.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

Level 2 Certificate
FURTHER MATHEMATICS

Paper 1 Non-Calculator

Tuesday 11 June 2024 Afternoon Time allowed: 1 hour 45 minutes

Materials
For this paper you must have:
- mathematical instruments
- the Formulae Sheet (enclosed).

You must **not** use a calculator.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22	
TOTAL	

J U N 2 4 8 3 6 5 1 0 1

IB/M/Jun24/G4003/V5

8365/1

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Work out the value of $\sqrt{\frac{t}{20}}$ where $t = 2.42 \times 10^3$

[2 marks]

$2 \overline{)2.42}$ gives 1.21×10^2

To divide by 20, divide by 2 then divide by 10. The 2.42 can be divided by 2 to give 1.21 and the 10^3 can be divided by 10 to give 10^2

$\sqrt{121}$

1.21 multiplied by 10 twice gives 121. Then doing the square root

Answer 11

$11^2 = 121$ so $\sqrt{121} = 11$

2 Factorise $x^2 - y^2$

[1 mark]

Answer $(x + y)(x - y)$

Factorised using difference of two squares

Do not write outside the box

3 The nth term of a sequence is $\frac{3n+4}{n}$

Circle the limiting value of $\frac{3n+4}{n}$ as $n \to \infty$

[1 mark]

1   (3)   4   7

As n approaches infinity, the +4 becomes insignificant so it is basically $3n/n$, which simplifies to 3

4 The equations of two straight lines are

$$y - 3x = 4 \quad \text{and} \quad 6y = 18x - 5$$

Show that the lines are parallel.

[2 marks]

The general equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept. It is helpful to rearrange both of the equations into this form

$y = 3x + 4$ ← Adding $3x$ to both sides of the first equation eliminates the $-3x$ on the left and gets y on its own. It is now in the form $y = mx + c$

$y = 3x - \frac{5}{6}$ ← Dividing both sides of the second equation by 6 eliminates the 6 on the left and gets y on its own. It is now in the form $y = mx + c$

Both lines have gradient of 3 so are parallel ← As the value of m in both equations is 3

Turn over for the next question

6

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5 $$y = \frac{4x^3 + x^7}{x^4}$$

Work out $\frac{dy}{dx}$

[3 marks]

$y = 4x^{-1} + x^3$ ← Dividing both terms on the numerator by x^4. $a^w/a^y = a^{w-y}$ so $4x^3/x^4 = 4x^{3-4} = 4x^{-1}$ and $x^7/x^4 = x^{7-4} = x^3$

$\frac{dy}{dx} =$ $-4x^{-2} + 3x^2$

Differentiated by multiplying each term by the power then subtracting 1 from the power

6 Points A (–12, 1) and B (12, –1) lie on a circle.

AB is a diameter of the circle.

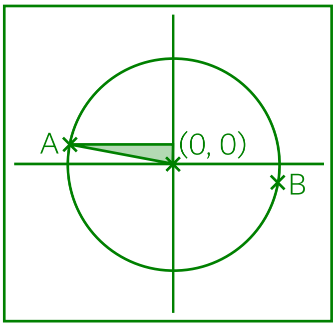
Work out the equation of the circle.

[3 marks]

The midpoint of the diameter is the centre of the circle. Halfway between -12 and 12 is 0 and halfway between 1 and -1 is 0. So the coordinates of the centre of the circle is (0, 0)

$12^2 + 1^2$ ← The radius^2 can be found by using Pythagoras' theorem. The distance in the x-direction between (0, 0) and (-12, 1) is 12 and the distance in the y-direction is 1. The radius is the hypotenuse of the right-angled triangle formed by the distance of 1 and 12 (see diagram below). $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. c is the radius so substituting the distances of 1 and 12 into a and b

$144 + 1$ ← $12^2 = 144$ and $1^2 = 1$. So the radius$^2 = 145$



Answer $x^2 + y^2 = 145$

The equation of a circle with its centre at (0, 0) is $x^2 + y^2 = \text{radius}^2$

Do not write outside the box

7 A point P (x, y) is transformed using the transformation represented by $\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix}$

The image of P is $(-8, 7)$

Work out the values of x and y.

[3 marks]

$\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ ← Multiplying the transformation matrix by the point P as a matrix will give the image P as a matrix

$4x = -8$ ← $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$ ← So $4 \times x + 0 \times y$ must be -8

Dividing both sides by 4 gives $x = -2$

$4 + 3y = 7$ ← $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$ ← So $-2 \times x + 3 \times y$ must be 7. Substituting -2 for x and $-2 \times -2 = 4$

$3y = 3$ ← Subtracting 4 from both sides to get the y term on its own

Dividing both side by 3 gives $y = 1$

$x =$ -2 $y =$ 1

9

Turn over ▶

Do not write outside the box

8 Solve by factorising $2x^3 - 9x^2 - 5x = 0$

[3 marks]

$x(2x^2 - 9x - 5)$ ← Bringing out x as a factor on the left side of the equation

$2x^2 - 10x + x - 5$ ← Factorising the $2x^2 - 9x - 5$. It is in the form $ax^2 + bx + c$. Multiplying a by c gives -10. Two numbers which multiply to this -10 and add to b (which is -9) are -10 and 1. Splitting the middle x term into these numbers of x

$2x(x - 5) + 1(x - 5)$ ← Factorising the left two terms and right two terms separately. +1 must be brought out as a factor as there is no other common factor

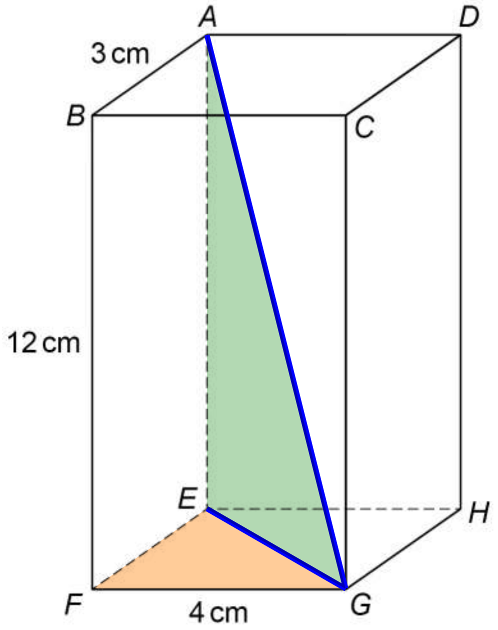
$x(2x + 1)(x - 5) = 0$ ← Bringing together the $2x$ and +1 and writing the $(x - 5)$ once. Putting the factorised quadratic back into the equation

Answer $x = 0, x = -0.5, x = 5$

Three values are multiplied together to give 0 so one of them must be 0. So either $x = 0$ or $2x + 1 = 0$ or $x - 5 = 0$. Rearranging these equations to make x the subject gives the solutions

Do not write outside the box

9 *ABCDEFGH* is a cuboid.



Work out the length *AG*.

[3 marks]

$4^2 + 3^2 = EG^2$ ← Using Pythagoras' Theorem in the orange right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting 4 cm for a, 3 cm for b and EG for c

$16 + 9$ ← $4^2 = 16$ and $3^2 = 9$

$\sqrt{25} = 5$ ← $16 + 9 = 25$, which is EG^2. Square rooting finds that EG = 5 cm

$5^2 + 12^2 = AG^2$ ← Using Pythagoras' Theorem in the green right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting 5 cm for a, 12 cm for b and AG for c

$25 + 144$ ← $5^2 = 25$ and $12^2 = 144$

$\sqrt{169}$ ← $25 + 144 = 169$, which is AG^2. Square rooting finds that AG = 13 cm

AG = 13 cm

6

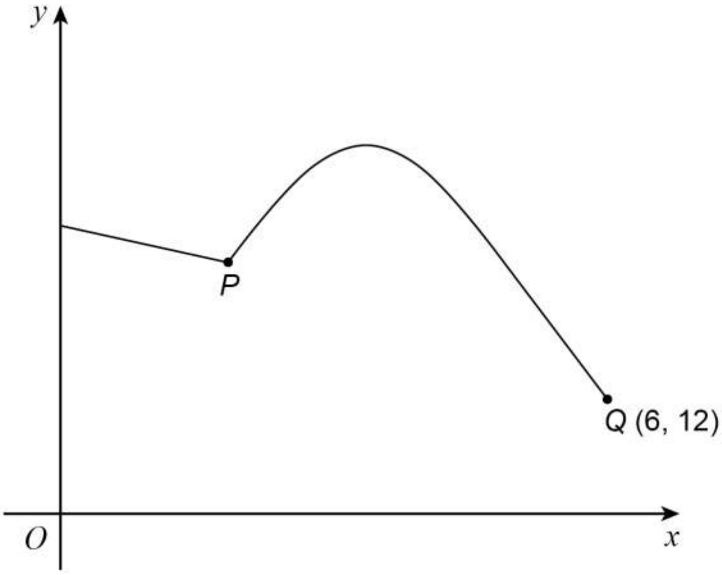
Do not write outside the box

10 A function f is given by

$$f(x) = -\frac{1}{2}x + 21 \qquad 0 \leqslant x \leqslant 2$$

$$= ax^2 + bx \qquad 2 < x \leqslant 6$$

A sketch of $y = f(x)$ is shown.



Work out the values of a and b.

[5 marks]

$-\frac{1}{2}(2) + 21$ ← Substituting the x-coordinate of P into x in the first f(x) works out that the y-coordinate of P is 20

P(2, 20) ← So P has coordinates (2, 20)

$a(2)^2 + b(2)$ ← Substituting the x-coordinate of P into x in the second f(x) expresses the y-coordinate of P

$4a + 2b = 20$ ← Simplifying and setting equal to the 20. This forms the 1st equation

$a(6)^2 + b(6)$ ← Substituting the x-coordinate of Q into x in the second f(x) expresses the y-coordinate of Q

$36a + 6b = 12$ ← Simplifying and setting equal to the 12. This forms the 2nd equation

$12a + 6b = 60$ ← Multiplying the 1st equation by 3 gets the same number of b as the 2nd equation. This forms the 3rd equation

$24a = -48$ ← Solving simultaneous equations by subtracting the 3rd equation from the 2nd equation. This cancels out the b term

Dividing both sides by 24 gives $a = -2$

$-8 + 2b = 20$ ← Substituting -2 for a in the 1st equation. $4 \times -2 = -8$

$2b = 28$ ← Adding 8 to both sides gets the b term on its own

Dividing both sides by 2 gives $b = 14$

$a =$ -2 $\qquad b =$ 14

Do not write outside the box

11 (a) $(3x - 7)$ is a factor of $3x^3 - 4x^2 - 13x + 14$

Work out the other two linear factors.

[3 marks]

$$\begin{array}{r} x^2 + x - 2 \\ 3x - 7 \overline{)\, 3x^3 - 4x^2 - 13x + 14} \\ \underline{3x^3 - 7x^2} \\ 3x^2 - 13x \\ \underline{3x^2 - 7x} \\ -6x + 14 \end{array}$$

Using algebraic long division to divide the $3x^3 - 4x^2 - 13x + 14$ by the $(3x - 7)$. This works out that it can be written as $(3x - 7)(x^2 + x - 2)$

Answer $(x - 1)$ and $(x + 2)$

Factorising the $x^2 + x - 2$ gives $(x - 1)(x + 2)$. So it can be written as $(3x - 7)(x - 1)(x + 2)$ as three linear factors

11 (b) $(x - 2)$ is a factor of $ax^4 - 3ax^3 + 5x - 22$

Work out the value of a.

[3 marks]

$a(2)^4 - 3a(2)^3 + 5(2) - 22$ ← The factor theorem can be used. $(x - 2)$ is a factor so substituting 2 into the expression will give 0

$16a - 24a + 10 - 22$ ← Simplifying

$-8a - 12 = 0$ ← Collecting like terms then setting equal to 0 (because of the factor theorem)

$-12 = 8a$ ← Adding $8a$ to both sides to get the a term positive and on its own

$a = -\frac{12}{8}$

Dividing both sides by 8 gets a on its own

11

Turn over ▶

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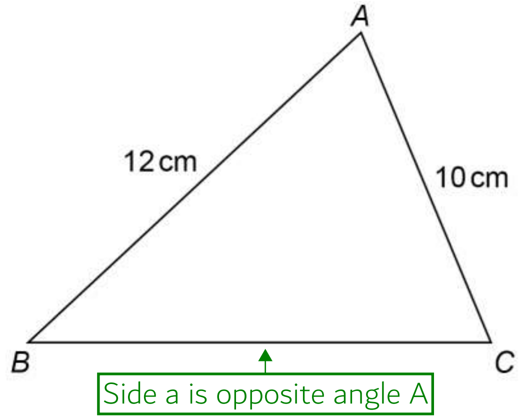
12 In triangle *ABC*, $\cos A = \frac{3}{4}$

Not drawn accurately



Work out the length *BC*.

[3 marks]

$BC^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \times \frac{3}{4}$ ← Using the cosine rule. $a^2 = b^2 + c^2 - 2bc\cos A$. Substituting BC for a, 10 for b, 12 for c and 3/4 for cosA

$= 244 - 180$ ← $10^2 = 100$ and $12^2 = 144$, then $100 + 144 = 244$. $12 \times 3/4 = 9$, then $9 \times 20 = 90$, then $2 \times 90 = 180$

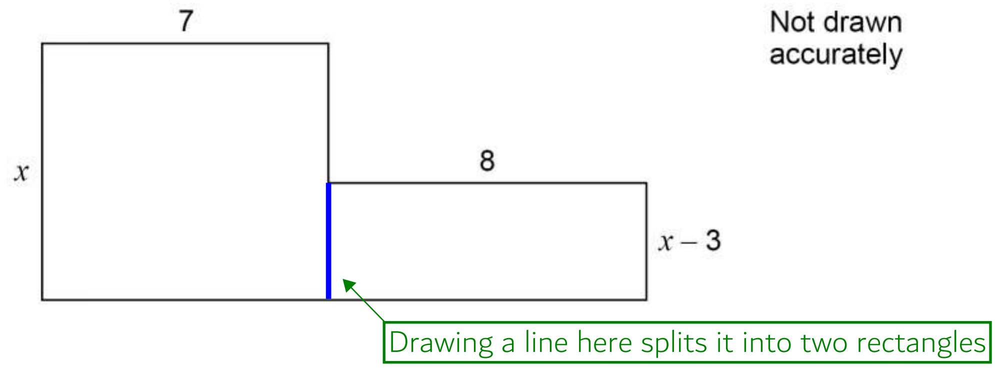
$BC = \sqrt{64}$ ← $244 - 180 = 64$. Then square rooting both sides gets BC on its own. Length cannot be negative so ignoring the negative square root

Answer 8 cm

Do not write outside the box

13 A garden patio is made from two rectangles.

All lengths are in metres.



The area of the patio is **less than** 51 m^2

Work out the range of possible values of x.

Give your answer in the form $a < x < b$ where a and b are both integers.

You **must** show your working.

[4 marks]

$8(x - 3)$ ← Expressing the area of the right rectangle. Area of rectangle = length × width

$8x - 24 + 7x$ ← Expanding the bracket and adding the area of the left rectangle. This expresses the area of the patio

$15x - 24 < 51$ ← Collecting like terms and setting less than the 51 m^2

$15x < 75$ ← Adding 24 to both sides to get the x term on its own

15, 30, 45, 60, 75 ← Dividing both sides by 15 by counting in 15s until it reaches 75. So $x < 5$

Answer 3 $< x <$ 5

x must be greater than 3 otherwise x - 3 would be a negative length

7

Turn over ▶

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14 (a) Write $2x^2 - 16x + 7$ in the form $k(x + m)^2 + n$ where k, m and n are integers.

[3 marks]

$2(x^2 - 8x) + 7$ ← Bringing out 2 as a factor on the first two terms

$2(x - 4)^2 + 7 - 32$ ← Completing the square. Halving the coefficient of x, putting this in a bracket with x and squaring the bracket. $(-4)^2 = 16$ then $16 \times 2 = 32$, which needs to be subtracted from the end

Answer $2(x - 4)^2 - 25$

$7 - 32 = -25$

14 (b) Solve $(x - 1)^2 - 5 = 0$

[1 mark]

$(x - 1)^2 = 5$ ← This can be solved by rearranging as there is only one power of x. First adding 5 to both sides

$x - 1 = \pm\sqrt{5}$ ← Then doing the positive and negative square root to the right to eliminate the 2 on the left

Answer $x = 1 \pm\sqrt{5}$

Then adding 1 to both sides gets x on its own

Do not write outside the box

15 (a) Matrix **M** represents a reflection in the line $y = -x$

Write down matrix **M**

[1 mark]

$$\mathbf{M} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

See the next page for a method of how to work out the matrix without having to memorise them

15 (b) $\mathbf{N} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Describe geometrically the single transformation represented by $\mathbf{N}^2$

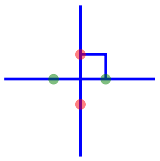
[2 marks]

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ ← N^2 means N × N

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$ ← So N^2 is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ ← $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$ ← Multiplying N^2 by (0, 1) as a matrix gives the coordinates (0, -1)

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ ← $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$ ← Multiplying N^2 by (1, 0) as a matrix gives the coordinates (-1, 0)

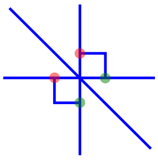


← So (0, 1) transforms to (0, -1), shown in red, and (1, 0) transforms to (-1, 0), shown in green

Rotation 180° centre (0, 0)

Turn over for the next question

7



Drawing a unit square on a graph with coordinates (0, 0), (0, 1), (1, 0), (1, 1). Drawing the line of $y = -x$ (which goes diagonally down through the origin). Reflecting the unit square in this line. The corner in red (0, 1) transforms to (-1, 0) and the corner in green (1, 0) transforms to (0, -1)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Multiplying the transformation matrix by the coordinates (0, 1) as a matrix must give (-1, 0) as a matrix

$b = -1$

$d = 0$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

So $a(0) + b(1) = b = -1$ and $c(0) + d(1) = d = 0$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

Multiplying the transformation matrix by the coordinates (1, 0) as a matrix must give (0, -1) as a matrix

$a = 0$

$c = -1$

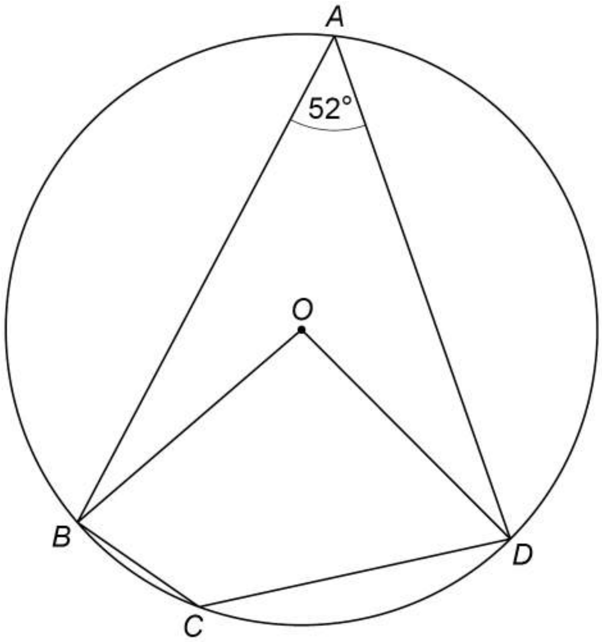
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

So $a(1) + b(0) = a = 0$ and $c(1) + d(0) = c = -1$

The values of a, b, c, d can now be filled into the matrix

16 *A*, *B*, *C* and *D* are points on a circle, centre *O*.

angle *OBC* : angle *ODC* = 5 : 3



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Work out the size of angle *OBC*.

You **must** show your working.

[4 marks]

BCD = 180 - 52 = 128 ← As opposite angles in a cyclic quadrilateral add up to 180°

BOD = 52 × 2 = 104 ← As the angle at the centre is double the angle at the circumference

128 + 104 ← Adding angles BCD and BOD works out that there are 232° in quadrilateral OBCD so far

360 - 232 ← There are 360° in a quadrilateral. Subtracting the 232° so far in the quadrilateral from 360° works out that there are 128° left in the quadrilateral. This is the total of angles OBC and ODC

0 1 6
8 | 1 [1]2 [4]8 ← 5 + 3 = 8 parts in total in the ratio which represent the total of angles OBC and ODC. So dividing the 128° by 8 works out that 1 part of the ratio is worth 16°

16 × 5 ← Multiplying the value of 1 part of the ratio by the 5 parts which represent angle OBC works out that angle OBC is 80°

Answer 80 °

Do not write outside the box

17 Show that $\dfrac{21x}{3x^2 - 2x - 8} - \dfrac{7}{x - 2}$ simplifies to $\dfrac{k}{3x^2 - 2x - 8}$ where k is an integer.

[3 marks]

$3x^2 + 4x - 6x - 8$ ← Factorising the denominator of the first fraction. It is in the form $ax^2 + bx + c$. Multiplying a by c gives -24. Two numbers which multiply to this and add to b (which is -2) are 4 and -6. Splitting the middle x term into these numbers of x

$x(3x + 4) - 2(3x + 4)$ ← Factorising the left two terms separately to the right two terms

$\dfrac{21x}{(x - 2)(3x + 4)} - \dfrac{7(3x + 4)}{(x - 2)(3x + 4)}$ ← Bringing together the x and -2 and writing the repeated $(3x + 4)$ once gives $(x - 2)(3x + 4)$. Multiplying both the numerator and denominator of the second fraction by $(3x + 4)$ gives it the same denominator as the first fraction

$\dfrac{21x - 21x - 28}{(x - 2)(3x + 4)}$ ← Fractions with the same denominator can be subtracted by subtracting the numerators. Expanding the bracket on the numerator of the second fraction gives $21x + 28$. Subtracting all of this from the numerator of the first fraction

$\dfrac{-28}{3x^2 - 2x - 8}$ ← $21x - 21x$ cancels out leaving -28 as the numerator. The denominator can be written back as it was before it was factorised

Turn over for the next question

7

Do not write outside the box

18 Rationalise the denominator and simplify fully

$$\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

[4 marks]

$$\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

Multiplying the fraction by $\sqrt{5}+\sqrt{3}$ (which is $\sqrt{5}-\sqrt{3}$ with the sign in the middle flipped) over itself

$$\frac{15+3\sqrt{15}+\sqrt{15}+3}{5+\sqrt{15}-\sqrt{15}-3}$$

Expanding the numerators and denominators like they were brackets. $\sqrt{5}\times\sqrt{5}=5$ and $\sqrt{3}\times\sqrt{3}=3$. $3\times 5 = 15$

$$\frac{18+4\sqrt{15}}{2}$$

Collecting like terms

Answer $9+2\sqrt{15}$

Dividing both terms on the numerator by 2

Do not write outside the box

19 The equation of a curve is $y = x^3 - 3x^2 + 5$

Work out the stationary points of the curve and determine their nature.

You **must** show your working.

[5 marks]

$3x^2 - 6x = 0$ ← Differentiated by multiplying each term on the right side of the equation by the power then subtracting 1 from the power. Setting equal to 0 as differentiating gives an expression of the gradient and the gradient at stationary points is 0

$3x(x - 2) = 0$ ← Solving for x by factorising the left side of the equation. Either $3x = 0$ (so $x = 0$) or $x - 2 = 0$ (so $x = 2$)

$(0)^3 - 3(0)^2 + 5$ ← Substituting 0 for x in the equation of the curve works out that $y = 5$ when $x = 0$. So a stationary point is at (0, 5)

$(2)^3 - 3(2)^2 + 5$ ← Substituting 2 for x in the equation of the curve works out that $y = 1$ when $x = 2$. So a stationary point is at (2, 1)

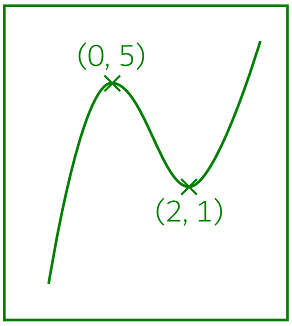
$6(0) - 6 = -6$

$6(2) - 6 = 6$

← Differentiating again by multiplying each term of $3x^2 - 6x$ by the power then subtracting 1 from the power gives $6x - 6$, which is an expression for the rate of change of the gradient. Substituting 0 for x gives -6, meaning that the gradient is decreasing and that the point (0, 5) must be a maximum. Substituting 2 for x gives 6, meaning that the gradient is increasing and that the point (2, 1) must be a minimum

Stationary point (0 , 5) Nature Maximum

Stationary point (2 , 1) Nature Minimum



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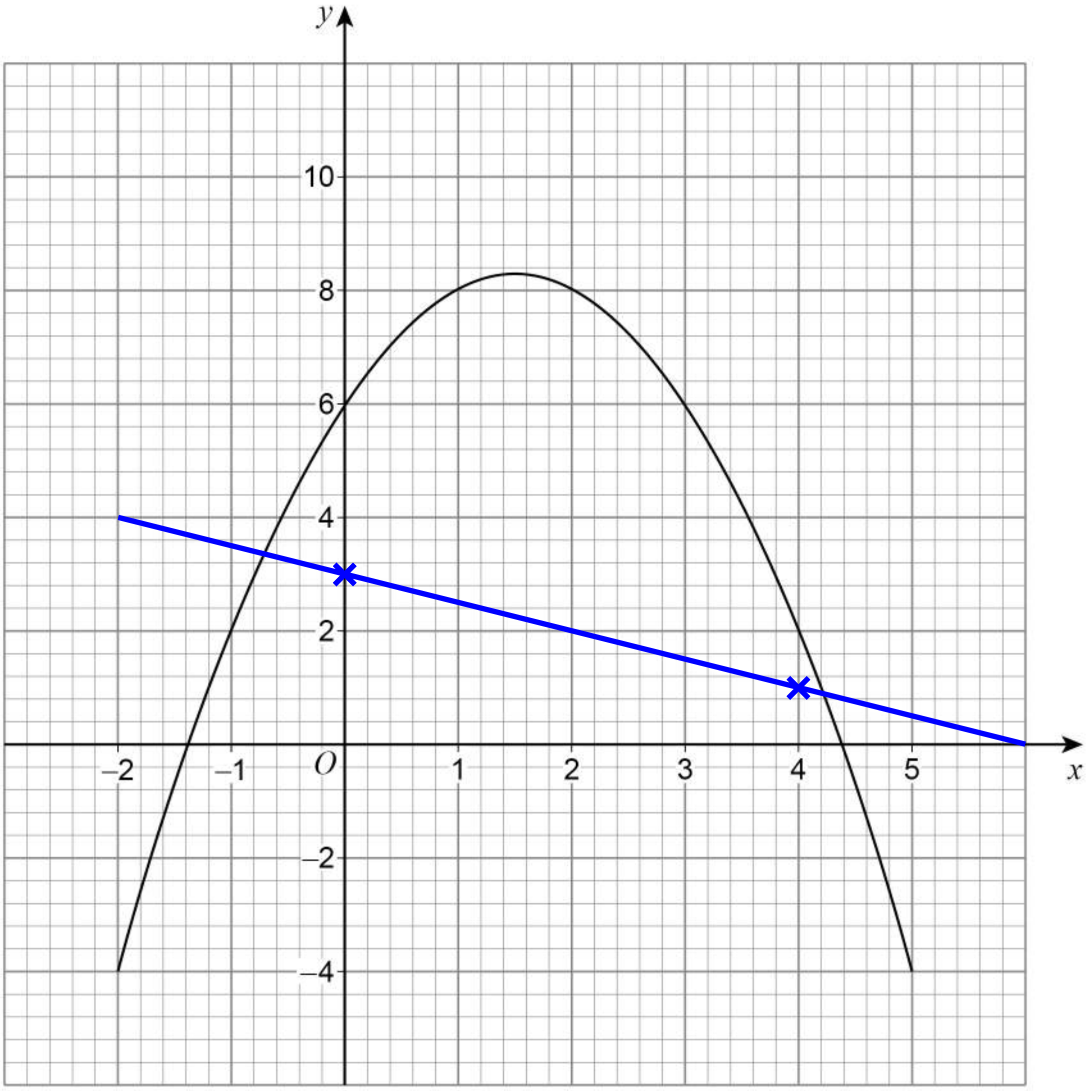
Turn over ►

Do not write outside the box

20 Here is the graph of $y = -x^2 + 3x + 6$ for $-2 \leqslant x \leqslant 5$



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By drawing a suitable **straight line**, work out approximate solutions to

$$x^2 - \frac{7}{2}x - 3 = 0$$

[4 marks]

$y = -x^2 + 3x + 6$ ← Copying down the equation of the curve. This is the 1st equation

$y = mx + c$ ← Writing down the general equation of a straight line. This is the 2nd equation

$0 = x^2 + (m - 3)x + c - 6$ ← Doing simultaneous equations between the 1st and 2nd equation. Subtracting the 1st equation from the 2nd equation to get positive x^2 and cancel out the y. $mx - 3x = (m - 3)x$

$m - 3 = -\frac{7}{2}$ ← Equating the coefficients of the x terms in equation above and the equation which we are trying to solve

$m = -\frac{1}{2}$ ← Adding 3 (or 6/2) to both sides to get m on its own

$c - 6 = -3$ ← Equating the constants

$c = 3$ ← Adding 6 to both sides to get c on its own

So the graph which needs to be drawn must be $y = -1/2\, x + 3$. When $x = 0$, $y = -1/2\,(0) + 3 = 3$, so plotting the point (0, 3). When $x = 4$, $y = -1/2\,(4) + 3 = 1$, so plotting the point (4, 1). Drawing a straight line through these two points

The solutions to the equation are the x-coordinates of the two point which the straight line and the curve meet

Answer -0.7 and 4.2

4

Turn over ▶

Do not write outside the box

21 $\left(2^{x+3}\right)^{4x+1} = 8^{x-1}$

Work out the possible values of x.

[5 marks]

$2^{(x+3)(4x+1)} = (2^3)^{x-1}$ ← $(a^w)^y = a^{wy}$, so multiplying the two powers on the left. Expressing 8 as 2^3

$2^{4x^2 + x + 12x + 3} = 2^{3x-3}$ ← Expanding the brackets on the left power. $(a^w)^y = a^{wy}$, so multiplying the two powers on the right

$4x^2 + 13x + 3 = 3x - 3$ ← Setting the power on the left equal to the power on the right. Collecting like terms on the left

$4x^2 + 10x + 6 = 0$ ← Rearranging into the quadratic form $ax^2 + bx + c = 0$ by subtracting $3x$ from both sides and adding 3 to both sides

$2x^2 + 5x + 3 = 0$ ← Dividing all terms on both sides by 2 to get smaller whole numbers

$2x^2 + 3x + 2x + 3 = 0$ ← Factorising the left side. Multiplying a by c gives 6. Two numbers which multiply to this 6 and add to b (which is 5) are 3 and 2. Splitting the middle x term into these numbers of x

$x(2x + 3) + 1(2x + 3) = 0$ ← Factorising the left two terms separately to the right two terms

$(x + 1)(2x + 3) = 0$ ← Bringing together the x and +1. Writing the $(2x + 3)$ once. The left side is now factorised

Answer -1 and -1.5

Either $x + 1 = 0$ (so $x = -1$) or $2x + 3 = 0$ (so $2x = -3$ and $x = -1.5$)

Do not write outside the box

22 $$f(x) = \frac{3\sin x\cos x + \sin^2 x}{12\cos^2 x + 4\sin x\cos x}$$

Simplify f(x) and hence solve $f(x) = -\frac{\sqrt{3}}{4}$ for $90° < x < 180°$

You **must** show your working.

[4 marks]

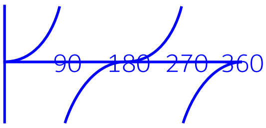
$$\frac{\sin x(3\cos x + \sin x)}{4\cos x(3\cos x + \sin x)}$$ ← Factorising the numerator and denominator

$$\frac{\tan x}{4} = -\frac{\sqrt{3}}{4}$$ ← Cancelling out the ($3\cos x + \sin x$). $\sin x/\cos x = \tan x$. Setting the simplified f(x) equal to the $-\sqrt{3}/4$

$\tan x = -\sqrt{3}$ ← Multiplying both sides by 4 cancels out the denominators

0	30	45	60	90
0	1	2	3	4
4	3	2	1	0

← Listing the angles of 0°, 30°, 45°, 60°, 90°. Listing 0, 1, 2, 3, 4 under these for the sin values and 4, 3, 2, 1, 0 under these for the cos values. Square rooting them and putting them over 2 works out the sin and cos values. Dividing the sin values by the cos values gives the tan values. $\tan 60 = \sqrt{3}$ as $\sin 60 = \sqrt{3}/2$ and $\cos 60 = 1/2$ and $\sqrt{3}/2 \div 1/2 = \sqrt{3}$



← Sketching the graph of $y = \tan x$ to visualise where $\tan x = -\sqrt{3}$

180 - 60 ← If $\tan 60 = \sqrt{3}$, x must be 60° before 180°

$x =$ 120°

Turn over for the next question

9

Do not write outside the box

23 The nth term of a quadratic sequence is $an^2 - 5n + c$ where a and c are integers.

The first four terms of the sequence are

2 x 16 y

Work out the values of x and y.

[5 marks]

$a(1)^2 - 5(1) + c = 2$ ← On the 1st term n = 1. Substituting 1 for n in the nth term must give 2

$a - 5 + c = 2$ ← Simplifying

$a + c = 7$ ← Adding 5 to both sides forms the 1st equation

$a(3)^2 - 5(3) + c = 16$ ← On the 3rd term n = 3. Substituting 3 for n in the nth term must give 16

$9a - 15 + c = 16$ ← Simplifying

$9a + c = 31$ ← Adding 15 to both sides forms the 2nd equation

$8a = 24$ ← Doing simultaneous equations. Subtracting the first equation from the 2nd equation cancels out c

$a = 3$ ← Dividing both sides by 8 gets a on its own

$c = 4$ ← Substituting 3 for a in the 1st equation gives 3 + c = 7. Subtracting 3 from both sides gets c on its own. So the nth term is $3n^2 - 5n + 4$

$3(2)^2 - 5(2) + 4$ ← Substituting 2 for n in the nth term gives the 2nd term, x

$3(4)^2 - 5(4) + 4$ ← Substituting 4 for n in the nth term gives the 4th term, y

$x =$ 6 $y =$ 32

END OF QUESTIONS

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