AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

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I declare this is my own work.

Level 2 Certificate
FURTHER MATHEMATICS

Paper 1 Non-Calculator

Thursday 8 June 2023 Morning Time allowed: 1 hour 45 minutes

Materials
For this paper you must have:
- mathematical instruments
- the Formulae Sheet (enclosed).

You must **not** use a calculator.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
TOTAL	

J U N 2 3 8 3 6 5 1 0 1

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue should be written in the exam.

Anything written in green in a rectangle doesn't have to be written in the exam.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Do not write outside the box

Answer **all** questions in the spaces provided.

1 The function f is given by $f(x) = 2x + 1$

1 (a) Work out x when $f(x) = –5$

[2 marks]

$2x + 1 = -5$ ← Setting f(x) equal to -5

$2x = -6$ ← Subtracting 1 from both sides to get the x term on its own

$x =$ -3

Dividing both sides by 2 gets x on its own

1 (b) The function g is given by $g(x) = x^2$

Work out fg(3)

[2 marks]

3^2 ← Substituting 3 for x in g(x) gives 9

$2(9) + 1$ ← Substituting 9 for x in f(x) gives 19

Answer 19

2 Factorise fully $6x^2y + 21xy$

[2 marks]

3 is the highest common factor of 6 and 21. xy is the highest power of the letters in common to both terms. So bringing 3xy out as a factor, dividing both terms by this and leaving the result in a bracket

Answer $3xy(2x + 7)$

3 (a) Circle the transformation matrix that represents a reflection in the line $y = -x$

[1 mark]

[✓ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$] $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

See the next page for a method of how to work out the matrix without having to memorise them

3 (b) Show that

$$\begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}\begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = k\,\mathbf{I} \quad \text{where } k \text{ is an integer.}$$

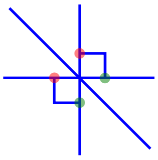
[2 marks]

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \leftarrow \boxed{\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}} \leftarrow \boxed{\begin{array}{l} 2 \times -3 + 4 \times 1 = -6 + 4 = -2 \\ 2 \times -4 + 4 \times 2 = -8 + 8 = 0 \\ -1 \times -3 + -3 \times 1 = 3 - 3 = 0 \\ -1 \times -4 + -3 \times 2 = 4 - 6 = -2 \end{array}}$$

$$-2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \leftarrow \boxed{\text{Expressing it as -2} \times \text{the identity matrix, } \mathrm{I}}$$

$-2\mathrm{I}$

9



Drawing a unit square on a graph with coordinates (0, 0), (0, 1), (1, 0), (1, 1). Drawing the line of $y = -x$ (which goes diagonally down through the origin). Reflecting the unit square in this line. The corner in red (0, 1) transforms to (-1, 0) and the corner in green (1, 0) transforms to (0, -1)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Multiplying the transformation matrix by the coordinates (0, 1) as a matrix must give (-1, 0) as a matrix

b = -1

d = 0

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

So a(0) + b(1) = b = -1 and c(0) + d(1) = d = 0

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

Multiplying the transformation matrix by the coordinates (1, 0) as a matrix must give (0, -1) as a matrix

a = 0

c = -1

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

So a(1) + b(0) = a = 0 and c(1) + d(0) = c = -1

4 S (7, 2) and T (5, –4) are points on a straight line.

4 (a) Work out the gradient of the line.

[2 marks]

$\frac{-4 - 2}{5 - 7}$ ← Gradient = (change in y)/(change in x)

$\frac{-6}{-2}$

Answer 3

4 (b) Work out the distance between S and T.

Give your answer in the form $a\sqrt{b}$ where a and b are both integers greater than 1

[3 marks]

$6^2 + 2^2 = ST^2$ ← Pythagoras' Theorem can be used to work out the distance between two points. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. The change in y was -6, which is a distance of 6. The change in x was -2, which is a distance of 2. Substituting 6 for a, 2 for b and ST for c

$ST = \sqrt{40}$ ← $6^2 = 36$ and $2^2 = 4$. Then 36 + 4 = 40. Square rooting both sides eliminates the 2 as a power on ST. There is no need to do the negative square root as length cannot be negative

$= \sqrt{4}\sqrt{10}$ ← $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, so $\sqrt{40}$ can be split into $\sqrt{4} \times \sqrt{10}$. $\sqrt{4}$ was chosen as this can be square rooted to give an integer

$\sqrt{4} = 2$

Answer $2\sqrt{10}$ units

5 X_n and Y_n are the nth terms of two sequences.

$$X_n = (n - 1)(n + 1)$$

$$Y_n = (n + 1)(n + 2)$$

Prove that every term of the sequence with nth term $Y_n - X_n$ is a multiple of 3

[3 marks]

$(n^2 + 2n + n + 2) - (n^2 + n - n - 1)$ Expanding the brackets for Y_n and X_n and subtracting them

$n^2 + 2n + n + 2 - n^2 - n + n + 1$ Flipping the sign of all the terms in the second bracket as the bracket is negative

$3n + 3$ Collecting like terms

$3(n + 1)$ Bringing 3 out as a factor proves that it is a multiple of 3

Turn over for the next question

8

6 The equation of a curve is $y = x^6 + 4x^2 - 7$

Work out the equation of the tangent to the curve at the point (1, –2)

Give your answer in the form $y = mx + c$

[4 marks]

$6x^5 + 8x$ ← Differentiated by multiplying each term by the power and then subtracting 1 from the power. The -7 is basically $-7x^0$ so becomes 0. This is an expression of the gradient

$6(1)^5 + 8(1)$ ← Substituting 1 for x in the expression of the gradient works out that the gradient at point (1, -2) is 14

$y - -2 = 14(x - 1)$ ← The equation of a straight line going through point (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$

$y = 14x - 14 - 2$ ← Expanding the bracket and adding -2 to both sides to make y the subject

Answer $y = 14x - 16$

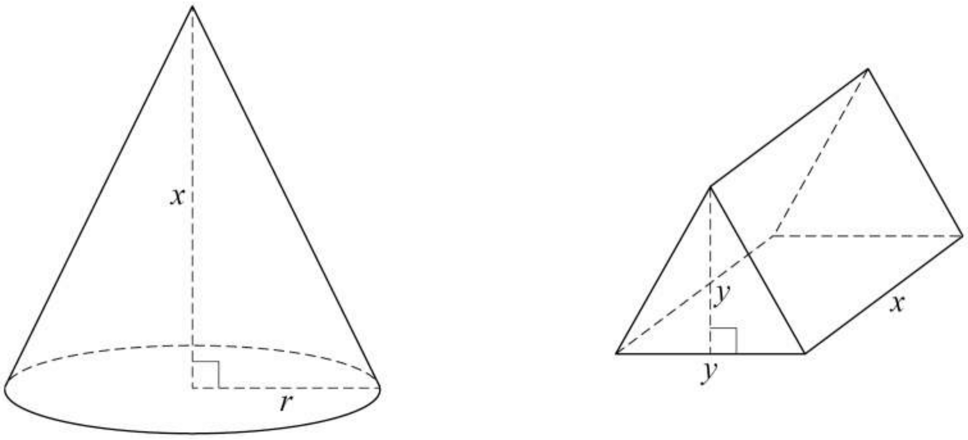
7 The diagram below shows a cone and a prism.

All measurements are in cm

The cone has base radius r and perpendicular height x.

The prism has a triangular cross section with base y and perpendicular height y.

The length of the prism is x.



Volume of a cone = $\frac{1}{3}$ × area of base × perpendicular height

The volume of the cone is **four** times the volume of the prism.

Express r in terms of y.

[4 marks]

$$\frac{1}{3} \times \pi r^2 \times x = 4 \times \frac{1}{2} \times y \times y \times x$$

Setting the volume of the cone equal to four times the volume of the prism. The base of the cone is a circle. Area of circle = π × radius². Volume of prism = area of cross section × length. The cross section is a triangle. Area of triangle = 1/2 × base × height

$$r^2 = \frac{6y^2}{\pi}$$

First dealing with the numbers: 4 × 1/2 = 2. Then multiplying both sides by 3 to eliminate the 1/3 on the left gets 6 on the right.

Then dealing with the letters: dividing both sides by x cancels it out on both sides. y × y = y².

Then dividing both sides by π to get r² on its own

$r = \sqrt{\frac{6y^2}{\pi}}$

Doing the square root of both sides to cancel out the 2 as a power on the left. Not doing the negative square root as length cannot be negative

$\frac{\quad}{8}$

8 A circle has centre (0, 0) and radius 5

A straight line has equation $2y = x + 5$

Work out the coordinates of the **two** points where the circle and straight line intersect.

Do **not** use trial and improvement.

You **must** show your working.

[6 marks]

$x^2 + y^2 = 25$ ← The equation of a circle with its centre at the origin (0, 0) is $x^2 + y^2 = \text{radius}^2$

$x = 2y - 5$ ← Rearranging the equation of the straight line to make x the subject by subtracting 5 from both sides

$(2y - 5)^2$ ← Expressing x^2 in terms of y

$4y^2 - 20y + 25 + y^2 = 25$ ← Expanding the brackets by squaring the first term, doubling the product of the two terms, squaring the last term. Putting this back into the equation of the circle in place of x^2

$5y^2 - 20y = 0$ ← Collecting like terms and subtracting 25 from both sides

$5y(y - 4) = 0$ ← Factorising the left side. Either $5y = 0$ (so $y = 0$) or $y - 4 = 0$ (so $y = 4$)

$x = 2(0) - 5$ ← Substituting 0 for y in $x = 2y - 5$ finds that $x = -5$ when $y = 0$

$x = 2(4) - 5$ ← Substituting 4 for y in $x = 2y - 5$ finds that $x = 3$ when $y = 4$

Answer (-5 , 0) and (3 , 4)

9 Rearrange $w = \dfrac{y^2 + 5}{y^2 - 2}$ to make y the subject.

[4 marks]

$wy^2 - 2w = y^2 + 5 \leftarrow$ Multiplying both sides by $y^2 - 2$ to eliminate the denominator involving y

$wy^2 - y^2 = 2w + 5 \leftarrow$ Subtracting y^2 from both sides and adding 2w to both sides to get all terms involving y on the same side and all the other terms on the other side

$y^2(w - 1) = 2w + 5 \leftarrow$ Factorising the left side to get y^2 out of the terms

$y^2 = \dfrac{2w + 5}{w - 1} \leftarrow$ Dividing both sides by w - 1 to get y^2 on its own

Answer $y = \pm\sqrt{\dfrac{2w + 5}{w - 1}}$

$\uparrow$ Doing the positive and negative square root to cancel out the 2 as a power and get y on its own

Turn over for the next question

10

Do not write outside the box

10 Rationalise the denominator and simplify fully

$$\frac{1+\sqrt{5}}{3-\sqrt{5}}$$

[4 marks]

$$\frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

Flipping the sign in the middle of 3 - √5 gives 3 + √5.
Multiplying both the numerator and denominator by this

$$\frac{3+\sqrt{5}+3\sqrt{5}+5}{9+3\sqrt{5}-3\sqrt{5}-5}$$

Expanding the numerators and denominators. √5 × √5 = 5

$$\frac{8+4\sqrt{5}}{4}$$

Collecting like terms

Answer $2+\sqrt{5}$

Dividing both terms on the numerator by the 4

Do not write outside the box

11 $y = \frac{1}{12}x^4 + 3x^2 + 4$

Work out the **positive** value of x for which $\frac{d^2y}{dx^2} = 55$

[3 marks]

$\frac{1}{3}x^3 + 6x$ ← Differentiated by multiplying each term by the power then subtracting 1 from the power. 4 is basically $4x^0$ so becomes 0

$x^2 + 6 = 55$ ← Differentiated again by multiplying each term by the power then subtracting 1 from the power. $6x^1$ becomes $6x^0$, which is 6. Setting equal to the 55

$x^2 = 49$ ← Subtracting 6 from both sides to get x^2 on its own

$x =$ 7

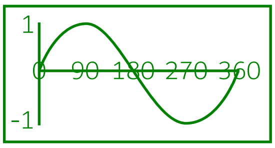
Square rooting both sides eliminates the 2 as a power on the left and gets x on its own. Not doing the negative square root as this gives a negative value of x

Turn over for the next question

7

12 (a) Write down the value of x for $0° \leqslant x \leqslant 360°$ when $\sin x = -1$

[1 mark]



$x =$ 270°

12 (b) Work out the values of y for $0° \leqslant y \leqslant 360°$ when $\sqrt{3} \tan y = 1$

[3 marks]

$\tan y = \frac{1}{\sqrt{3}}$

Dividing both sides by √3 to get tany on its own

0	30	45	60	90
0	1	2	3	4
4	3	2	1	0

Listing out the angles 0°, 30°, 45°, 60°, 90°. Listing 0, 1, 2, 3, 4 under these for the sin values. Listing 4, 3, 2, 1, 0 under these for the cos values. Square rooting them and putting them over 2 works out the trig values. Dividing the sin values by the cos values gives the tan values. sin30 = 1/2 and cos30 = √3/2. tan30 = 1/√3 as sin30 ÷ cos30 = 1/√3

Answer 30°, 210°

A tan graph repeats every 180° so adding 180° to the 30° gives another value in the range

13 Write $\frac{2x-3}{x} - \frac{1}{3x} + 1$ as a single fraction.

Give your answer in its simplest form.

[3 marks]

$$\frac{6x - 9}{3x} - \frac{1}{3x} + \frac{3x}{3x}$$

Multiplying both the numerator and denominator of the 1st fraction by 3 and multiplying both the numerator and denominator of the 3rd fraction (1/1) by 3x makes it so that all three fractions have the same denominator

Answer $\frac{9x - 10}{3x}$

6x - 9 - 1 + 3x = 9x - 10. The denominator stays the same

Turn over for the next question

7

Turn over ▶

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14 Solve $\frac{8}{x} + 3x \leqslant 10$ where x is positive.

[4 marks]

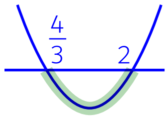
$8 + 3x^2 \leq 10x$ ← Multiplying all terms on both sides by x to eliminate it as a denominator

$3x^2 - 10x + 8 \leq 0$ ← Subtracting 10x from both sides to get everything on the same side and 0 on the other side

$3x^2 - 6x - 4x + 8$ ← Factorising the left side. It is in the form $ax^2 + bx + c$. Multiplying a by c gives 24. Two numbers which multiply to this 24 and add to b are -6 and -4. Splitting the middle x-term into these numbers of x

$3x(x - 2) - 4(x - 2)$ ← Factorising the left two terms separately to the right two terms

$(3x - 4)(x - 2) = 0$ ← Bringing the 3x and -4 together and writing the (x - 2) once. The quadratic is now factorised. Setting it equal to 0. Either 3x - 4 = 0 (so x = 4/3) or x - 2 = 0 (so x = 2)



← Sketching a graph and showing the x-coordinates where the curve meets the x-axis. It is u-shaped as it is positive x^2

Answer $\frac{4}{3} \leq x \leq 2$

It is less than or equal to 0 when x is greater than or equal to 4/3 and less than or equal to 2

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15 Solve $\left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)^2 = x^2 + x$

[4 marks]

$x - 2x^2 + x^3 = x^2 + x$ ← Expanding the square bracket by squaring the first term, doubling the product of the two terms, squaring the last term. $(a^w)^y = a^{wy}$ and $a^w \times a^y = a^{w+y}$. 1/2 × 2 = 1, 1/2 + 3/2 = 4/2 = 2, 3/2 × 2 = 3

$x^3 - 3x^2 = 0$ ← Subtracting x^2 and x from both sides to get everything on the same side equal to 0. Collecting like terms

$x^2(x - 3) = 0$ ← Factorising the left side

Answer $x = 0$

$x = 3$

Either $x^2 = 0$ (so x = 0) or x - 3 = 0 (so x = 3)

16 The expansions of $(1 + 12x)^4$ and $(a + 4x)^3$ have the same coefficient of x^2

Work out the value of a.

[4 marks]

See the next page for the method

$a =$ 18

12

Turn over ▶

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Writing Pascal's Triangle up to the 5th line to help work out the coefficients of each term of the binomial expansions

$$1(1)^4(12x)^0 + 4(1)^3(12x)^1 + 6(1)^2(12x)^2 \ldots$$

Doing a binomial expansion of $(1 + 12x)^4$ up to the x^2 term. The coefficients are found on the 5th line of Pascal's Triangle. The power of 1 decreases by 1 and the power of 12x increases by 1 between each term

$$1(a)^3(4x)^0 + 3(a)^2(4x)^1 + 3(a)^1(4x)^2 \ldots$$

Doing a binomial expansion of $(a + 4x)^3$ up to the x^2 term. The coefficients are found on the 4th line of Pascal's Triangle. The power of a decreases by 1 and the power of 4x increases by 1 between each term

$$\begin{array}{r} 144 \\ \times \quad 6 \\ \hline \underset{2\ 2\ \ }{864} \end{array} = 48a$$

$6(1)^2(12x)^2 = 6 \times 12^2 \times x^2 = 6 \times 144 \times x^2 = 864x^2$ so the coefficient of the x^2 term in the binomial expansion of $(1 + 12x)^4$ is 864. $3(a)^1(4x)^2 = 3 \times 4^2 \times a \times x^2 = 3 \times 16 \times a \times x^2 = 48ax^2$, so the coefficient of the x^2 term in the binomial expansion of $(a + 4x)^3$ is 48a. These coefficients are equal

$$\begin{array}{r} 0\ \ 1\ \ 8 \\ 48\overline{\smash{)}\,8\ {}^{8}6\ {}^{38}4} \end{array}$$

48, 96, 144, 192, 240, 288, 336, 384

Dividing both sides by 48 finds that a = 18.
Listing the 48 times table helps with the division

17 The curve $y = ax^3 + bx^2 + 7$ has a stationary point at (–2, 11)

Work out the values of a and b.

[5 marks]

$11 = a(-2)^3 + b(-2)^2 + 7$	The point (-2, 11) is on the curve so its coordinates must satisfy the equation. Substituting -2 for x and 11 for y
$-8a + 4b = 4$	$(-2)^3 = -8$ and $(-2)^2 = 4$. Subtracting 11 from both sides. This forms the 1st equation
$3ax^2 + 2bx$	Differentiating to give an expression of the gradient by multiplying each term by the power then subtracting 1 from the power. 7 is basically $7x^0$ so becomes 0
$3a(-2)^2 + 2b(-2) = 0$	Substituting -2 for x and setting equal to 0 as the gradient at a stationary point is 0
$12a - 4b = 0$	Simplifying. This forms the 2nd equation
$4a = 4$	Doing simultaneous equations to solve the 1st and 2nd equations. Adding the 1st equation to the 2nd equation cancels out the b term. Then dividing both sides by 4 finds that a = 1
$12 - 4b = 0$	Substituting 1 for a in the 2nd equation
$4b = 12$	Adding 4b to both sides to make it positive. Then dividing both sides by 4 finds that b = 3

$a =$ 1 $b =$ 3

18 Solve the simultaneous equations

$2x + y = 13$ 1st equation
$x + 3z = 2$ 2nd equation
$z - 2y = -7$ 3rd equation

Do **not** use trial and improvement.

You **must** show your working.

[5 marks]

$2x + 6z = 4$	Multiplying all terms in the 2nd equation by 2 to get the same magnitude of x as the 1st equation. This forms the 4th equation
$6z - y = -9$	Subtracting the 1st equation from the 4th equation cancels out the x terms to get an equation just in terms of z and y. This forms the 5th equation
$12z - 2y = -18$	Multiplying all terms in the 5th equation by 2 to get the same magnitude of y as the 3rd equation. This forms the 6th equation
$11z = -11$	Subtracting the 3rd equation from the 6th equation cancels out the y terms to get an equation just in terms of z. Then dividing both sides by 11 finds that z = -1
$-1 - 2y = -7$	Substituting -1 for z in the 3rd equation
$6 = 2y$	Adding 2y to both sides make the y term positive. Adding 7 to both sides gets the y term on its own. Then dividing both sides by 2 finds that y = 3
$2x + 3 = 13$	Substituting 3 for y in the 1st equation
$2x = 10$	Subtracting 3 from both sides gets the x term on its own. Then dividing both sides by 2 finds that x = 5

$x =$ 5 $y =$ 3 $z =$ -1

10

Turn over ▶

19 $8x^2 + 20x + n \equiv c(x + d)^2 + 3$ where c, d and n are constants.

Work out the values of c, d and n.

[3 marks]

$$8(x^2 + \frac{5}{2}x) + n \longleftarrow$$

Bringing 8 out as a factor on the first two terms. 20/8 = 5/2

$$\underset{\overset{\uparrow}{\boxed{c}}}{8}(x + \underset{\overset{\uparrow}{\boxed{d}}}{\frac{5}{4}})^2 + n - 12.5 \longleftarrow$$

Completing the square by halving the coefficient of x, putting this in a bracket with x and squaring the bracket, then subtracting $8(5/4)^2$ from the end. $(5/4)^2$ = 25/16 then 8(25/16) = 25/2 = 12.5

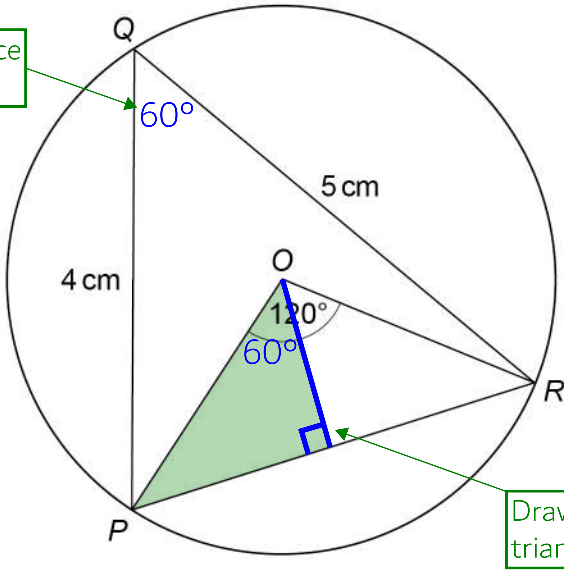
$$n - 12.5 = 3 \longleftarrow$$

Equating the constants. Adding 12.5 to both sides finds that n = 15.5

$c =$ 8 $d =$ $\frac{5}{4}$ $n =$ 15.5

20 P, Q and R are points on a circle, centre O.

Angle $POR = 120°$ $PQ = 4$ cm $QR = 5$ cm



Not drawn accurately

The angle at the circumference is half the angle at the centre

Drawing the line of symmetry on the isosceles triangle POR forms a right-angled triangle

Work out the radius of the circle.

Give your answer in the form $\sqrt{k}$ where k is an integer.

[6 marks]

$PR^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos 60$

Using the cosine rule in triangle QPR. $a^2 = b^2 + c^2 - 2bc\cos A$. Side a is opposite angle A. Substituting PR for a, 4 for b, 5 for c and 60 for A

0	30	45	60	90
4	3	2	1	0

Listing the angles 0°, 30°, 45°, 60°, 90°. Listing 4, 3, 2, 1, 0 under these for the cos values. Square rooting them and putting them over 2 finds that cos60 = 1/2

$PR^2 = 16 + 25 - 20$

$4^2 = 16$ and $5^2 = 25$ and $-2 \times 4 \times 5 \times 1/2 = -20$

$PR = \sqrt{21}$

Square rooting both sides finds length PR

$$\begin{array}{ccc} \overset{\checkmark}{O}\checkmark & A\checkmark & \overset{\checkmark}{O} \\ S\ H & C\ H & T\ A \end{array}$$

Doing right-angled trigonometry on the green triangle. OP is a radius and is the hypotenuse so ticking H. Half of PR is the opposite so ticking O. There are two ticks on the SOH formula triangle so this one can be used

0	30	45	60	90
0	1	2	3	4

Listing the angles 0°, 30°, 45°, 60°, 90°. Listing 0, 1, 2, 3, 4 under these for the sin values. Square rooting them and putting them over 2 finds that sin60 = $\sqrt{3}/2$

$$\frac{1}{2}\sqrt{21} \div \frac{\sqrt{3}}{2}$$

Covering H in the SOH formula triangle finds that hypotenuse = opposite ÷ (sin of the angle). The opposite is half of PR

$$\frac{1}{2}\sqrt{21} \times \frac{2}{\sqrt{3}}$$

To divide by a fraction, change the division to a multiplication and flip the fraction

The 1/2 and 2 cancel out leaving $\sqrt{21}/\sqrt{3} = \sqrt{7}$

Answer $\sqrt{7}$ cm

END OF QUESTIONS

9