

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

H

Higher Tier Paper 1 Non-Calculator

Thursday 25 May 2017

Morning

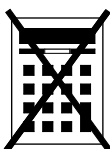
Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- mathematical instruments.

You must **not** use a calculator.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Advice

- In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

- 1 Simplify $2^5 \times 2^3$
Circle your answer.

[1 mark]

4^8

2^8

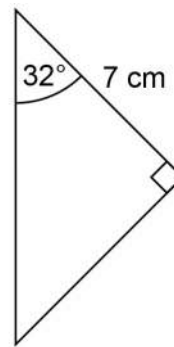
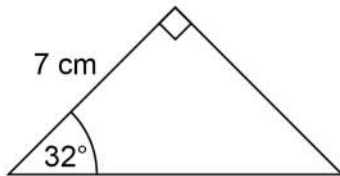
2^{15}

4^{15}

$$a^x \times a^y = a^{x+y}$$

$$2^5 \times 2^3 = 2^{5+3} = 2^8$$

- 2



Not drawn
accurately

Circle the reason why these triangles are congruent.

[1 mark]

SSS

SAS

ASA

RHS

Two angles and a side (which is opposite the same angle) are the same in both triangles. Angle-side-angle, or ASA, is the reason they are congruent (the same shape and size)

- 3 Which of these is a geometric progression?
Circle your answer.

[1 mark]

2, 4, 6, 8, 10

2, 3, 5, 8, 12

2, 6, 18, 54, 162

2, 6, 10, 14, 18

Geometric progression means that each term is multiplied by the same amount to get the next term. Each term in this sequence is multiplied by 3 to get the next term



4 $a : b = 4 : 3$

Circle the correct statement.

[1 mark]

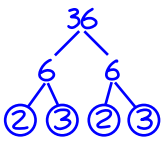
 b is $\frac{4}{7}$ of a b is $\frac{3}{7}$ of a b is $\frac{4}{3}$ of a b is $\frac{3}{4}$ of a

a could be 4 and b could be 3. This is the only statement which works with these numbers as 3 is $\frac{3}{4}$ of 4

5 Write 36 as a product of prime factors.

Give your answer in index form.

[3 marks]



Doing a factor tree for 36 by splitting each number into factors. Circling and stopping at the prime numbers (which are only divisible by themselves and 1)

Writing the prime numbers multiplied together gives it as a product of prime factors. $2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$ in index form

Answer _____ $2^2 \times 3^2$ _____

Turn over for the next question



- 6 The table shows information about the times for 10 people to complete a task.

Time, t (minutes)	Frequency		
$0 < t \leq 20$	1	0	20
$20 < t \leq 40$	6	120	240
$40 < t \leq 60$	3	120	180
		240	440

These statements are about the mean and range of the actual times.

Tick the correct box for each statement.

[4 marks]

	True	False	
The mean could be less than 20 minutes	<input type="checkbox"/>	<input checked="" type="checkbox"/>	A
The mean could be more than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>	B
The mean could be less than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>	C
The range could be more than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>	D
The range could be less than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>	E
The range could be more than 60 minutes	<input type="checkbox"/>	<input checked="" type="checkbox"/>	F

See next page for an explanation



A: Working out the lower bound of the mean. Mean = total \div number, where total is the total time and number is the number of people. Taking the lowest time in each interval and multiplying it by the frequency works out the lower bound of the total for each interval. $0 \times 1 = 0$. $20 \times 6 = 120$. $40 \times 3 = 120$. Adding all these totals gives an overall total of 240. Dividing this by the 10 people works out that the lower bound of the mean is 24. So the mean could not be less than 20 minutes.

B: Working out the upper bound of the mean. Mean = total \div number, where total is the total time and number is the number of people. Taking the greatest time in each interval and multiplying it by the frequency works out the upper bound of the total for each interval. $20 \times 1 = 20$. $40 \times 6 = 240$. $60 \times 3 = 180$. Adding these totals gives an overall total of 440. Dividing this by the 10 people works out that the upper bound of the mean is 44. So the mean could be more than 40 minutes.

C: The lower bound of the mean is 24 as calculated for A. So the mean could be less than 40 minutes.

D: The upper bound of the range is when the times are as spread out as possible. This happens when the lowest time is 0 and the greatest time is 60. $60 - 0 = 60$ so the upper bound of the range is 60. So the range could be more than 60 minutes.

E: The lower bound of the range is when the times are as close together as possible. This happens when the lowest time is 20 and the greatest time is 40. $40 - 20 = 20$ so the lower bound of the range is 20. So the range could be less than 40 minutes.

F: The upper bound of the range is 60 as calculated in D. So the range could not be more than 60 minutes

7 $\frac{3}{5}$ of a number is 162

Work out the number.

[2 marks]

$$\begin{array}{r} 054 \\ 3 \overline{)162} \end{array}$$

Dividing the 162 by 3 works out that $\frac{1}{3}$ of the the number is 54

$$\begin{array}{r} 54 \\ \times 5 \\ \hline 270 \end{array}$$

Multiplying the 54 by 5 works out that the whole number ($\frac{5}{3}$ of the number) is 270

Answer _____ 270 _____

8 x km/h = y mph

Use 8 km/h = 5 mph to write a formula for y in terms of x .

[2 marks]

x needs to be divided by 8 to work out how many lots of 8 km/h it is. Then multiplying by 5 as every lot of 8 km/h is a lot of 5 mph

Answer _____ $y = \frac{x}{8} \times 5$ _____

Turn over for the next question



9 (a) Density = $\frac{\text{mass}}{\text{volume}}$

The mass of solid A is 6 times the mass of solid B.

The volume of solid A is 3 times the volume of solid B.

Complete the sentence.

[1 mark]

The density of solid A is 2 times the density of solid B.

Multiplying the mass by 6 multiplies the density by 6. Multiplying the volume by 3 divides the density by 3 as it is the denominator. The overall effect is multiplying by 2

9 (b) Average speed = $\frac{\text{distance}}{\text{time}}$

If the distance is halved and the time is doubled, what happens to the average speed?

Circle your answer.

[1 mark]

$\times 2$

$\times 4$

no change

$\div 2$

$\div 4$

Halving the distance halves the speed. Doubling the time halves the speed as it is the denominator. The overall effect is dividing by 4



10 Solve the simultaneous equations.

$$2x + y = 18$$

Equation 1

$$x - y = 6$$

Equation 2

[3 marks]

$$3x = 24$$

Adding equation 2 to equation 1 cancels out the y terms and leaves an equation in terms of x. $2x + x = 3x$. $y + -y = 0$. $18 + 6 = 24$

Dividing both sides by 3 finds that $x = 8$

$$8 - y = 6$$

Substituting 8 for x in equation 2

Answer _____

$$x = 8$$

$$y = 2$$

2 must be subtracted from 8 to get 6, so $y = 2$

Turn over for the next question

Turn over ►



- 11 Billy wants to buy these tickets for a show.
4 adult tickets at £15 each
2 child tickets at £10 each

A 10% booking fee is added to the ticket price.
3% is then added for paying by credit card.

Work out the **total** charge for these tickets when paying by credit card.

[5 marks]

$$\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 60 \\ + 20 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \div 10 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 88 \div 100 \\ \hline \end{array}$$

$$\begin{array}{r} 0.88 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 2.64 \\ \hline \end{array}$$

$$\begin{array}{r} 88.00 \\ + 2.64 \\ \hline \end{array}$$

$$\begin{array}{r} 90.64 \\ \hline \end{array}$$

Multiplying the £15 by the 4 adult tickets works out that they cost £60.

Multiplying the £10 by the 2 child tickets works out that they cost £20

Adding the £60 and the £20 works out that the cost of the tickets is £80. Dividing the £80 by 10 works out that the 10% booking fee is £8

Adding the £8 to the £80 works out that the charge is now £88.

Dividing this £88 by 100 works out that 1% of £88 is £0.88

Multiplying £0.88 by 3 works out that the 3% for paying by credit card is £2.64

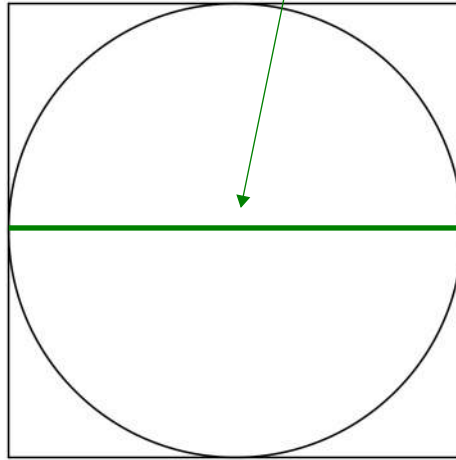
Adding this £2.64 to the £88 works out that the total charge is £90.64

Answer £ 90.64



12

Here is a circle touching a square.

This is the diameter
of the circleNot drawn
accuratelyThe area of the square is 64 cm^2

Work out the area of the circle.

Give your answer in terms of π .

[3 marks]

$\sqrt{64}$

Area of square = length². So length = $\sqrt{\text{area}}$. The length of the square is 8 cm

$8 \div 2$

The diameter of the circle is the same length as the length of the square. The radius is half of the diameter so dividing the 8 by 2 works out that the radius is 4 cm

$\pi \times 4^2$

Area of circle = $\pi \times \text{radius}^2$ Answer 16 π cm² $4^2 = 4 \times 4 = 16$. π is left in the answer as it asks for it in terms of π

Turn over for the next question

Turn over ►



13 Write the number six million five thousand two hundred in standard form.

[2 marks]

6005200

First writing it as an ordinary number. It is easier to start with the two hundred, then add the five thousand, then add the six million

Dividing it by ten 6 times gives a decimal which is at least 1 and less than 10. Dividing by ten can be worked out by moving the decimal point to the left. So the resulting decimal must be multiplied by 10^6 to keep it equal

Answer 6.0052 × 10⁶

14 Solve $-3x > 6$

[1 mark]

Dividing both sides by -3 gets x on its own. $6 \div 3 = 2$ so $6 \div -3 = -2$. When dividing by a negative the inequality sign needs to flip

Answer $x < -2$

15 $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$ are four fractions.

How many of these fractions convert to a recurring decimal?

Circle your answer.

[1 mark]

0 1 2 3 4

3

$$6 \overline{) 1.0000}$$

$$7 \overline{) 1.00000000}$$

$$8 \overline{) 1.0000}$$

$$9 \overline{) 1.000}$$

Converting all the fractions to decimals by dividing the numerators by the denominators. As soon as a remainder repeats it can be known that the decimal is recurring



- 17 A is the point (2, -5)
B is the point (4, -9)

17 (a) Show that the gradient of the straight line passing through A and B is -2

[2 marks]

$$\frac{-9 - (-5)}{4 - 2}$$

Gradient = (change in y)/(change in x)
Change in y = (y coordinate of B) - (y coordinate of A)
Change in x = (x coordinate of B) - (x coordinate of A)

$$\frac{-4}{2}$$

Subtracting a negative is a double negative so it becomes $-9 + 5 = -4$ and $4 - 2 = 2$

$$-2$$

$4/2 = 2$ so $-4/2 = -2$

17 (b) C is the point (-301, 601)

Does C lie on the straight line passing through A and B?

You **must** show your working.

[2 marks]

$$\frac{601 - (-5)}{-301 - 2}$$

Gradient = (change in y)/(change in x)
Change in y = (y coordinate of C) - (y coordinate of A)
Change in x = (x coordinate of C) - (x coordinate of A)

$$\frac{606}{-303}$$

Subtracting a negative is a double negative so it becomes $601 + 5 = 606$ and $-301 - 2 = -303$

$$-2$$

$606/303 = 2$ so $606/-303 = -2$. So the gradient of the straight line passing through A and C is -2

Answer Yes

The gradient of -2 through A and B continues through A and C and both involve point A so C must be on the straight line passing through A and B



18

Bottles of drink are for sale at three shops.
The normal price of a bottle is the same at each shop.

Let x be the normal price of a bottle

Shop A
Buy 1 bottle
Get 2 more bottles at half price

Shop B
Buy 2 bottles
Get 3 more bottles at half price

Shop C = $0.7x$
30% off a bottle

$100\% - 30\% = 70\%$,
which is 0.7 as a decimal

What is the cheapest way to buy **exactly** 8 bottles?

You can buy from more than one shop.

You **must** show your working.

[3 marks]

$x + \frac{1}{2}x + \frac{1}{2}x$ ← Expressing the price of using the offer from Shop A.
Collecting like terms finds that it is $2x$ for 3 bottles

$\begin{array}{r} 0.6 \\ 3 \overline{)2.0} \end{array}$ ← Dividing the $2x$ by the 3 bottles works out that
the average price per bottle is $0.6x$ from Shop A

$x + x + \frac{1}{2}x + \frac{1}{2}x + \frac{1}{2}x$ ← Expressing the price of using the offer from Shop B.
Collecting like terms finds that it is $3.5x$ for 5 bottles

$\begin{array}{r} 0.7 \\ 5 \overline{)3.5} \end{array}$ ← Dividing the $3.5x$ by the 5 bottles works out that
the average price per bottle is $0.7x$ from Shop B

The average price per bottle is the lowest from Shop A so as many bottles as possible should be bought from Shop A. They can be bought in lots of 3 and 2 lots of 3 go into 8 with a remainder of 2. As 2 lots of 3 is 6, there should be 6 bottles bought from Shop A. The remaining 2 bottles should be bought from Shop C as the average price per bottle is the same as Shop B but the offer from Shop B gets 5 bottles and this is too many

Answer 6 from A, 2 from C

7

Turn over ►

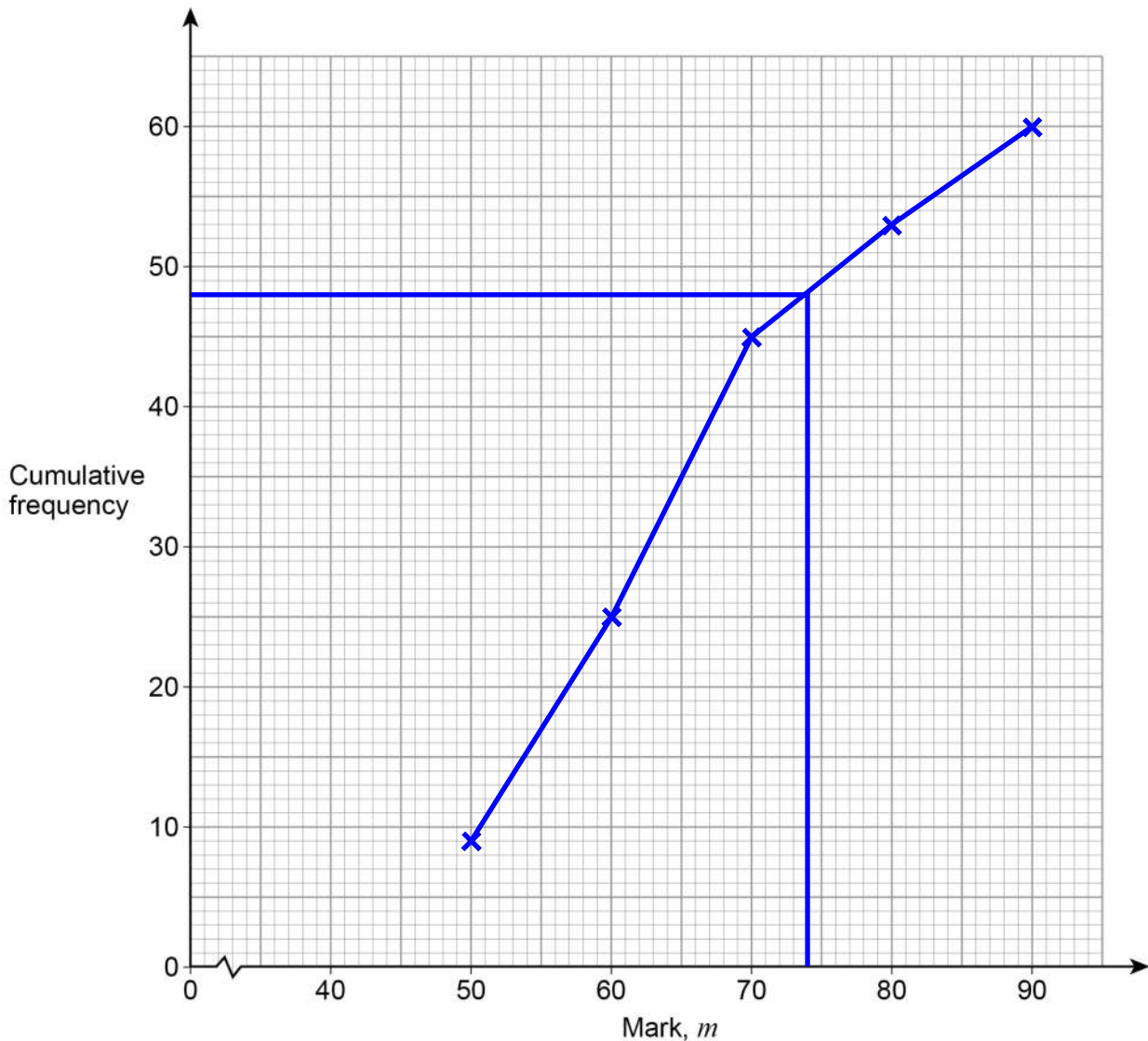


19 Here is some information about the marks of 60 students in a test.

Mark, m	Frequency	Cumulative Frequency
$40 < m \leq 50$	9	9
$50 < m \leq 60$	16	25 $\leftarrow 9 + 16$
$60 < m \leq 70$	20	45 $\leftarrow 25 + 20$
$70 < m \leq 80$	8	53 $\leftarrow 45 + 8$
$80 < m \leq 90$	7	60 $\leftarrow 53 + 7$

19 (a) On the grid, draw a cumulative frequency graph.

[3 marks]



The cumulative frequencies are plotted at the end points of each interval then joined with a series of straight lines



19 (b) Use your graph to estimate the lowest mark of the top 20% of students.

[2 marks]

$60 \div 10$

Dividing the 60 students by 10 finds that 10% of the students is 6

6×2

Multiplying the 6 by 2 finds that 20% of the students is 12

Answer _____ 74

Drawing across from 12 below 60 on the cumulative frequency axis to the line then down to the mark estimates the lowest mark of the top 12 students, which is the lowest mark of the top 20% of students

20 Work out the diameter of the circle $x^2 + y^2 = 64$

Circle your answer.

[1 mark]

8

16

32

128

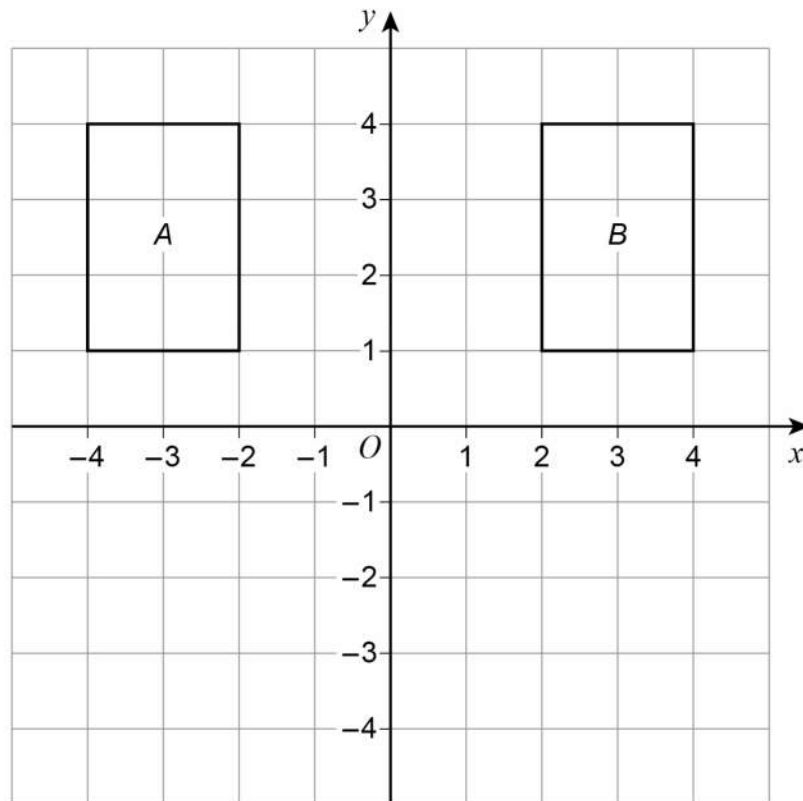
The equation of a circle with its centre at the origin is $x^2 + y^2 = r^2$, where r is the radius. $r^2 = 64$ so $r = 8$. Diameter is double the radius. $8 \times 2 = 16$

Turn over for the next question

Turn over ►



21 (a) The diagram shows rectangles A and B.



Rectangle A can be mapped to rectangle B by a **single** transformation.

Javed says,

“The **only** single transformation is a reflection in the y -axis because the rectangles are on opposite sides of the y -axis.”

Is he correct?

Tick a box.

Yes

No

Give a reason for your answer.

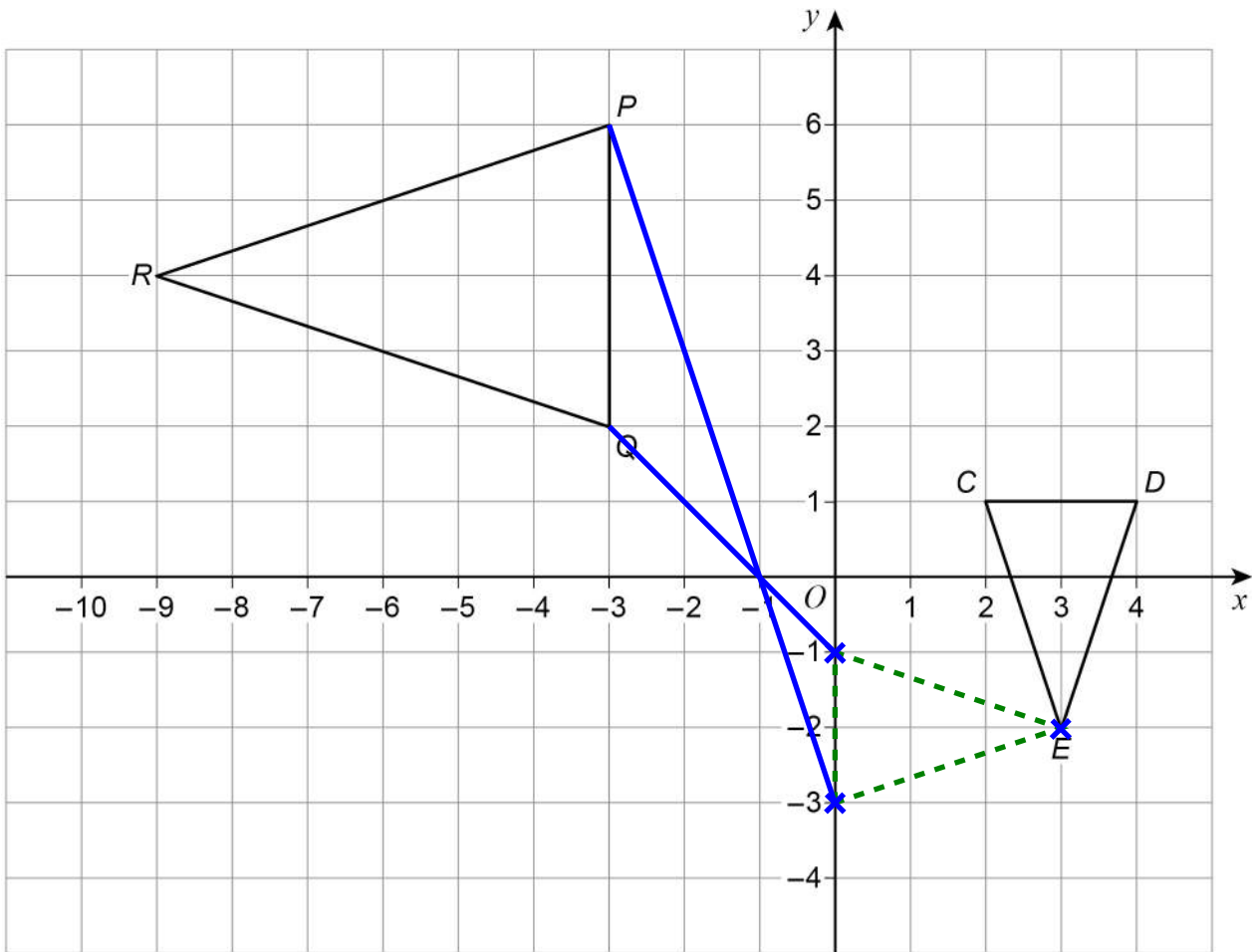
[1 mark]

It can be translated ←

Moving A 6 to the right translates it to B. So Javed is not correct as there are other single transformations



21 (b) This diagram shows triangles CDE and PQR .



CDE is mapped to PQR by combining two single transformations.

The first is a rotation of 90° anticlockwise about E .

Describe fully the second transformation.

[3 marks]

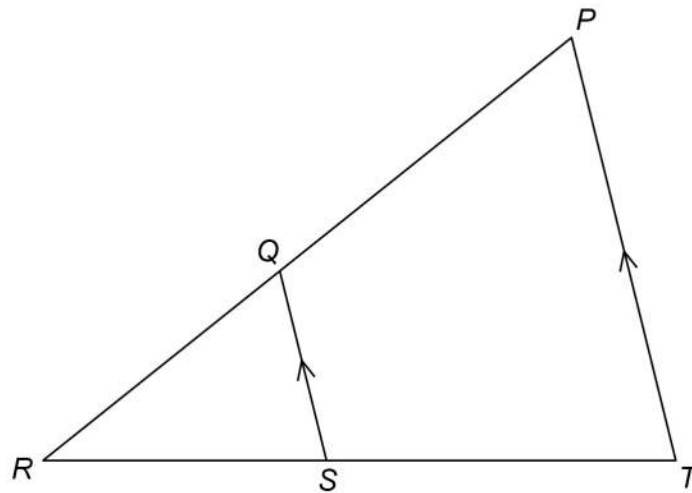
Enlargement, scale factor -2 , centre $(-1, 0)$

First rotating CDE 90° anticlockwise about E . Using tracing paper could help. The second transformation must be an enlargement as it has changed size. The scale factor is -2 as PQR is 2 times the size but has inverted. The centre of enlargement is found by drawing straight lines from the corners on the rotated triangle to the same corners on PQR and seeing where they meet

Turn over for the next question



22

 PRT and QRS are similar triangles.Not drawn
accuratelyWhich of these is equivalent to $\frac{QR}{PR}$?

Circle your answer.

[1 mark]

$$\frac{RS}{ST}$$

$$\frac{QS}{PT}$$

$$\frac{PT}{QS}$$

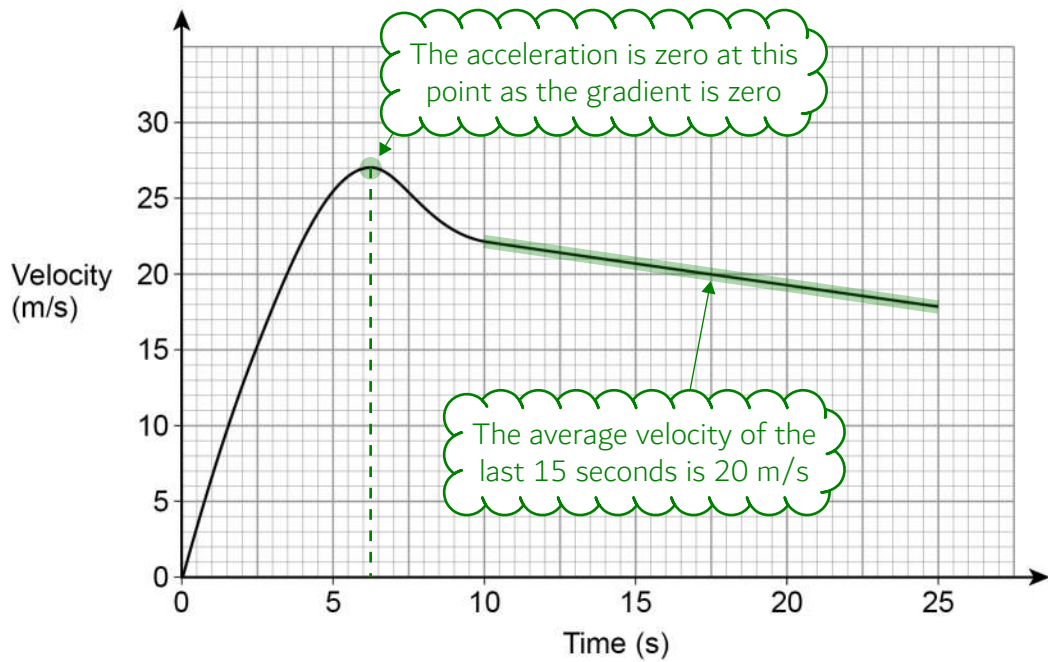
$$\frac{RT}{RS}$$

Triangle QRS is a smaller version of triangle PRT . They are the same shape but different size.
 QR/PR expresses the scale factor from triangle PRT to QRS as QR is the shorter version of PR .
 QS/PT also expresses the scale factor from triangle PRT to QRS as QS is the shorter version of PT .



23

Here is a velocity-time graph of a motorbike for 25 seconds.



23 (a) After how many seconds was the acceleration zero?

[1 mark]

Answer 6.25 seconds

The time scale goes up 5 over 10 small boxes. $5 \div 10 = 0.5$, so each small box is worth 0.5, 2 small boxes is worth 1 and half a small box is worth 0.25

23 (b) Work out the distance travelled in the last 15 seconds.

[2 marks]

 s^d_t

Writing the distance, speed, time formula triangle

 20×15 Covering d finds that distance = speed \times time. Using the average speed of 20 m/s and the time of 15 secondsAnswer 300 metres

$2 \times 15 = 30$, so $20 \times 15 = 300$

Turn over ►



24 (a) Work out $\sqrt{12\frac{1}{4}}$ as an improper fraction.

[1 mark]

$$\sqrt{\frac{49}{4}}$$

Converting the mixed number into an improper fraction by multiplying the 12 by the 4 to give 48 then adding this to the 1 to give 49. The denominator stays the same

Answer _____ $\frac{7}{2}$

A fraction can be square rooted by square rooting the numerator and square rooting the denominator. $\sqrt{49} = 7$ and $\sqrt{4} = 2$

24 (b) Work out $\sqrt[3]{16}$ as a power of 2

[2 marks]

2, 4, 8, 16

Going through the whole number powers of 2 by starting with 2 and keep multiplying by 2. 16 is the 4th in the list so $16 = 2^4$

Answer _____ $2^{\frac{4}{3}}$

The cube root can be written to the power of $1/3$. $(2^4)^{1/3} = 2^{4 \times 1/3} = 2^{4/3}$



25

In an office there are twice as many females as males. 2:1

Writing the ratio of
females to males

$\frac{1}{4}$ of the females wear glasses.

$\frac{3}{8}$ of the males wear glasses.

84 people in the office wear glasses.

Work out the number of people in the office.

[4 marks]

$$\frac{1}{4} \times \frac{2}{3}x + \frac{3}{8} \times \frac{1}{3}x$$

Let x be the total number of people in the office. $1 + 2 = 3$ so there are 3 parts in total in the ratio. 2 out of these 3 parts represent the females so $\frac{2}{3}$ of x is the number of females. $\frac{1}{4}$ of these females wear glasses. 1 out of these 3 parts represent the males so $\frac{1}{3}$ of x is the number of males. $\frac{3}{8}$ of these males wear glasses. Of means to multiply. Adding the number of females who wear glasses and the number of males who wear glasses expresses the number of people who wear glasses in terms of x

$$\frac{2}{12}x + \frac{3}{24}x$$

Multiplying the fractions by multiplying the numerators and multiplying the denominators

$$\frac{7}{24}x = 84$$

Adding the fractions by multiplying the numerator and denominator of $\frac{2}{12}$ by 2 to give $\frac{4}{24}$. Now the denominators are the same it can be added to the $\frac{3}{24}$ to give $\frac{7}{24}$. The simplified expression of the number of people who wear glasses must be equal to the 84 people in the office who wear glasses

$$x = 84 \times \frac{24}{7}$$

Dividing both sides by $\frac{7}{24}$ gets x on its own. To divide by a fraction: keep the first number, change the division to a multiply, flip the second fraction

$$\begin{array}{r} 12 \\ \times 24 \\ \hline 48 \\ 240 \\ \hline 288 \end{array}$$

To do a fraction multiplied by an amount: divide the amount by the denominator then multiply the result by the numerator. $84 \div 7 = 12$. Then multiplying this by the 24

Answer 288

$x = 288$ and x was the total number of people in the office

Turn over for the next question

Turn over ►



26

Expand and simplify $(x - 4)(2x + 3y)^2$ **[4 marks]**

$$(x-4)(4x^2+12xy+9y^2)$$

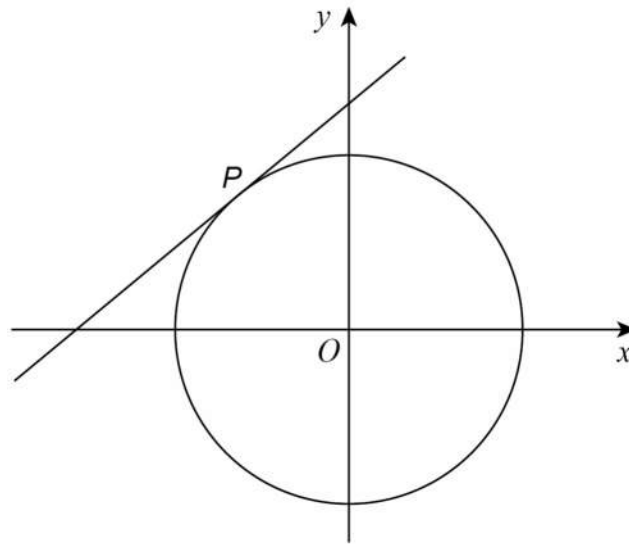
Expanded the square bracket by squaring the first term,
doubling the product of the two terms, squaring the last term

Answer $4x^3+12x^2y+9xy^2-16x^2-48xy-36y^2$

Expanded the brackets by first doing x multiplied by everything in the bracket then doing
-4 multiplied by everything in the bracket. There are no like terms so it cannot be simplified



27

 $P(-1, 4)$ is a point on a circle, centre O Not drawn
accuratelyWork out the equation of the tangent to the circle at P .Give your answer in the form $y = mx + c$ m is the gradient

[4 marks]

$$\frac{0-4}{0-1}$$

Expressing the gradient of the radius OP .Gradient = (change in y)/(change in x).Change in y = (y-coordinate of O) - (y-coordinate of P).Change in x = (x-coordinate of O) - (x-coordinate of P)

$$\frac{-4}{1}$$

 $0 - 4 = -4$. Subtracting a negative is a double negative so $0 - -1$ becomes $0 + 1 = 1$ The tangent is perpendicular to the radius so its gradient must be the negative reciprocal of $-4/1$, which is $1/4$

$$4 = \frac{1}{4}x - 1 + c$$

Substituting the coordinates of P for x and y and $1/4$ for m in the equation $y = mx + c$
 $1/4 x - 1 = -1/4$. Adding $1/4$ to both sides finds that $c = 4^{1/4}$.
 Substituting $1/4$ for m and $4^{1/4}$ for c in the equation $y = mx + c$
Answer $y = \frac{1}{4}x + 4\frac{1}{4}$

Turn over ►



29 Simplify $\frac{2 \sin 45^\circ - \tan 45^\circ}{4 \tan 60^\circ}$

Give your answer in the form $\frac{\sqrt{a} - \sqrt{b}}{c}$ where a , b and c are integers.

[4 marks]

0 30 45 60 90
0 1 2 3 4
4 3 2 1 0

Writing the angles of 0, 30, 45, 60, 90 degrees. Writing 0, 1, 2, 3, 4 under these for the sin values and 4, 3, 2, 1, 0 under these for the cos values. Square rooting them and putting them over 2 finds that $\sin 45 = \sqrt{2}/2$, $\cos 45 = \sqrt{2}/2$, $\sin 60 = \sqrt{3}/2$, $\cos 60 = 1/2$

$\frac{\sqrt{3}}{2} \div \frac{1}{2}$

Dividing $\sin 60$ by $\cos 60$ finds that $\tan 60 = \sqrt{3}$

$\frac{\sqrt{2}-1}{4\sqrt{3}}$

Dividing $\sin 45$ by $\cos 45$ finds that $\tan 45 = 1$.
Substituting in all the trig values. $2(\sqrt{2}/2) = \sqrt{2}$

Rationalising the denominator by multiplying both the numerator and denominator by $\sqrt{3}$

$\frac{\sqrt{6}-\sqrt{3}}{12}$

Answer _____

END OF QUESTIONS

