frac{7}{15}$ ← 1 x 7 x 1 x 1 = 7 and 5 x 3 x 1 = 15

Yes ← 7/15 is less than 1/2 as 7 is less than half of 15. So Riley is correct .... **[5]**

END OF QUESTION PAPER