AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE MATHEMATICS

H

Higher Tier   Paper 2 Calculator

Thursday 6 June 2019   Morning   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Do not write outside the box

Answer **all** questions in the spaces provided

1 Circle the point that lies on the curve $y = x^2 - 4x + 1$

[1 mark]

(–1, 4) (–1, –4) (–1, –2) (–1, 6)

Substituting in the x-coordinate of -1 (which is the same for all options) into the equation works out which y-coordinate is correct. $(-1)^2 - 4(-1) + 1 = 6$

2 The height of a tree is 12 metres, correct to the nearest metre.

Circle the error interval.

[1 mark]

$11.5 \text{ m} \leqslant \text{height} < 12.5 \text{ m}$ $11.5 \text{ m} \leqslant \text{height} \leqslant 12.5 \text{ m}$

$11.5 \text{ m} < \text{height} \leqslant 12.5 \text{ m}$ $11.5 \text{ m} < \text{height} < 12.5 \text{ m}$

The height can be equal to 11.5m as this rounds up to 12m but can't be equal to 12.5m as this rounds up to 13m

3 $2a$ is five times bigger than b.

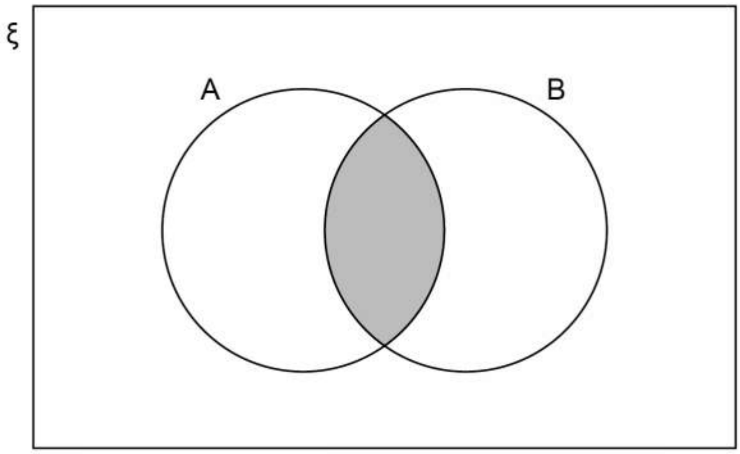
Circle the ratio $a : b$

[1 mark]

10 : 1 1 : 10 5 : 2 2 : 5

2a = 5b, so a could be 5 and b could be 2

4



Which of these represents the shaded region?

Circle your answer.

[1 mark]

$A \cup B$ $(A \cap B)'$ $A \cap B$ $A' \cup B'$

The shaded region is the intersection of A and B

Turn over for the next question

4

5 Using ruler and compasses, show the region inside the grid that is

less than 4 cm from *A*

and

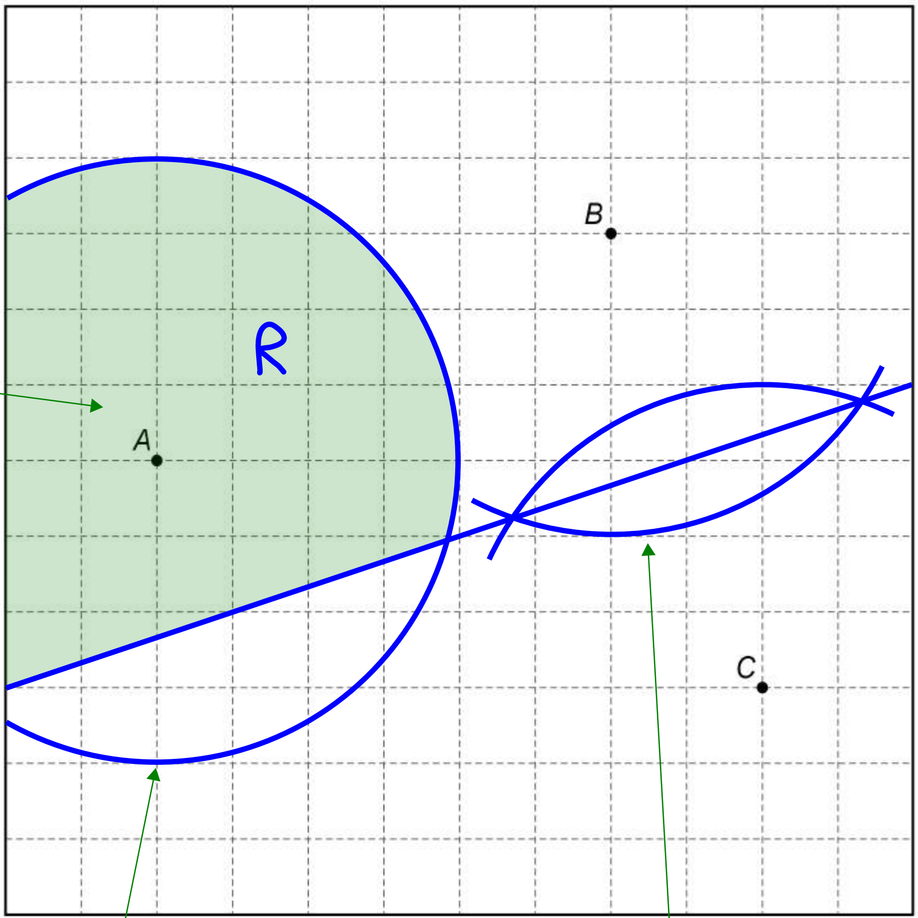
nearer to *B* than to *C*.

Label the region R.

Show all your construction lines.

[3 marks]

The region doesn't need to be shaded but is shown in green



Set the compass with a radius of 4 cm and scribe an arc around A to indicate all points which are 4cm from A

Construct a perpendicular bisector to line BC to indicate all points which are an equal distance from B and C. Set the compass to a radius which is greater than half of the distance from B to C then scribe arcs from B and C. Draw a straight line through both of the points where the arcs meet

Do not write outside the box

6 Beth drives 200 miles in 4 hours.

She drives the first 18 miles at an average speed of 36 mph

Work out her average speed for the rest of the journey.

[3 marks]

$s\,^{d}\,t$ ← Writing the formula triangle for speed, distance, time

$18 \div 36$ ← Covering t in the formula triangle finds that time = distance ÷ speed. So dividing the 18 miles by the 36 mph works out that the first part of the journey took 0.5 hours. The unit is hours as this is involved in miles per hour

$4 - 0.5 = 3.5$ ← Subtracting the 0.5 hours from the 4 hours works out that the rest of the journey took 3.5 hours

$200 - 18$ ← Subtracting the 18 miles from the 200 miles works out that the rest of the journey is 182 miles

$182 \div 3.5$ ← Covering s in the formula triangle finds that speed = distance ÷ time. So dividing the 182 miles by the 3.5 hours works out the speed for the rest of the journey in mph

Answer 52 mph

Turn over for the next question

6

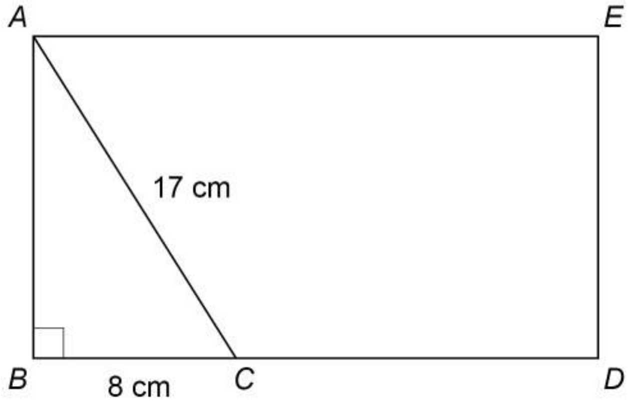
Do not write outside the box

7 The diagram shows rectangle *ABDE* and right-angled triangle *ABC*.

AC = 17 cm

BC = 8 cm

Not drawn accurately



BC : *CD* = 1 : 2

Work out the area of rectangle *ABDE*.

[4 marks]

$8^2 + AB^2 = 17^2$ ← Pythagoras' Theorem can be used in right-angled triangle ABC to work out side AB. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting 8 cm for a, AB for b and 17 cm for c

$AB^2 = 225$ ← Subtracting 8^2 from both sides to get AB^2 on its own

$AB = 15$ ← Square rooting both sides finds that AB is 15 cm

8×3 ← BC is represented by 1 part of the ratio and there are 3 parts in total in the ratio so multiplying 8 cm by 3 works out that the length of BD is 24 cm

24×15 ← Area of rectangle = base x height. BD is the base and AB is the height

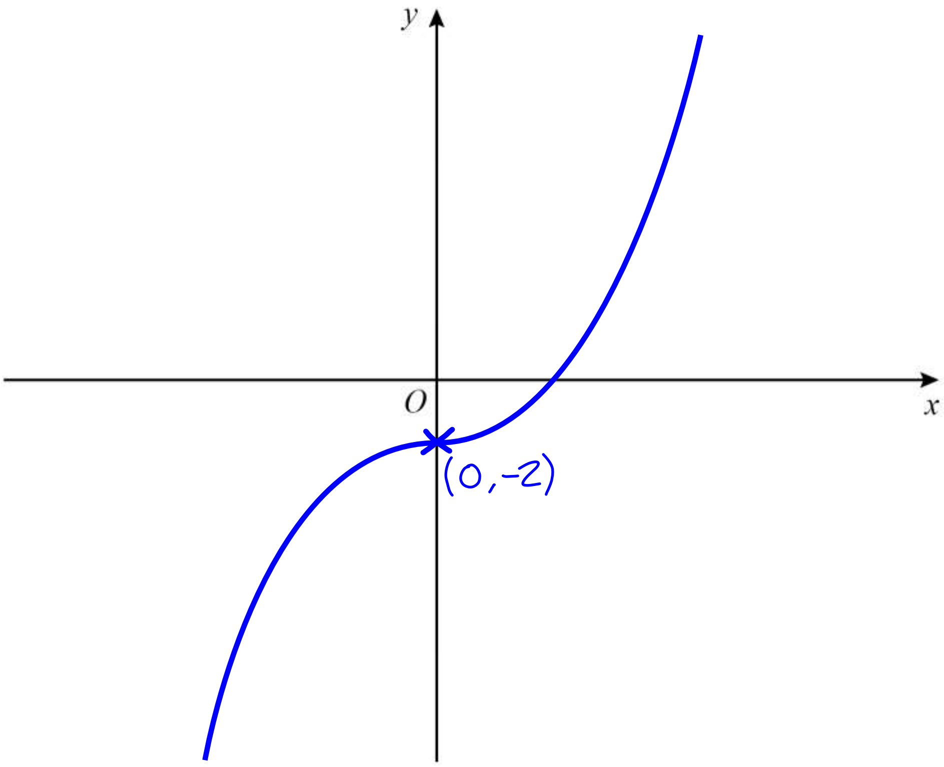
Answer 360 cm^2

Do not write outside the box

8 On the axes, sketch the curve $y = x^3 - 2$

You **must** show the coordinates of the y-intercept.

[2 marks]



$y = x^3$ is a typical graph. Subtracting 2 translates the graph downward 2 in the y direction.

Alternatively, using table mode on the calculator to create a table of values, roughly plotting the points on the graph then join them up with a curve. Set $f(x) = x^3 - 2$. Start: -5. End: 5 Step: 1

Turn over for the next question

6

Do not write outside the box

9 In a sport, injury time is added time played at the end of a match.

The table shows the injury time, t (minutes) played in 380 matches.

Injury time, t (minutes)	Frequency
$0 < t \leqslant 2$	59
$2 < t \leqslant 4$	158
$4 < t \leqslant 6$	106
$6 < t \leqslant 8$	45
$8 < t \leqslant 10$	12

9 (a) Circle the **two** words that describe the data.

[1 mark]

continuous   discrete   grouped   ungrouped

Time is continuous as it could be any value. The time is grouped into intervals ($0 < t \leqslant 2$, for example)

9 (b) Which class interval contains the median?

You **must** show your working.

[2 marks]

$\frac{380+1}{2}$

Using the formula $(n + 1)/2$, where n is the total number of matches, works out which value is halfway between the 1st and the 360th value and so which is the median. So the median will be halfway between the 190th and 191st value when put in order

$190.5 - 59 = 131.5$

Subtracting the first 59 matches from 190.5 works out that there are another 131.5 matches to count until the median is reached

Answer $2 < t \leqslant 4$

158 is greater than 131.5, so not all of the 158 can be counted before reaching the median. So the median must be in $2 < t \leqslant 4$

Do not write outside the box

9 (c) What percentage of the matches had **more than** 6 minutes of injury time?

[2 marks]

$45 + 12$

Both the $6 < t \leqslant 8$ and $8 < t \leqslant 10$ are more than 6 minutes. Adding the 45 and the 12 works out that 57 of the matches had more than 6 minutes of injury time

$\frac{57}{380} \times 100$

Putting the 57 over the 380 matches expresses the fraction of the matches which had more than 6 minutes of injury time. Multiplying this by 100 converts it into a percentage

Answer 15 %

10 x is an integer.

$$-4 < x \leqslant 2$$

and

$$2 \leqslant x + 3 < 9$$

Work out all the possible values of x.

[3 marks]

$-1 \leq x < 6$

Subtracting 3 from all sides of the second inequality gets x on its own in the middle

Answer $-1, 0, 1, 2$

The smallest integer which satisfies both inequalities is -1. The largest integer which satisfies both inequalities is 2. Listing these and all integers in between

8

11 Joe and Kyle share an amount of money in the ratio $7 : n$

Joe gets 35% of the money.

Work out the value of n.

[2 marks]

$35 \div 7 = 5$ — 7 parts of the ratio represent the 35%. So dividing 35% by 7 works out that 1 part of the ratio is worth 5%

$100 - 35$ — Percentage is out of 100 so subtracting the 35% from 100% works out that Kyle gets 65% of the money

$65 \div 5$ — Dividing the 65% by 5% works out that it is 13 lots of 5% and so is 13 parts as every 5% is 1 part

Answer 13

12 A biased coin is thrown 250 times.

The relative frequency of Heads is worked out after every 50 throws.

Total number of throws	50	100	150	200	250
Relative frequency	0.4	0.29	0.4	0.32	0.3

Circle the best estimate of the probability of Heads.

[1 mark]

0.3 0.32 0.342 0.4

The more times it is thrown, the more likely the relative frequency will be an accurate probability

Do not write outside the box

13 The amounts spent on clothes by 40 boys and 40 girls in one month were recorded.

The table shows information about the amounts spent by the boys.

Amount, x (£)	**Midpoint**	**Number of boys**	
$0 \leqslant x < 20$	10	22	220
$20 \leqslant x < 40$	30	9	270
$40 \leqslant x < 60$	50	6	300
$60 \leqslant x < 80$	70	3	210
		Total = 40	1000 $\div 40$

A B C D

The mean for the girls was £35

Estimate the mean for the girls as a percentage of the mean for the boys.

[5 marks]

A: Working out the midpoint of each interval by doing the mean of the upper and lower bounds. For $0 \leqslant x < 20$, 0 + 20 = 20 then 20 ÷ 2 = 10 so this is the midpoint. For $20 \leqslant x < 40$, 20 + 40 = 60 then 60 ÷ 2 = 30 so this is the midpoint. For $40 \leqslant x < 60$, 40 + 60 = 100 then 100 ÷ 2 = 50 so this is the midpoint. For $60 \leqslant x < 80$, 60 + 80 = 140 then 140 ÷ 2 = 70 so this is the midpoint.

B: Multiplying the midpoints by the number of boys for each interval works out an estimated total for each interval. 10 x 22 = 220 and 30 x 9 = 270 and 50 x 6 = 300 and 70 x 3 = 210.

C: Adding all of the estimated totals gives the overall estimated total. 220 + 270 + 300 + 210 = 1000.

D: Mean = total ÷ number, where total is the estimated total amount spent and number is the number of boys. So 1000 ÷ 40 = 25 which is the estimated mean for the boys

$\frac{35}{40} \times 100$

Expressing the mean of the girls as a fraction of the estimated mean of the boys then multiplying it by 100 to convert it into a percentage

Answer 140 %

8

Do not write outside the box

14 Ali and Mel are making 3-digit codes.

The digit 0 is **not** used.

Ali only uses odd digits.

Mel only uses even digits.

14 (a) Ali can make x more codes than Mel.

Assume that digits **cannot** be repeated.

Work out the value of x.

[3 marks]

$5 \times 4 \times 3 = 60$

Using the product rule for counting works out that Ali can make 60 codes. The odd digits are 1, 3, 5, 7, 9. There are 5 possibilities for the first digit. Given that one has already been chosen there are 4 possibilities for the second digit. Given that two have already been chosen there are 3 possibilities for the third digit. Multiplying all the possibilities for each digit gives the total number of possibilities for the 3-digit code for Ali

$4 \times 3 \times 2$

Using the product rule for counting works out that Mel can make 24 codes. The even digits not including 0 are 2, 4, 6, 8. There are 4 possibilities for the first digit. Given that one has already been chosen there are 3 possibilities for the second digit. Given that two have already been chosen there are 2 possibilities for the third digit. Multiplying all the possibilities for each digit gives the total number of possibilities for the 3-digit code for Mel

$60 - 24$

Subtracting the 24 possible codes for Mel from the 60 possible codes for Ali works out how many more codes Ali can make than Mel

Answer 36

14 (b) In fact, digits **can** be repeated.

What does this tell you about the actual value of x?

Tick **one** box.

[1 mark]

- [x] It is bigger than my answer to part (a)
- [ ] It is smaller than my answer to part (a)
- [ ] It is the same as my answer to part (a)

$5 \times 5 \times 5 = 125$

$4 \times 4 \times 4$

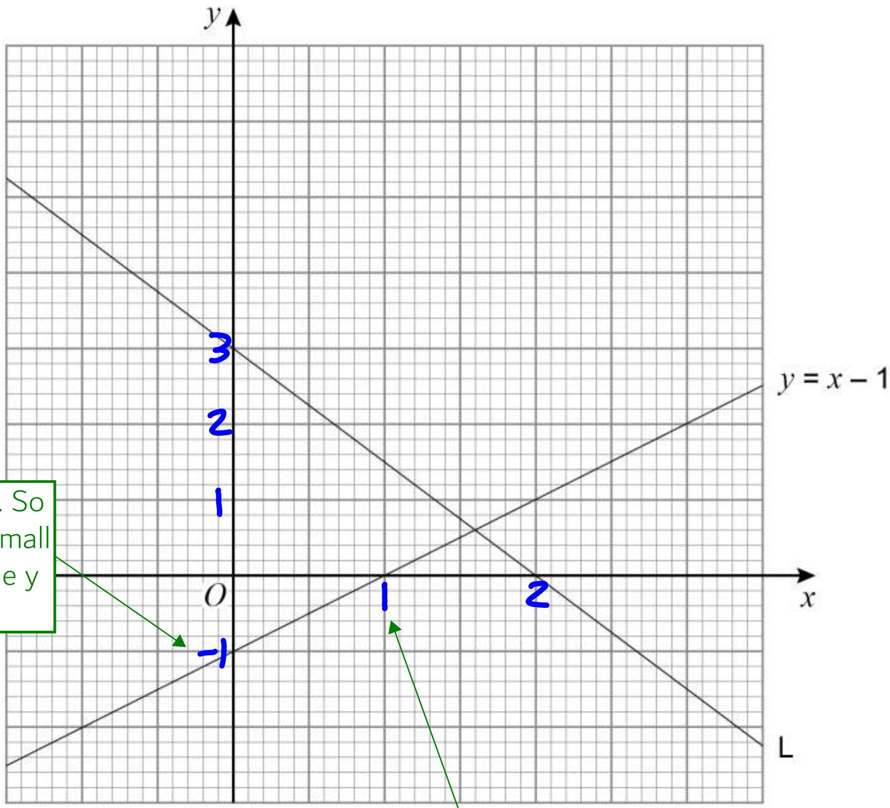
$125 - 64 = 61$

This time for Ali there are 5 possibilities for each digit and for Mel there are 4 possibilities for each digit. Using the product rule for counting again works out that Ali can now make 125 codes and Mel can now make 64 codes. Subtracting the 64 possible codes for Mel from the 125 possible codes for Ali works out that Ali can now make 61 more codes than Mel. This is bigger than the answer to part (a)

Do not write outside the box

15 Here is line L and the graph of $y = x - 1$

The scales of the axes are not shown.



x = 0 on the y-intercept. So y = 0 - 1 = -1. Every 5 small boxes represents 1 in the y direction

Work out the equation of line L.

[4 marks]

$0 = x - 1$

$x = 1$

y = 0 on the x-intercept. Substituting this into y = x - 1 and rearranging to find x shows that the line crosses the x-axis at 1. Every 10 small boxes represents 1 in the x direction

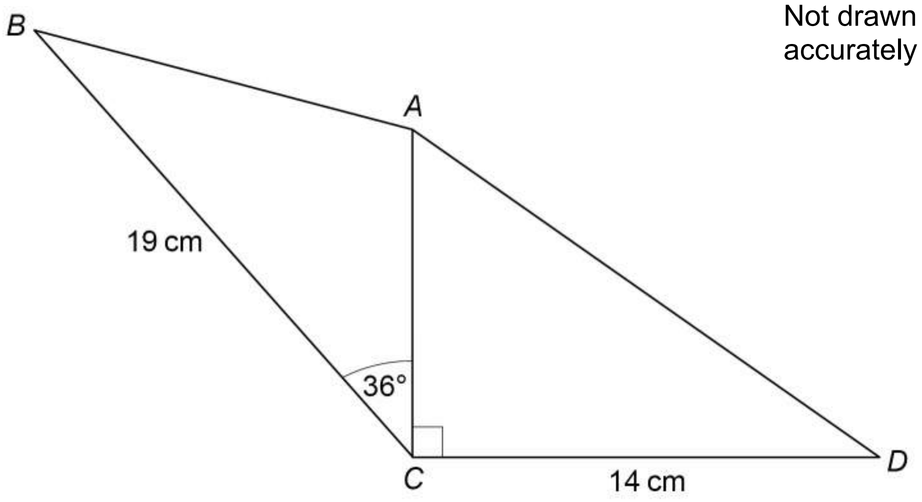
Answer $y = \frac{-3}{2}x + 3$

The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept. The gradient is (change in y)/(change in x). From the point (0, 3) to (2, 0), y has gone downward 3 so the change in y is -3. The change in x is 2. So the gradient is -3/2. By filling in the scale on the graph, the y-intercept is 3

8

Do not write outside the box

16 *ABC* and *ACD* are triangles.



The area of *ACD* is 80.5 cm^2

Work out the area of *ABC*.

Give your answer to 3 significant figures.

[4 marks]

$\frac{1}{2} \times 14 \times AC = 80.5$

Area of triangle = 1/2 x base x height. The base is 14 cm and the height is AC. Expressing the area of the triangle in terms of AC then setting this equal to the actual area of 80.5 cm^2

$AC = 11.5$

Dividing both sides by 1/2 x 14 finds that AC is 11/5 cm

$\frac{1}{2} \times 19 \times 11.5 \times \sin 36$

Area of triangle = 1/2 absinC, where a and b are two sides and C is the angle between them. Substituting 19 cm for a, 11.5 cm for b and 36 degrees for C

Answer 64.2 cm^2

64.21... is rounded to 3 significant figures

Do not write outside the box

17 $m = \dfrac{p - 2b}{2}$

p = 68.3 correct to 1 decimal place.

b = 8.7 correct to 1 decimal place.

Work out the lower bound for m.

[3 marks]

$68.3 - \frac{0.1}{2} = 68.25$ ← Subtracting half of the resolution from 68.3 works out that the lower bound for p is 68.25. The resolution is 0.1 as it is correct to 1 decimal place

$8.7 + \frac{0.1}{2}$ ← Adding half of the resolution to 8.7 works out that the upper bound for b is 8.75. The resolution is 0.1 as it is correct to 1 decimal place

$\frac{68.25 - 2 \times 8.75}{2}$ ← Substituting in the lower bound of p and the upper bound of b works out the lower bound for m. The upper bound of b is needed as subtracting more makes the value less

Answer 25.375

Turn over for the next question

7

Do not write outside the box

18 In a bag there are blue discs, green discs and white discs.

There are four times as many blue discs as green discs.

number of blue discs : number of white discs = 3 : 5

One disc is selected at random.

Work out the probability that the disc is either blue or white.

[3 marks]

B	G	W
4	1	
3		5
12	3	20

Writing the ratios of blue to green (which is 4 : 1 as there are are four times as many blue discs as green discs) and blue to white. Both ratios have blue in common. A common multiple of 4 and 3 is 12. Multiplying both sides of the first ratio by 3 gives 12 : 3. Multiplying both sides of the second ratio by 4 gives 12 : 20. These two ratios can now be combined as the number of parts representing blue is the same

$12+20=32$

Adding the 12 parts for blue and the 20 parts for white works out that there are 32 parts representing blue or white

$12+3+20$

Adding the 12 parts for blue, the 3 parts for green and the 20 parts for white works out that there are 35 parts in total

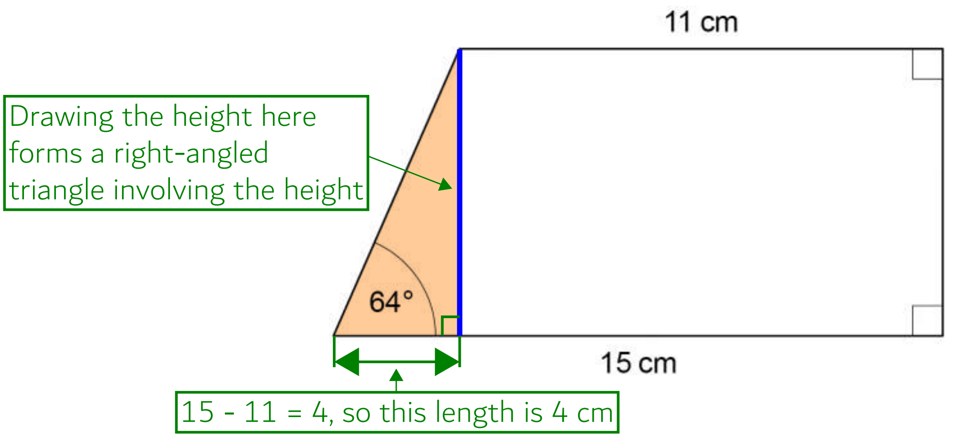
Answer $\frac{32}{35}$

32 out of the 35 parts are for blue or white

Do not write outside the box

19 Work out the area of the trapezium.

Not drawn accurately



[4 marks]

$\overset{\checkmark}{O}$ S H C $\overset{\checkmark}{A}$ H T $\overset{\checkmark}{O}$ $\overset{\checkmark}{A}$

Using right-angled trigonometry on the orange right-angled triangle to find the height of the trapezium. Ticking A as we have the adjacent and ticking O as we are looking for the opposite. There are two ticks on the TOA formula triangle so this one can be used

$\tan 64 \times 4$

Covering O in the TOA formula triangle finds that opposite = tan of the angle x adjacent. 64 degrees is the angle and 4 cm is the adjacent. So the height is 8.2... cm

$\frac{1}{2} \times (11 + 15) \times 8.2...$

Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them. Substituting 11 cm for a, 15 cm for b and 8.2... cm for h

Answer 106.6 cm^2

Turn over for the next question

7

Do not write outside the box

20 Expressions for consecutive triangular numbers are

$$\frac{n(n+1)}{2} \quad \text{and} \quad \frac{(n+1)(n+2)}{2}$$

Prove that the sum of two consecutive triangular numbers is always a square number.

[4 marks]

$$\frac{n^2+n+n^2+2n+n+2}{2}$$

The denominators of the fractions are the same so the numerators can be added and the denominator stays the same. Expanding the brackets

$$\frac{2n^2+4n+2}{2}$$

Collecting like terms on the numerator

$$n^2+2n+1$$

Dividing all the terms on the numerator by 2 to cancel out the denominator

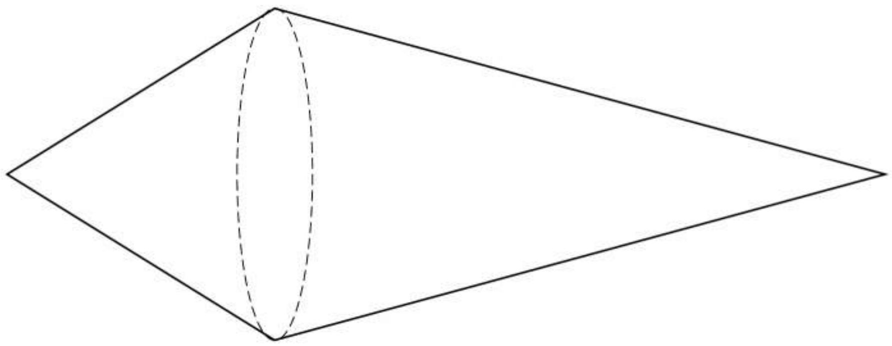
$$(n+1)^2$$

Factorising to express as a bracket squared (which is a square number). Two numbers which add to the 2 and multiply to the 1 are 1 and 1. Putting these in brackets with n but as both brackets are the same it can be squared instead

Do not write outside the box

21 A solid shape is made by joining two cones.

Each cone has the same radius.



One cone has slant height = 2 × radius

The other cone has slant height = 3 × radius

The total surface area of the shape is 57.8π cm^2

Curved surface area of a cone = $\pi r l$ where r is the radius and l is the slant height

Work out the radius.

[3 marks]

$\pi \times r \times 2r + \pi \times r \times 3r$ ← Expressing the total surface area of the shape in terms of the radius, r. The slant height of the one cone must be 2r and the slant height of the other cone must be 3r

$2\pi r^2 + 3\pi r^2$ ← Simplifying

$5\pi r^2 = 57.8\pi$ ← Collecting like terms and setting equal to the actual surface area of 57.8π cm^2

$r^2 = 11.56$ ← Dividing both sides by 5π to get r^2 on its own

Answer 3.4 cm

Square rooting both sides finds r

7

Turn over ▶

Do not write outside the box

22 Show that $(5\sqrt{3} - \sqrt{12})^2$ simplifies to an integer.

[3 marks]

$75 - 60 + 12 = 27$

Expanding the square bracket by squaring the first term, doubling the product of the two terms and squaring the last term.'
$(5\sqrt{3})^2 = 5 \times 5 \times \sqrt{3} \times \sqrt{3} = 25 \times 3 = 75$
$2 \times 5\sqrt{3} \times - 12 = -10\sqrt{36} = -10 \times 6 = -60$
$(-\sqrt{12})^2 = 12$
Then adding the results gives 27, which is an integer

23 A and B are similar cuboids.

surface area of A : surface area of B = 16 : 25

Work out volume of A : volume of B

Circle your answer.

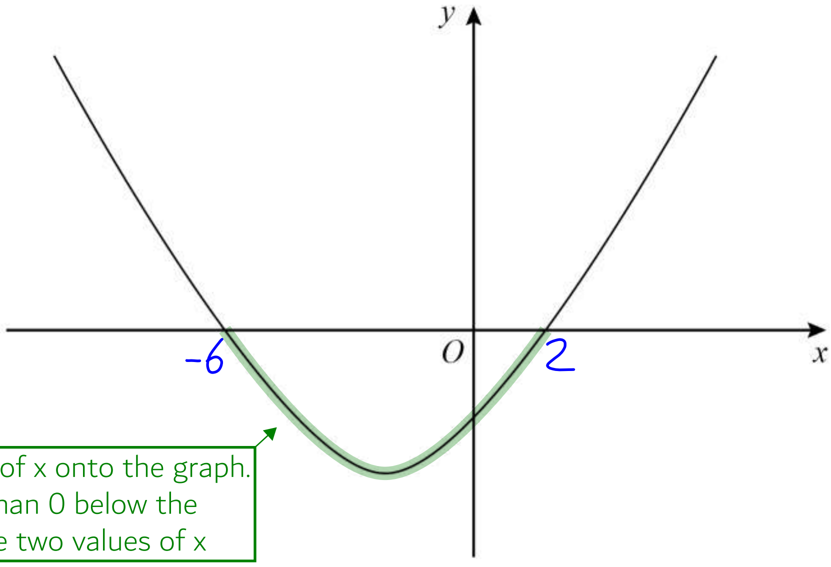
[1 mark]

4 : 5 16 : 25 64 : 125 256 : 625

Square rooting both sides of the ratio to get the ratio of the lengths gives 4 : 5.
Cubing both sides of the ratio gives 64 : 125, which is the ratio of the volumes

Do not write outside the box

24 Here is a sketch of the curve $y = x^2 + 4x - 12$



Marking the solutions of x onto the graph. The quadratic is less than 0 below the x-axis and between the two values of x

Work out the values of x for which $x^2 + 4x - 12 < 0$

Give your answer as an inequality.

[3 marks]

$(x+6)(x-2)=0$

Factorising to solve when the quadratic is equal to 0. Two numbers which multiply to get the -12 and add to get the 4 are 6 and -2. Putting these in brackets with x. Either x + 6 = 0 or x - 2 = 0. x = -6 or x = 2

Answer $-6<x<2$

To the right of -6 is greater than -6. To the left of 2 is less than 2. x cannot be equal to either -6 or 2

7

25 A sample of 50 eggs is taken from Farm A.

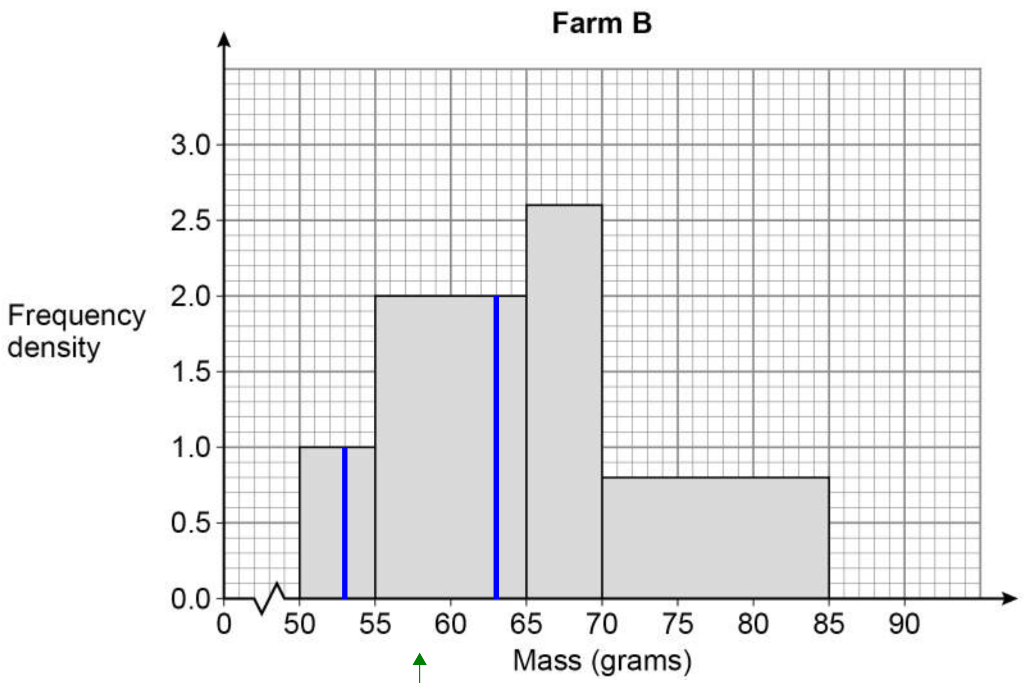
The table shows information about the masses of the eggs from Farm A.

Farm A

Mass, m (grams)	Frequency
$53 < m \leqslant 58$	8
$58 < m \leqslant 63$	19
$63 < m \leqslant 68$	15
$68 < m \leqslant 73$	8

A sample of 50 eggs is taken from Farm B.

The histogram shows information about the masses of the eggs from Farm B.



Splitting the bars at 53 g and 63 g to estimate how many eggs are 53 g < mass ⩽ 63 g

Do not write outside the box

For medium eggs, 53 g < mass ⩽ 63 g

The Farm A sample has more medium eggs than the Farm B sample.

Using the table and the histogram, estimate how many more.

You **must** show your working.

[4 marks]

$8+19=27$

From the table for Farm A, both the first two intervals have masses which are 53 g < mass ⩽ 63 g. Adding the frequencies for these finds that 27 eggs in the Farm A sample are medium eggs

$2\times1=2$

8×2

Frequency on a histogram is the area of each bar. Area of rectangle = base x height. The base of the bar 53 g < mass ⩽ 55 g is 2 and the height is 1 and multiplying these estimates the frequency is 2. The base of the bar 55 g < mass ⩽ 63 g is 8 and the height is 2 and multiplying these estimates the frequency of 16

$2+16$

Adding the frequencies of 2 and 16 works out that an estimated 18 eggs are medium in the Farm B sample

$27-18$

Subtracting the estimated 18 medium eggs in the Farm B sample from the 27 medium eggs in the Farm A sample estimates how many more medium eggs are in the Farm A sample

Answer 9

Turn over for the next question

4

Do not write outside the box

26 $(x + 5)(x + 2)(x + a) \equiv x^3 + bx^2 + cx - 30$

Work out the values of the integers a, b and c.

[3 marks]

$x^2 + 2x + 5x + 10$ ← Expanding the first two brackets on the left side

$(x^2 + 7x + 10)(x + a)$ ← Collecting like terms and writing multiplied by the third bracket

$x^3 + ax^2 + 7x^2 + 7ax + 10x + 10a$ ← Expanding the brackets

$x^3 + (7 + a)x^2 + (7a + 10)x + 10a$ ← Collecting like terms to put in the same form as the right side of the identity

$10a = -30$ ← Equating the constants on the left and right sides of the identity

$a =$ -3 ← Dividing both sides by 10 finds that a = -3

$b =$ 4 ← Equating coefficients of x^2 on the left and right sides of the identity. 7 + a = b so 7 - 3 = b

$c =$ -11 ← Equating coefficients of x on the left and right sides of the identity. 7a + 10 = c so 7(-3) + 10 = c

Do not write outside the box

27 $f(x) = \frac{2x}{5} - 1$

Work out the value of $f^{-1}(3) + f(-0.5)$

[5 marks]

$x = \frac{2y}{5} - 1$ ← Swapping f(x) with x and x with y then rearranging to make y the subject finds the inverse function $f^{-1}(x)$

$2y = 5x + 5$ ← Multiplying both sides by 5 to eliminate the denominator on the left

$y = \frac{5x+5}{2}$ ← Dividing both sides by 2 to get y on its own. The right side is the inverse function $f^{-1}(x)$

$\frac{5(3)+5}{2} + \frac{2(-0.5)}{5} - 1$ ← Substituting 3 for x in $f^{-1}(x)$ and -0.5 for x in f(x)

Answer 8.8

END OF QUESTIONS

8