AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

GCSE
MATHEMATICS

H

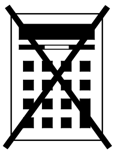
Higher Tier   Paper 1 Non-Calculator

Tuesday 21 May 2019   Morning   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- mathematical instruments

You must **not** use a calculator.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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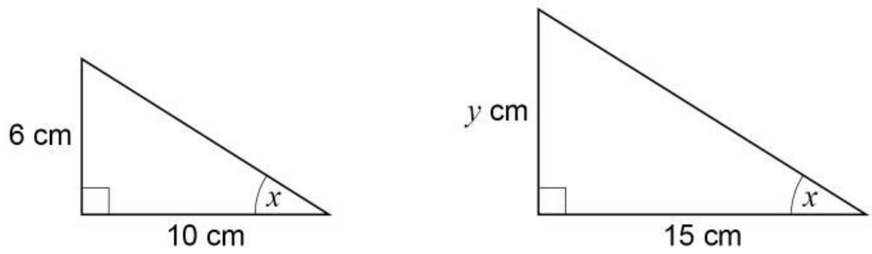
Answer **all** questions in the spaces provided

1 Here are two right-angled triangles.

Not drawn accurately



Circle the value of y.

[1 mark]

11   7.5   9   4

The triangles are similar as all the angles in both triangles are the same. The 15 cm is the bigger version of the 10 cm as they are both opposite the missing angle so dividing these gives the scale factor. 15 ÷ 10 = 1.5, which is the scale factor. y is the bigger version of the 6 cm so multiplying the 6 cm by the scale factor gives y. 6 x 1.5 = 9

2 Work out the value of $\left(1\frac{2}{3}\right)^2$

Circle your answer.

[1 mark]

$1\frac{4}{9}$ $3\frac{1}{3}$ $2\frac{4}{9}$ $2\frac{7}{9}$

$\left(\frac{5}{3}\right)^2$ ← Converted the mixed number into an improper fraction by multiplying the 1 by the 3 and adding the result to the 2

$\frac{25}{9}$ ← Squaring the fraction by squaring the numerator and squaring the denominator

Dividing the 25 by the 9 gives 2 as the whole number with a remainder of 7 which is left in the fraction

3 Work out the arc length, in metres, of a semicircle of radius 6 metres.

Circle your answer.

[1 mark]

3π 6π 12π 18π

The arc on a semicircle is half of the circumference of the full circle. Circumference = π x diameter. The diameter is double the radius. 6 x 2 = 12, which is the diameter. Then 12 x π = 12π, which is the circumference. Dividing this by 2 gives 6π, which is the arc length

Do not write outside the box

4 Circle the fraction that is equivalent to 4.625

[1 mark]

$\frac{39}{8}$ $\frac{37}{8}$ $\frac{185}{4}$ $\frac{17}{4}$

$8 \overline{)3\,9.{}^{7}0}$ = 4.8

39/8 is 4.8... so can't be 4.625

Converting the fractions into decimals by dividing the numerators by the denominators

$8 \overline{)3\,7.{}^{5}0{}^{2}0{}^{4}0}$ = 4.625

37/8 converts into 4.625

5 (a) Write 0.000 97 in standard form.

[1 mark]

Answer 9.7×10^{-4}

It is multiplied by 10 4 times to get 9.7, a number at least 1 and less than 10. So it must be multiplied by 10^{-4} (which basically means divide by 10 4 times) to keep it equal

5 (b) Work out $\dfrac{3 \times 10^5}{4 \times 10^3}$

Give your answer as an ordinary number.

[2 marks]

0.75×10^2

3/4 = 0.75
Using $a^x/a^y = a^{x-y}$
$10^5/10^3 = 10^{5-3} = 10^2$

Answer 75

0.75 multiplied by 10 twice gives 75

7

Turn over ▶

6 Anna plays a game with an ordinary, fair dice.

If she rolls 1 she wins.

If she rolls 2 or 3 she loses.

If she rolls 4, 5 or 6 she rolls again.

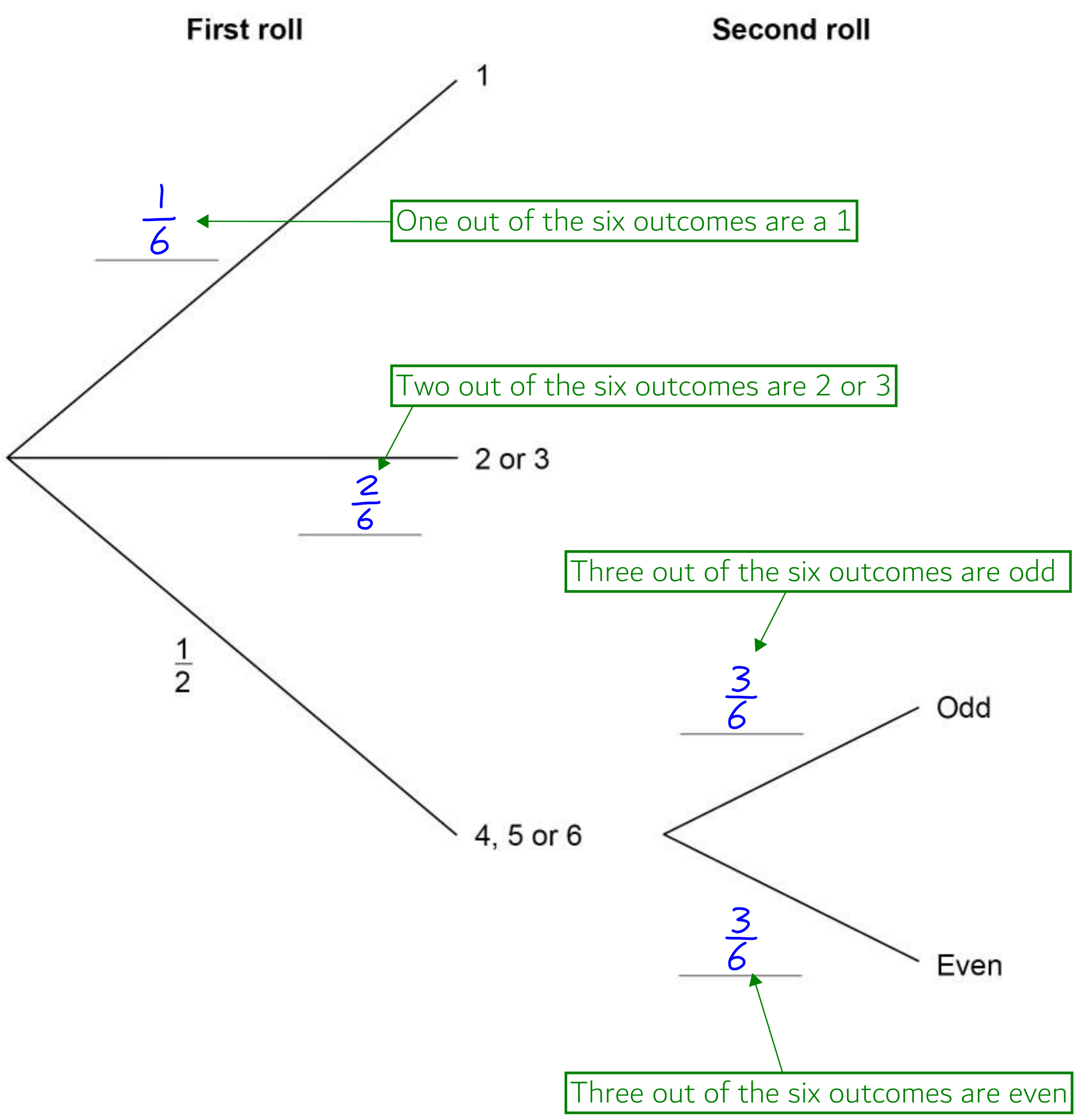
When she has to roll again,

if she rolls an odd number she wins

if she rolls an even number she loses.

6 (a) Complete the tree diagram with the four missing probabilities.

[2 marks]



Do not write outside the box

6 (b) Is Anna more likely to win or to lose?

You **must** work out the probability that she wins.

[4 marks]

$\frac{1}{6} + \frac{1}{2} \times \frac{3}{6}$

AND means to multiply, OR means to add. To win, roll a 1 OR roll 4, 5, 6 AND odd. Substituting in the probabilities from the tree diagram gives this

$\frac{2}{12} + \frac{3}{12} = \frac{5}{12}$

1/2 x 3/6 = 3/12 as the numerators are multiplied and the denominators are multiplied. 1/6 is converted into 2/12 by multiplying the numerator and denominator by 2 to make the denominator the same as the 3/12 so they can be added to get 5/12

Lose

The only outcomes for the game are win or lose. 5/12 is less than half as half of 12 is 6 and 5 is less than this. Therefore the probability of losing must be more than half so will be more likely

Turn over for the next question

6

Do not write outside the box

7 Three friends arrive at a party.

Their arrival increases the number of people at the party by 20%

In total, how many people are now at the party?

[2 marks]

3×6

20% of the people is 3. As it has increased by 20%, the number of people is now at 120%. Multiplying 20% by 6 gives 120% so 3 is also multiplied by 6 to find out how many people there are

Answer 18

8 Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

[3 marks]

$3^7 \div 3^3 = 3^4$

$a^x / a^y = a^{x-y}$

$a^x \times a^y = a^{x+y}$

$3^{12} / 3^5 = 3^{12-5} = 3^7$

$3^2 \times 3^1 = 3^{2+1} = 3^3$

$3^7 / 3^3 = 3^{7-3} = 3^4$

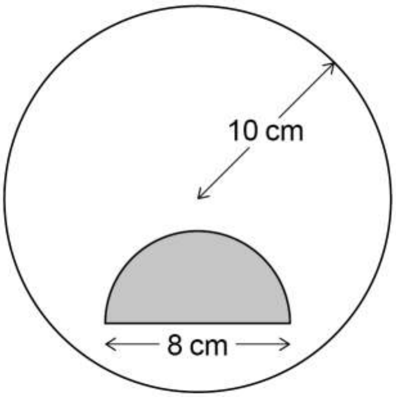
Answer 81

$3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$

Do not write outside the box

9 A shaded semicircle is inside a circle as shown.

Not drawn accurately



The **radius** of the circle is 10 cm

The **diameter** of the semicircle is 8 cm

How many times bigger is the unshaded area than the shaded area?

[4 marks]

$\frac{1}{2} \times \pi \times 4^2 = 8\pi$

Area of circle = π x radius². 8 cm is the diameter of the semicircle so halving this gets the radius of 4 cm. As it is a semicircle, finding half of the area of the full circle finds its area. So the area of the shaded semicircle is 8π cm²

$\pi \times 10^2 = 100\pi$

Area of circle = π x radius². 10 cm is the radius of the circle. So the area of the circle is 100π cm²

$100\pi - 8\pi = 92\pi$

Subtracting the area of the shaded semicircle from the area of the circle works out that the unshaded area is 92π cm²

$\frac{92\pi}{8\pi} = \frac{92}{8}$

Dividing the unshaded area by the shaded area works out how many times larger it is. π cancels out from the numerator and denominator

$8 \overline{)9{}^{1}2.{}^{4}0}$ = 11.5

Using short division to convert 92/8 to a decimal

Answer 11.5

Turn over for the next question

9

Do not write outside the box

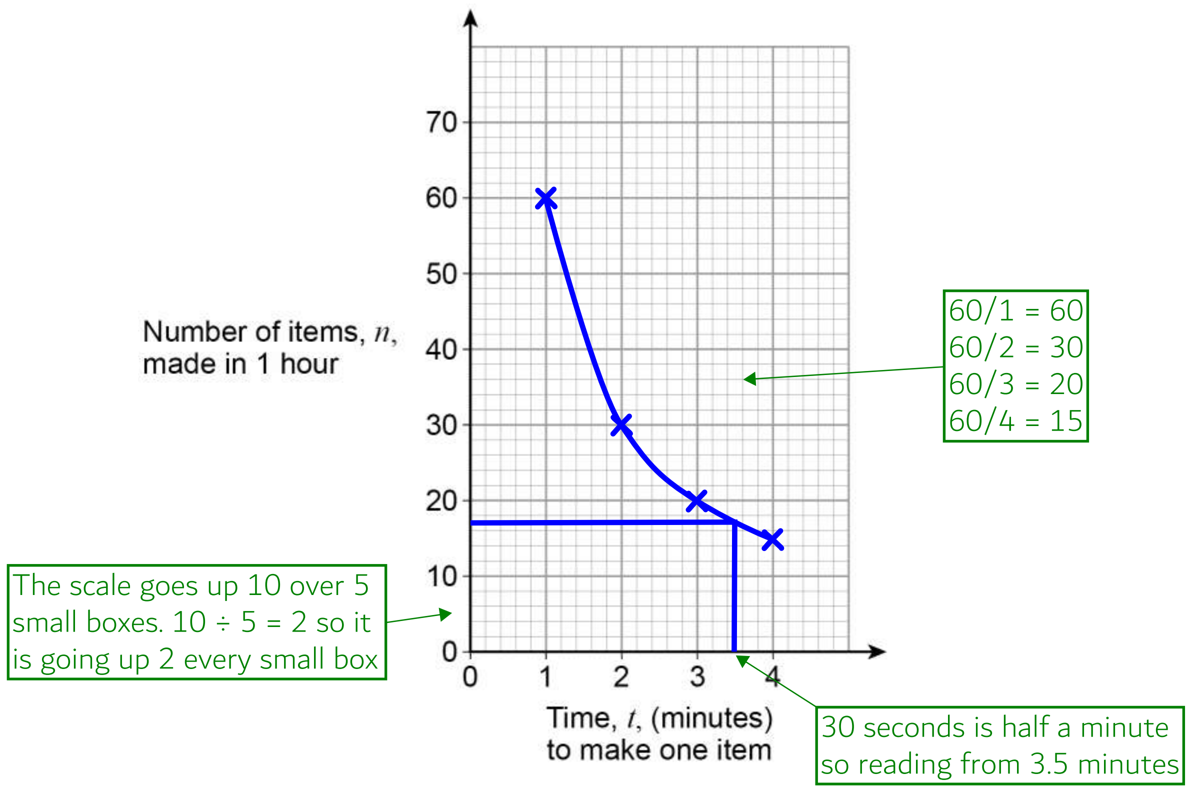
10 The number of items, n, made in 1 hour by a machine is given by $n = \dfrac{60}{t}$

t is the time in minutes the machine takes to make one item.

The value of t changes for different types of item.

10 (a) On the grid below, draw the graph of $n = \dfrac{60}{t}$ for values of t from 1 to 4

[2 marks]



10 (b) The machine takes 3 minutes 30 seconds to make one item.

Use your graph to estimate the value of n.

[2 marks]

Answer 17

Do not write outside the box

11 Ed and Fay shared £330 in the ratio 7 : 4

Ed gives Fay some of his money.

Fay now has the same amount as Ed.

How much does Ed give Fay?

[3 marks]

$11 \overline{)330}$ = 30

7 + 4 = 11 so there are 11 parts in total which represent the total of £330. Dividing the £330 by 11 works out that 1 part of the ratio is worth £30

$30 \times 7 = 210$

Multiplying the value of 1 part of the ratio by 7 works out that Ed had £210

$30 \times 4 = 120$

Multiplying the value of 1 part of the ratio by 4 works out that Fay had £120

$210 - 120 = 90$

Subtracting Fay's money from Ed's money works out that the difference between them was £90

$2 \overline{)90}$ = 45

Diving the difference by 2 halves it and therefore works out that Ed gave Fay £45

Answer £ 45

12 The next term of a sequence is made by adding the previous two terms.

Which of these sequences follows this rule?

Circle your answer.

[1 mark]

–9 2 –7 –5 –12

–3 5 –2 3 1

0 –3 –3 0 –3

–1 –1 –2 –3 1

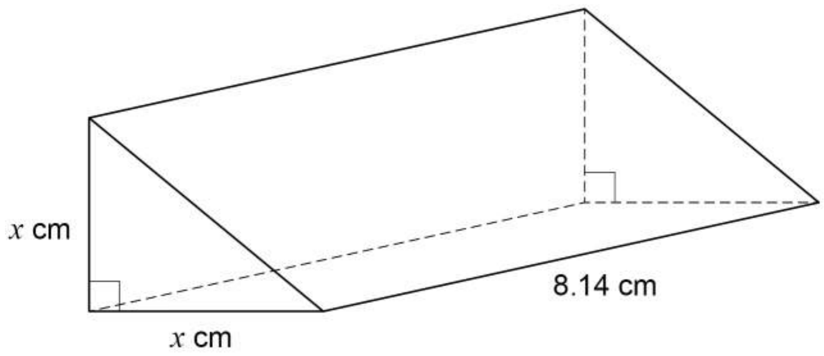
-9 + 2 = -7
2 + -7 = -5
-7 + -5 = -12

8

Turn over ►

Do not write outside the box

13 The triangular cross section of a prism is an isosceles right-angled triangle.



The volume of the prism is 102 cm^3

Use approximations to estimate the value of x.

You **must** show your working.

[3 marks]

$\frac{1}{2} \times x \times x \times 8$ ← This is an expression of the volume of the prism in terms of x. Volume of prism = cross sectional area x length. The cross section is a triangle. Area of triangle = 1/2 x base x height. Both the base and height are x. The length is approximately 8 cm

$4x^2 = 100$ ← Simplifying the expression and setting equal to 100 as this is approximately the volume

$x^2 = 25$ ← Dividing both sides by 4 to get the x^2 on its own

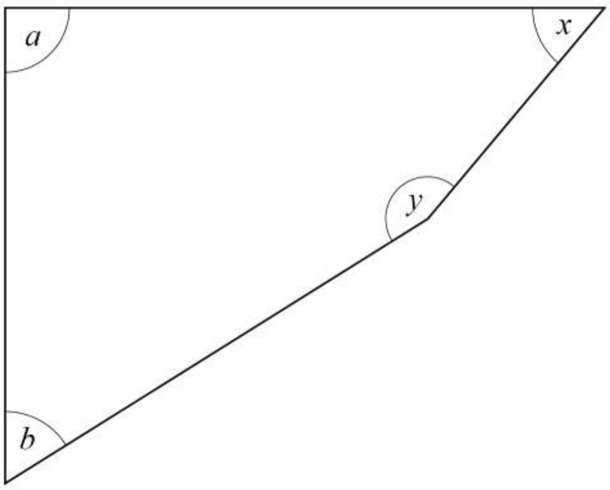
Answer 5

Doing the square root of both sides finds an estimate of x

Do not write outside the box

14 Here is a quadrilateral.

Not drawn accurately



$a = 90°$ and $a : b = 5 : 3$

$x : y = 1 : 3$

Show that $b = x$

[3 marks]

$5 \overline{)90}$ = 18

5 parts of the first ratio represents a, which is 90 degrees. Dividing 90 degrees by the 5 parts works out that 1 part of the ratio is 18 degrees

$18 \times 3 = 54$

Multiplying the value of 1 part of the first ratio by 3 works out that b is 54 degrees

$360 - 90 = 270$

$270 - 54 = 216$

Angles in a quadrilateral add up to 360 degrees so subtracting both angle a and b from 360 degrees works out that the total of angles x and y must be 216 degrees

$4 \overline{)216}$ = 54

3 + 1 = 4 so there are 4 parts in total in the second ratio which represent the total of angles x and y. Dividing the 216 degrees by the 4 parts works out that the value of 1 part of the second ratio is 54 degrees. x is 1 part so x must be 54 degrees

$b = 54$, $x = 54$

Showing that both angles b and x are 54 degrees shows that they are equal

6

Turn over ▶

Do not write outside the box

15 Here is some information about the test marks of 120 students.

Mark, m	$0 < m \leqslant 10$	$10 < m \leqslant 20$	$20 < m \leqslant 30$	$30 < m \leqslant 40$	$40 < m \leqslant 50$
Frequency	20	28	40	20	12

15 (a) Complete the cumulative frequency table.

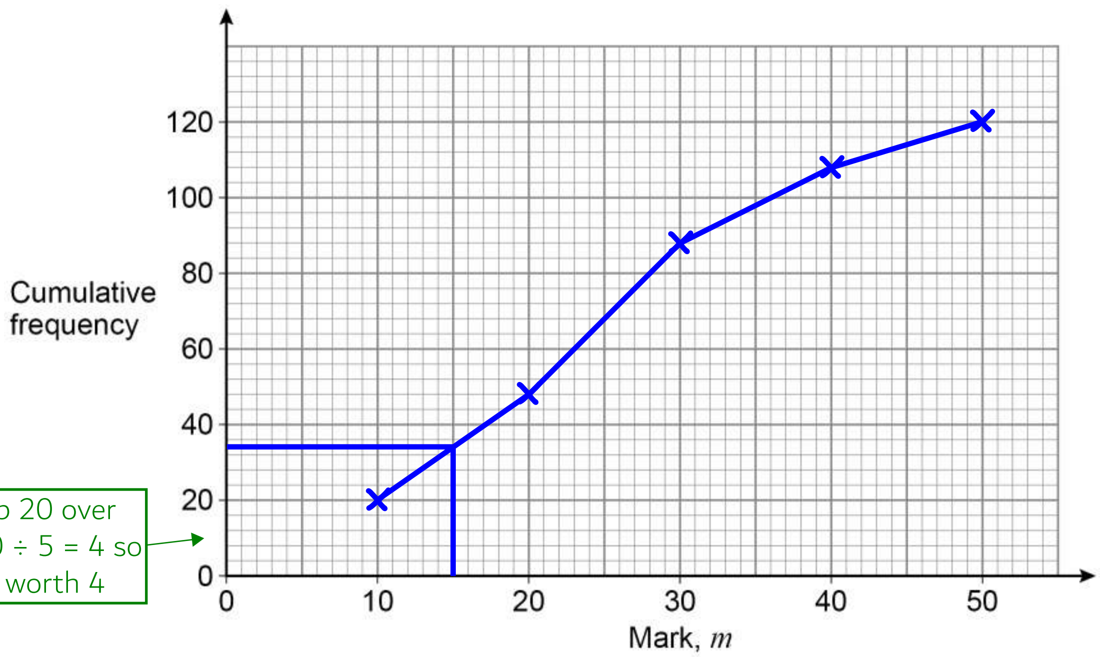
[1 mark]

Mark, m	$m \leqslant 10$	$m \leqslant 20$	$m \leqslant 30$	$m \leqslant 40$	$m \leqslant 50$
Cumulative frequency	20	48	88	108	120

Cumulative frequency means to add up the frequencies as they go. 20 + 28 = 48. 48 + 40 = 88. 88 + 20 = 108. 108 + 12 = 120

15 (b) Draw a cumulative frequency graph.

[2 marks]



The scale goes up 20 over 5 small boxes. 20 ÷ 5 = 4 so each small box is worth 4

Plotting the cumulative frequencies at the end point of each interval. Then joining them up with a series of straight lines

Do not write outside the box

15 (c) Students who scored 15 marks or fewer take another test.

Use your graph to estimate how many students take another test.

[2 marks]

Answer 34

Reading up from 15 marks to the line then across to the cumulative frequency

16 Simplify fully $\dfrac{4x - 8x^2}{12x - 6}$

[3 marks]

$\dfrac{-4x(2x-1)}{6(2x-1)}$

Factorising the numerator and denominator. The -8x² is the highest order of x on the numerator so has priority: it is negative so a negative factor is taken out

Answer $\dfrac{-2x}{3}$

(2x - 1) cancels out from the numerator and denominator to give -4x/6. Then the numerator and denominator can be divided by 2

Turn over for the next question

8

Do not write outside the box

17 Toby is forming and solving equations.

17 (a)

The product of half of a number and three more than the number
is the same as
the square of the number

Toby uses y to represent the number.

Write an equation that Toby could form.

[2 marks]

Product means multiplied together

'The same as' means equals

Answer $\frac{1}{2}y(y+3) = y^2$

Half of a number

3 more than the number

The square of the number

17 (b) Toby forms another equation.

$$x = \frac{9}{8x}$$

He wants to work out the values of x.

Here is his working.

$$x = \frac{9}{8x}$$

$$8x^2 = 9$$

$$8x = 3 \text{ or } 8x = -3$$

$$x = \frac{3}{8} \text{ or } x = -\frac{3}{8}$$

This line is incorrect as if square rooting the left, the 8 also needs to be square rooted

What error has he made in his working?

[1 mark]

Did not square root the 8

The third line should be $\sqrt{8}x = 3$ or $\sqrt{8}x = -3$. Alternatively he could have divided both sides by 8 before square rooting

Do not write outside the box

18 Here is an identity.

$$x^2 - y^2 \equiv (x + y)(x - y)$$

18 (a) Use the identity to work out the value of $193^2 - 7^2$

You **must** show your working.

[2 marks]

$(193+7)(193-7)$

x is 193 and y is 7

$$\begin{array}{r} 186 \\ \times 200 \\ \hline 37200 \end{array}$$

193 + 7 = 200
193 - 7 = 186

Answer 37200

18 (b) Factorise $100a^2 - 81b^2$

[1 mark]

Answer $(10a + 9b)(10a - 9b)$

$x^2 = 100a^2$
$x = 10a$
$y^2 = 81b^2$
$y = 9b$

19 Circle the fraction that is equivalent to $0.\dot{1}$

[1 mark]

$\frac{1}{9}$ $\frac{1}{99}$ $\frac{1}{10}$ $\frac{11}{100}$

It can't be this one as it is nearly 1/100, which equals to 0.01

It can't be this one as 1/10 = 0.1

It can't be this one as 11/100 = 0.11

Alternatively the numerators could be divided by the denominators to convert them into decimals

7

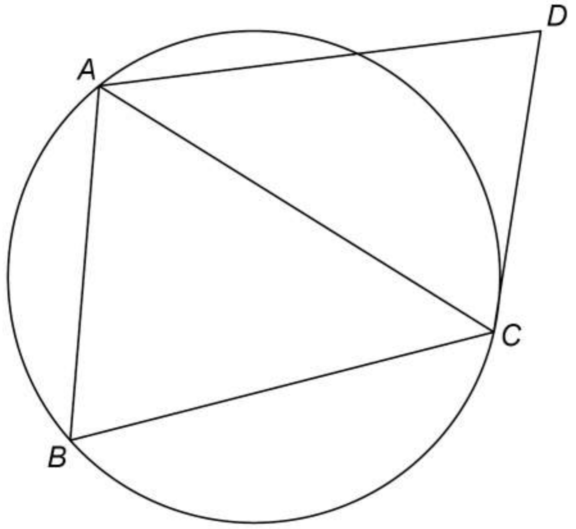
Turn over ▶

Do not write outside the box

20 *A*, *B* and *C* are points on a circle.

CD is a tangent.

Not drawn accurately



20 (a) Assume that triangle *ABC* is isosceles with *AC* = *BC*

Prove that *AB* is parallel to *DC*.

[4 marks]

Angles ABC = BAC as the base angles of isosceles triangles are equal

Angles ABC = ACD due to the alternate segment theorem

The angle between a tangent and a chord is equal to the interior opposite angle

Angles BAC = ACD so there are alternate angles

Therefore AB is parallel to DC

Do not write outside the box

20 (b) In fact, triangle *ABC* is equilateral.

Tick the **two** boxes for the statements that **must** be correct.

[1 mark]

- [x] *AB* is parallel to *DC*

This was proven in (a) as the angles in the triangle will still be equal and the alternate segment theorem will still apply

- [x] *AC* bisects angle *BCD*

Both halves of BCD (BCA and ACD) are both equal as all the angles in an equilateral triangle are equal and the alternate segment theorem

- [ ] *AC* bisects angle *BAD*

CD could have any length so angle CAD could be anything. It doesn't have to be the same as BAC so it doesn't have to be true that AC bisects BAD

21 Solve the simultaneous equations

$2x + 3y = 5p$ Equation 1

$y = 2x + p$ Equation 2

where p is a constant.

Give your answers in terms of p in their simplest form.

[4 marks]

$2x + 3(2x + p) = 5p$ Substituting the right side of Equation 2 for y in Equation 1

$2x + 6x + 3p = 5p$ Expanding the bracket

$8x = 2p$ Collecting the 2x and 6x to get 8x and subtracting the 3p from both sides

$x = \frac{2p}{8} = \frac{p}{4}$ Dividing both sides by 8 then simplifying the fraction

$y = 2\left(\frac{p}{4}\right) + p$ Substituting p/4 for x in Equation 2

$x =$ $\frac{p}{4}$ $y =$ $1\frac{1}{2}p$

2 x p/4 = p/2. Adding half of p to p gets this

9

Turn over ►

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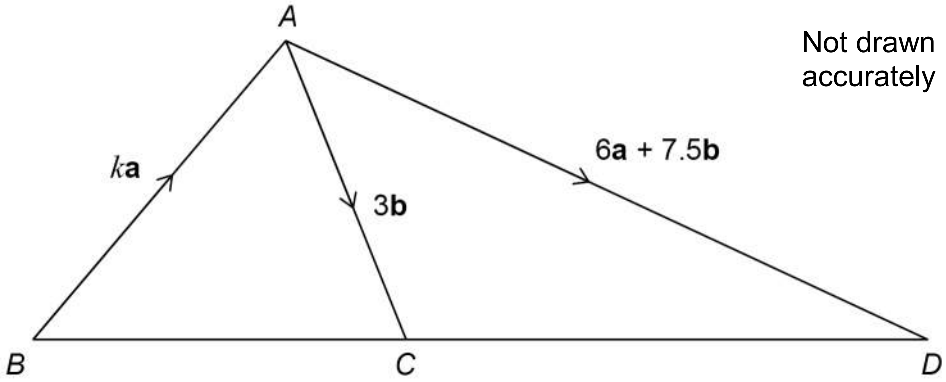
22 *ABC* and *ACD* are triangles.

k is a constant.



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22 (a) Show that $\overrightarrow{CD} = 6\mathbf{a} + 4.5\mathbf{b}$

[1 mark]

$-3b + 6a + 7.5b$

$\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$
$\overrightarrow{CA} = -3b$ as it is in the opposite direction to $\overrightarrow{AC}$

$6a + 4.5b$

Collecting like terms

22 (b) *BCD* is a straight line.

Work out the value of k.

You **must** show your working.

[3 marks]

$\overrightarrow{BC} = ka + 3b$

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$3 \overline{)4.5}$ = 1.5

As BCD is a straight line, BC must be in the same direction as $\overrightarrow{CD}$. So $\overrightarrow{BC}$ is a scaled down version of $\overrightarrow{CD}$. Dividing the coefficient of b in $\overrightarrow{CD}$ by the coefficient of b in $\overrightarrow{BC}$ works out that the scale factor must be 1.5

$\frac{6}{1.5} = \frac{60}{15}$

Dividing the coefficient of a in $\overrightarrow{CD}$ by the scale factor of 1.5 gives the coefficient of a in $\overrightarrow{BC}$. Multiplying the numerator and denominator by 10 to get rid of the decimal then 60/15 = 4

Answer 4

23 Simplify $8^4 \div 32^{\frac{2}{5}}$

Give your answer in the form 2^m where m is an integer.

[3 marks]

$(2^3)^4 \div (2^5)^{\frac{2}{5}}$ ← Expressing both 8 and 32 as powers of 2

$2^{12} \div 2^2$ ← $(a^x)y = a^{xy}$. So multiplying the powers

Answer 2^{10}

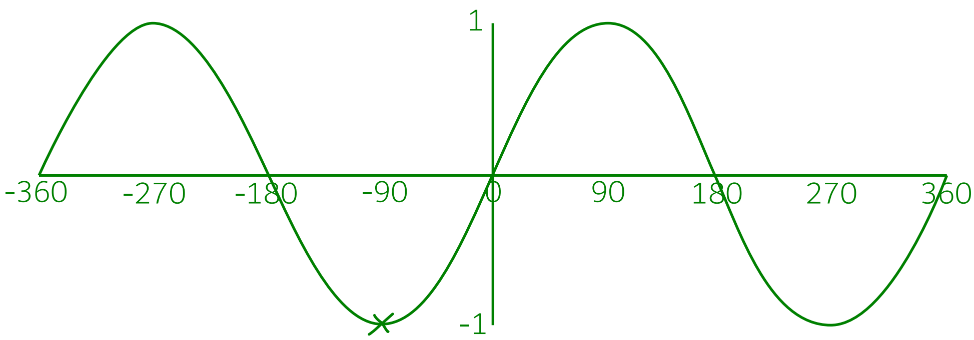
$a^x/a^y = a^{x-y}$. So subtracting the powers

24 $f(x) = \sin(x - 90°)$

Circle the value of $f(0°)$

[1 mark]

1   0 $-\frac{1}{2}$ -1



The sine curve keeps repeating every 360 degrees. Substituting 0 into f(x) gives sin(-90). Looking at the graph above, this equals to -1

Turn over for the next question

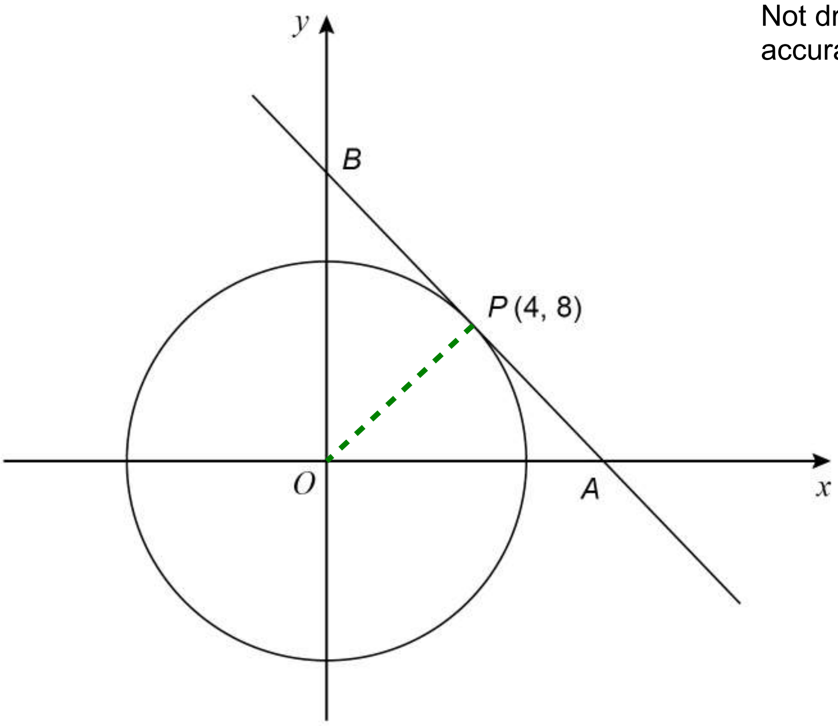
8

Do not write outside the box

25 *P* (4, 8) is a point on a circle, centre *O*.

The tangent at *P* intersects the axes at points *A* and *B*.

Not drawn accurately



25 (a) Show that the gradient of the tangent is $-\frac{1}{2}$

[2 marks]

$\frac{8-0}{4-0}=2$

Working out the gradient of the radius OP. Gradient = (change in y)/(change in x). y changes from 0 to 8 so the change in y is 8 - 0, which is 8. x changes from 0 to 4 so the change in x is 4 - 0, which is 4. Dividing 8 by 4 gives 2, so this is the gradient of radius OP

$-1\div2=-\frac{1}{2}$

The tangent is perpendicular to the radius OP so its gradient is the negative reciprocal of the gradient of the radius OP. Negative reciprocal means -1 divided by

Do not write outside the box

25 (b) Work out the length *AB*.

Give your answer in the form $a\sqrt{5}$ where a is an integer.

You **must** show your working.

[4 marks]

$y=-\frac{1}{2}x+c$ The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept. The gradient of the tangent is -1/2 so substituting this in for m

$8+\frac{1}{2}(4)=c$ Adding 1/2 x to both sides and substituting the x and y-coordinates from P (4, 8)

$c=10$ 1/2 (4) = 2, then 8 + 2 = 10. c is the y-intercept so the coordinates of B are (0, 10)

$0=-\frac{1}{2}x+10$ The y-coordinate at A is 0 so substituting this into the equation of the tangent. Also substituting 10 for c

$\frac{1}{2}x=10$ Adding 1/2 x to both sides

$x=20$ Multiplying both sides by 2 to get x on its own. So the coordinates of A must be (20, 0)

$20^2+10^2=AB^2$ Pythagoras' Theorem can be used to work out the distance between two points. $a^2+b^2=c^2$, where a and b are the shorter sides and c is the longest side. Substituting 20 for a as this is the distance between A (20, 0) and B (0, 10) in the x-direction. Substituting 10 for b as this it the distance between A (20, 0) and B (0, 10) in the y-direction. Substituting AB for c

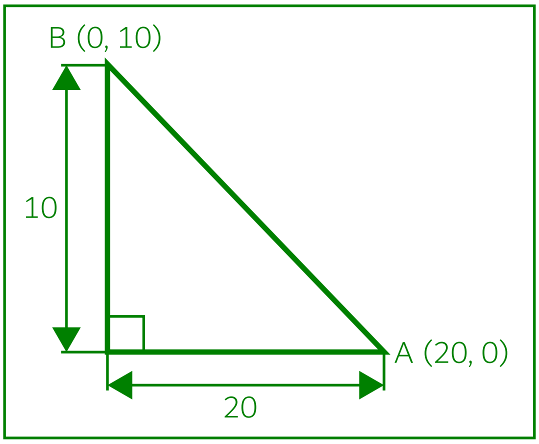
$AB^2=400+100$ $20^2 = 400$ and $10^2 = 100$

$AB=\sqrt{500}$ 400 + 100 = 500. Then square rooting both sides to get AB

$=\sqrt{100}\times\sqrt{5}$ $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

Answer $10\sqrt{5}$ units

$\sqrt{100} = 10$



Turn over for the next question

6

Do not write outside the box

26 The turning point of the graph $y = (x + a)^2 + b$ has x-coordinate –2

(3, 1) is another point on the graph.

Work out the y-coordinate of the turning point.

[3 marks]

$-2 + a = 0$ The turning point is where the square bracket has the minimum value, which is 0 (the lowest a squared value can be is 0). Substituting in the x-coordinate of the turning point

$a = 2$ Adding 2 to both sides to get a on its own

$b = 1 - (3+2)^2$ Rearranging the equation to get b on its own by subtracting $(x + a)^2$ from both sides, substituting in the x and y values from the point (3, 1) and 2 for a

Answer -24

When the square bracket is 0 (at the minimum point, which is the turning point), y = b and b = 1 - 5^2 = 1 - 25 = -24

Do not write outside the box

27 Angle x is acute.

$$\cos x = \sin 60° \times \tan 30°$$

Work out the size of angle x.

You **must** show your working.

[3 marks]

0 30 45 60 90
0 1 2 3 4
4 3 2 1 0

Listing the angles of 0, 30, 45, 60, 90 degrees. Listing 0, 1, 2, 3, 4 under these for the sin values. Listing 4, 3, 2, 1, 0 under these for the cos values. Square rooting the 3 (for sin under the 60) and putting it over 2 works out that sin60 = √3/2. Square rooting the 1 (for sin under the 30) and putting it over 2 works out that sin30 = 1/2. Square rooting the 3 (for cos under the 30) and putting it over 2 works out that cos30 = √3/2

$\frac{1}{2} \div \frac{\sqrt{3}}{2}$

Dividing sin30 by cos30 works out tan30

$\frac{1}{2} \times \frac{2}{\sqrt{3}}$

To divide by a fraction: keep the first number, change the division to a multiplication, flip the second fraction. To multiply fractions: multiply the numerators and multiply the denominators. So tan30 = 2/2√3 = 1/√3

$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$

Multiplying sin60 by tan30 gives 1/2

Answer 60 degrees

cos60 = 1/2 so x = 60

END OF QUESTIONS

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