AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE
MATHEMATICS

H

Higher Tier Paper 1 Non-Calculator

Tuesday 6 November 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- mathematical instruments

You must **not** use a calculator.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

1 Simplify $\left(5^4\right)^2$

Circle your answer.

[1 mark]

5^6 [✓ 5^8] 25^6 25^8

$(a^x)^y = a^{xy}$
So multiply the powers together

2 Circle the volume, in cm^3, of a cylinder with radius 5 cm and height 8 cm

[1 mark]

40π 80π [✓ 200π] 1600π

A cylinder can be treated like a prism, so volume = cross-sectional area x length.
The cross section is a circle and the length is the height.
$\pi \times 5^2 \times 8 = (25 \times 8)\pi = 200\pi$

3 Simplify $16a^2 \div a + 3a \times 2$

Circle your answer.

[1 mark]

[✓ $22a$] $8a$ $38a$ $2a$

Follow the order of operations (BIDMAS).
Division is first, then multiplication, then addition.
$16a^2 \div a = 16a$
$3a \times 2 = 6a$
$16a + 6a = 22a$

4 Circle the value of cos 30°

[1 mark]

$\frac{1}{2}$ [✓ $\frac{\sqrt{3}}{2}$] 0 1

0	30	45	60	90
4	3	2	1	0

Listing the angles of 0, 30, 45, 60, 90 degrees. Listing 4, 3, 2, 1, 0 under these for the cos values. Square rooting them and putting them over 2 works out the exact cos values. Square rooting the 3 and putting it over 2

5 Work out $8\frac{1}{2} \div 2\frac{2}{3}$

Give your answer as a mixed number.

[4 marks]

$\frac{17}{2} \div \frac{8}{3}$

Converting the mixed fractions into improper fractions by multiplying the whole number by the denominator then adding the result to the numerator

$\frac{17}{2} \times \frac{3}{8}$

To divide by a fraction, keep the first number, change the division to a multiplication, flip the second fraction

$$\begin{array}{r} 17 \\ \times\ 3 \\ \hline 5_{2}1 \end{array}$$

To multiply fractions, multiply the numerators and multiply the denominators. 2 x 8 = 16

$\frac{51}{16}$

Expressing the answer as an improper fraction

16, 32, 48, 64

Listing out the multiples of 16 to work out how many lots of 16 go into 51. 3 lots fit in with a remainder of 3

Answer $3\frac{3}{16}$

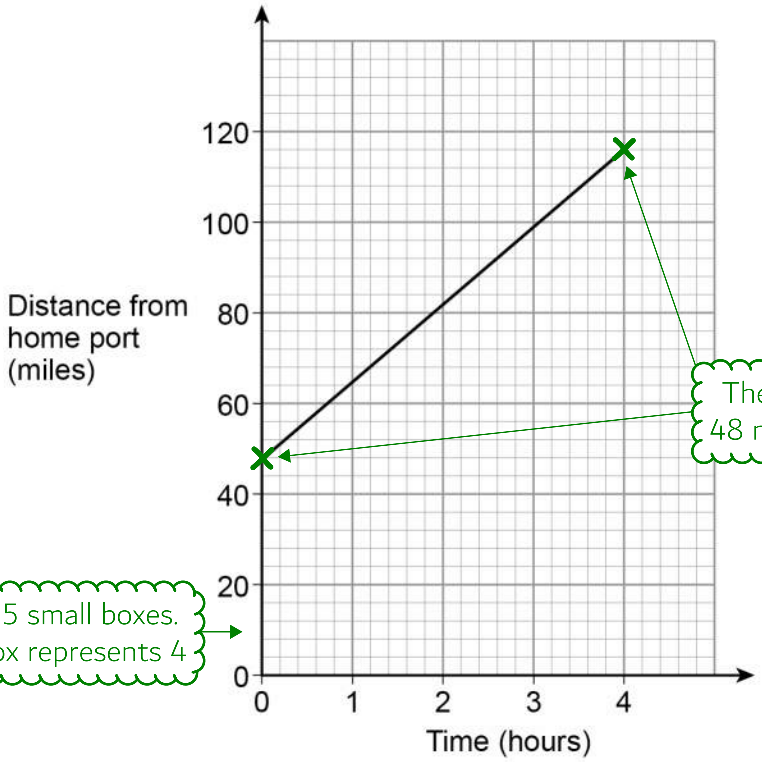
The 3 is the whole number and leaving the remainder of 3 in the fraction

8

Turn over ▶

6 A ship is sailing in a straight line from its home port.

The distance-time graph shows 4 hours of the journey.



The ship started at a distance of 48 miles and finished at 116 miles

The scale goes up 20 over 5 small boxes. 20 ÷ 5 = 4 so each small box represents 4

Work out the speed of the ship during these 4 hours.

[3 marks]

$$\begin{array}{r} {}^{0}\cancel{1}{}^{1 0}\cancel{1}{}^{1}6 \\ -\ 48 \\ \hline 68 \end{array}$$

Subtracting the 48 miles the ship started at from the 116 miles the ship finished at works out that the ship travelled 68 miles during these 4 hours

$$\begin{array}{r} 1\,7 \\ 4\overline{)6\,{}^{2}8} \end{array}$$

The unit of miles per hour tells us that we need to divide the miles by the hours

Answer 17 mph

7 The sum of the angles in any quadrilateral is 360°

For example, in a rectangle 4 × 90° = 360°

Zak writes,

5 × 90° = 450° so the sum of the angles in any pentagon must be 450°

Pentagons have 5 sides

Is he correct?

Tick a box.

☐ Yes ☑ No

Show working to support your answer.

[2 marks]

$(5-2) \times 180$

Sum of interior angles = (n - 2) x 180, where n is the number of sides of the polygon

$$\begin{array}{r} 180 \\ \times \quad 3 \\ \hline 5_{2}40 \end{array}$$

5 - 2 = 3. Then multiplying this by 180 works out that there are 540 degrees in a pentagon

Turn over for the next question

8 Kim works at an airport in the UK.

She records the number of planes landing between 10 am and 2 pm each day.

The table shows the data for the first 10 days in January.

Day	1	2	3	4	5	6	7	8	9	10
Number of planes	148	151	147	155	153	147	155	102	151	154

8 (a) The airport was affected by fog on one of the days.

Which day do you think it was?

Give a reason for your answer.

[1 mark]

Day 8

Reason It is an outlier ← All of the other days are around 150. Day 8 isn't close to this

8 (b) Kim uses the data to predict how many planes will land at the airport in a year.

In her method, she

uses an estimate of 150 planes in each 4-hour period throughout the day

assumes the same number of planes each day.

Work out her prediction.

[3 marks]

$$\begin{array}{r} 150 \\ \times\quad 6 \\ \hline 9_{3}00 \end{array}$$

← There are 24 hours in a day. There are 6 4-hour periods each day. So multiplying the 150 planes in each 4-hour period by 6 works out that there are 900 planes each day

$$\begin{array}{r} 365 \\ \times 900 \\ \hline 3_{5}28_{4}500 \end{array}$$

← Multiplying the 365 days in a year by the 900 planes each day works out the prediction

Answer 328500

8 (c) In fact,

fewer planes land in winter than in summer
fewer planes land at night than during the day.

What does this tell you about Kim's prediction?

Tick **one** box.

☐ Her prediction is too low

☐ Her prediction is too high

☑ Her prediction could be too low or too high

Give a reason for your answer.

[2 marks]

Fewer in winter makes it too low. Fewer at night makes it too high.

The prediction was based on the data collected from 10 am to 2 pm in January. This is in the day and in the Winter

Turn over for the next question

6

9 $\sqrt{6^2 + 8^2} = \sqrt[3]{125a^3}$

Work out the value of a.

[4 marks]

$$\begin{array}{r} 36 \\ +64 \\ \hline 100 \end{array}$$ ← 6^2 = 36 and 8^2 = 64

$10 = 5a$ ← The square root of 100 is 10. The cube root of 125 is 5 and the cube root of a^3 is a

Answer 2

Dividing both sides by 5 gets 2 = a

10 Work out the percentage increase from 80 to 280

[3 marks]

$280 - 80 = 200$ ← Working out the increase

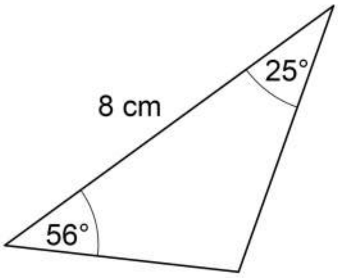
$\frac{200}{80} = \frac{20}{8} = \frac{10}{4} = \frac{250}{100}$ ← Expressing the increase as a fraction of the original then simplifying the fraction (by dividing the numerator and denominator by the same amount) until the denominator is 4, which can be multiplied by 25 to get 100

Answer 250 %

Percentage is out of 100 so 250/100 is 250%

11 Here are four triangles.

A



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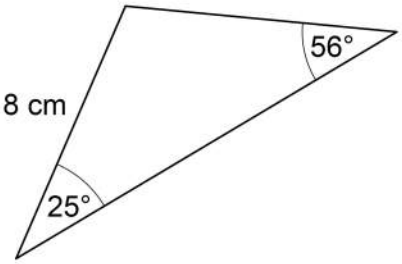

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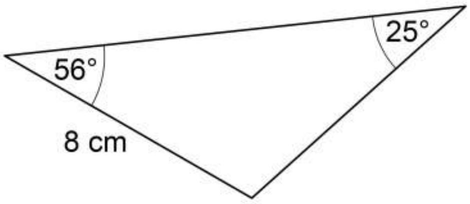
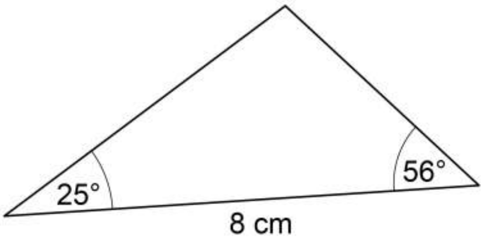
accurately

C



D



Which **two** triangles are congruent?

Circle **two** letters below.

[1 mark]

[✓ A] B C [✓ D]

All the triangles have the same angles and a side of 8cm. But the 8cm is opposite the unknown angle in A and D so these must be the congruent ones

Turn over for the next question

8

12 Solve $x^2 - x - 12 = 0$

[3 marks]

$1 \times 12, 2 \times 6, 3 \times 4$

Looking for two numbers which multiply to the -12 and add to the -1 (which is the coefficient of x). Listing out the factor pairs of 12 until they add to -1 (when one of the pair is negative in order to multiply to a negative)

$(x+3)(x-4)=0$

3 and -4 multiply to the -12 and add to the -1. Putting these in brackets with x factorises the left side

Answer $x=-3, x=4$

Either x + 3 = 0 or x - 4 = 0 (as the only way of multiplying two brackets together and getting 0 is if one of them is equal to 0). Rearranging gives these solutions

13 $e : f = 2 : 3$ and $f : g = 5 : 4$

Work out $e : g$

Give your answer in its simplest form.

[3 marks]

$10 : 15 : 12$

Combining the ratios together into the ratio e : f : g. f is in common to both ratios so they must have the same number of parts for f in order to be compatible. 15 is a common multiple of 3 and 5. Multiplying both sides of the ratios by the same amount converts them. The 3 parts for f are multiplied by 5 to get 15 so the 2 parts for e also need to be multiplied by 5 to get 10. The 5 parts for f are multiplied by 3 to get 15 so the 4 parts for g also need to be multiplied by 3 to get 12

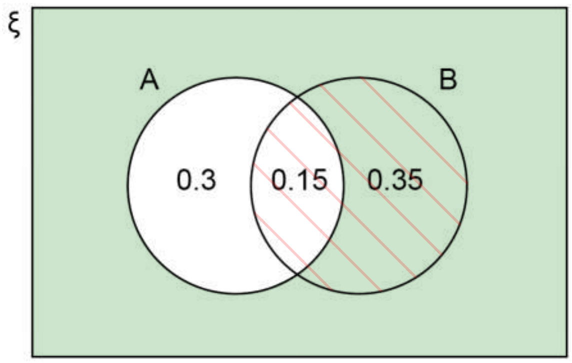
Answer 5 : 6

The ratio e : g is 10 : 12 and this can be simplified by dividing both sides by 2

14 A and B are two events.

Some probabilities are shown on the Venn diagram.

A' is highlighted in green. B is lined in pink. The union is any part which is highlighted or lined



Work out P(A′ U B) ← U means union

[2 marks]

1 − 0.3 ← As everything apart from 0.3 is in A' U B and the probabilities must add up to 1, subtracting the 0.3 from 1 finds the probability

Answer 0.7

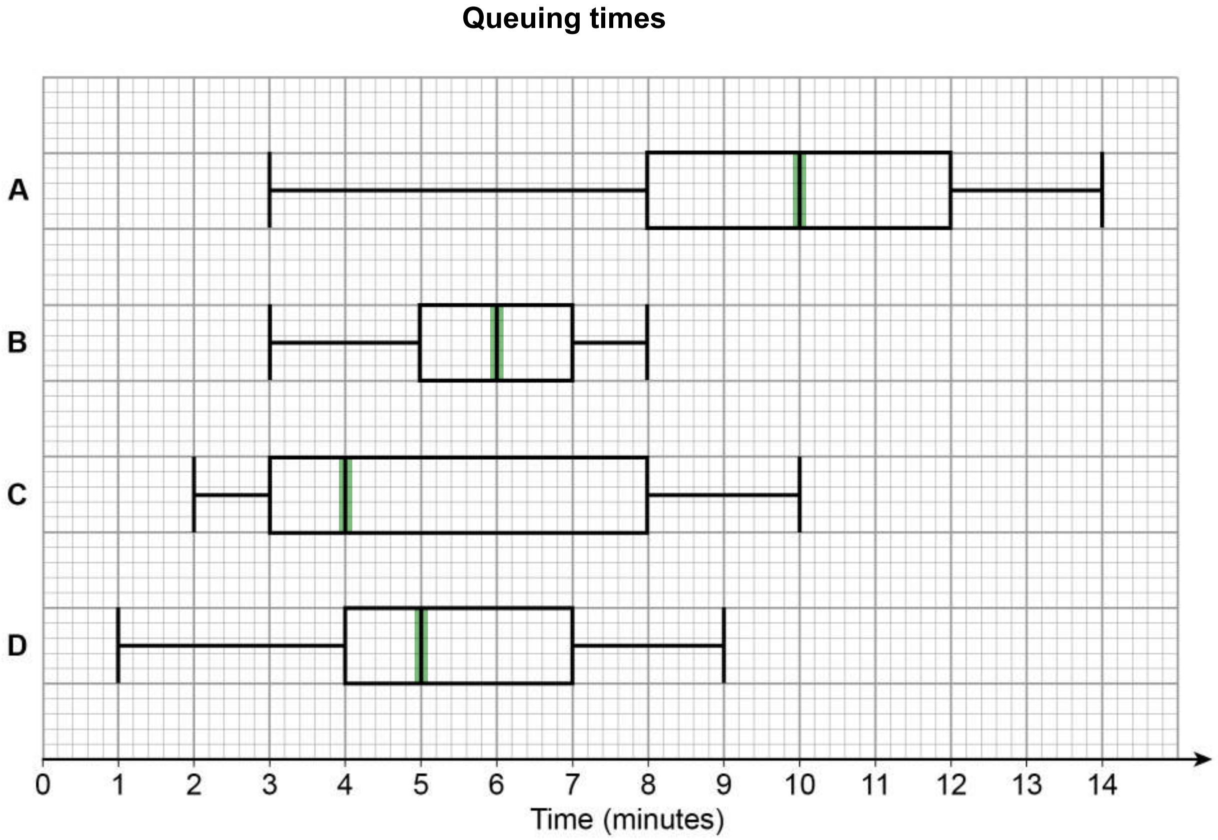
Turn over for the next question

8

15 In a survey, queuing times at supermarket checkouts were recorded.

One morning, samples of 50 customers were taken at supermarkets A, B, C and D.

The box plots represent the results.



15 (a) On average, which supermarket had the lowest queuing times?

Give a reason for your answer.

[2 marks]

Supermarket C

Reason Lowest median

The medians are highlighted in green

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15 (b) At which supermarket were the queuing times most consistent?

Give a reason for your answer.

[2 marks]

Supermarket B

Reason Lowest interquartile range

The wider the box, the greater the interquartile range and the less consistent. The box for B is the least wide

16 Circle the number that is closest to the value of 29^3

[1 mark]

[✓ 27 000]   90   2700   9000

An estimate is 30^3

$3^3 = 27$ so $30^3 = 27 \times 10^3$

17 Work out the exact value of $\left(\frac{3}{4}\right)^{-3}$

[2 marks]

$\frac{27}{64}$ ← First cubing 3 and 4, ignoring the negative part of the power for now

Answer $\frac{64}{27}$

The negative means to do the reciprocal (flip the fraction in this case)

Turn over for the next question

7

18 Beth and Mia translate documents from Spanish into English.

A set of documents that would take Beth 8 days would take Mia 10 days.

Beth starts to translate the documents.

After 2 days Beth and Mia both work on translating the documents.

How many **more** days will it take to complete the work?

You **must** show your working.

[4 marks]

$1 - \frac{2}{8} = \frac{6}{8}$

As it would take Beth 8 days, she would complete 1/8 of the documents per day. In two days she would have completed 2/8. Subtracting this from 1 set of documents leaves 6/8 of the documents which are still left to be translated

$\frac{1}{8} + \frac{1}{10} = \frac{10}{80} + \frac{8}{80} = \frac{18}{80}$

As it would take Mia 10 days, she would complete 1/10 of the documents per day. Adding together the amounts of documents they complete each day gives the total amount of documents completed each day when both Beth and Mia are working on them

$\frac{6}{8} \div \frac{18}{80} = \frac{6}{8} \times \frac{80}{18}$

Time = distance/speed. Distance is basically how much of the documents still need to be translated. Speed is basically how much of the documents are completed each day

Answer $\frac{10}{3}$ days

Cancelling out the 8 with the 80 leaves 10 and cancelling out the 6 with the 18 leaves 3

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19 In a chess club, there are x boys and y girls.

19 (a) If 5 more boys and 8 more girls join, there would be half as many boys as girls.

Show that $y = 2x + 2$

[2 marks]

$x+5=\frac{1}{2}(y+8)$ ← x + 5 expresses the number of boys there would be in the club. y + 8 expresses the number of girls there would be in the club. As the number of boys would be half the number of girls, the number of girls needs to be halved to make them equal

$2x+10=y+8$ ← Multiplying both sides by 2 to get rid of the 1/2

$y=2x+2$ ← Subtracting 8 from both sides makes y the subject and gets what we are trying to show

19 (b) If instead,

10 more boys and 1 more girl join, there would be the same number of boys and girls.

Work out x and y.

[3 marks]

$x+10=y+1$ ← x + 10 expresses how many boys there would be. y + 1 expresses how many girls there would be. These expressions are equal as there would be the same number of boys and girls

$=2x+2+1$ ← Substituting y for 2x + 2 as y = 2x + 2. This eliminates the y and makes an equation only in terms of x

$7=x$ ← Subtracting x and (2 + 1) from both sides. So x = 7

$y=2(7)+2$ ← Substituting 7 for x in y = 2x + 2. So y = 16

$x =$ 7

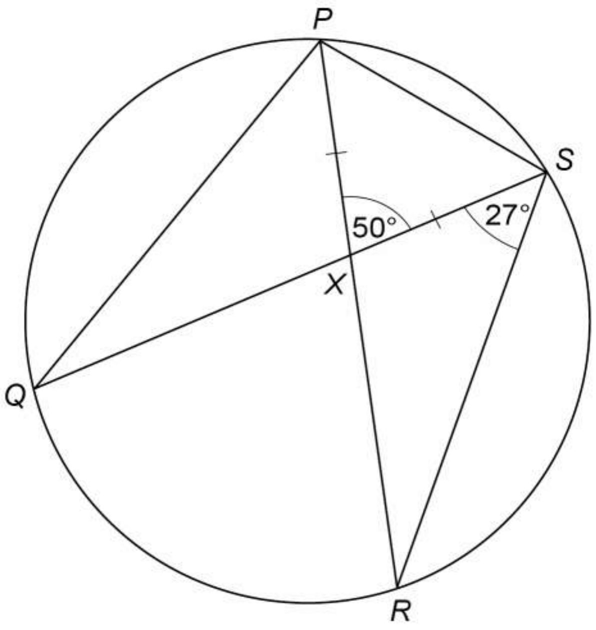
$y =$ 16

9

20 P, Q, R and S are points on a circle.

PXR and QXS are straight lines.

PX = SX



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Prove that QS is **not** a diameter of the circle.

[4 marks]

Angle QPR = 27 as angles in the same segment are equal. ← Angles QPR = QSR

Angle XPS = 65 as there are 180° in a triangle and base angles of an isosceles triangle are equal.

180 - 50 works out that there are 130 degrees left in triangle XPS. Dividing 130 by 2 gives 65, which is the base angles of the isosceles triangle XPS, which is isosceles as it has two equal sides

Angle QPS = 27 + 65 = 92 so QS is not a diameter as angles in a semicircle are 90 degrees.

Adding angles QPR and XPS gives angle QPS. Angle QPS must be 90 degrees if it is in a semicircle. QS would be the diameter of the semicircle

Do not write outside the box

21 Here are the first four terms of a quadratic sequence.

It will be in the form $an^2 + bn + c$

11 26 45 68

15 19

4

The difference between 11 and 26 is 15. The difference between 26 and 45 is 19

The second difference (difference of the 15 and 19) is 4

Work out an expression for the *n*th term.

[3 marks]

$2n^2$: 2 8

Halving the second difference gives 2, which is a. Listing the first two terms of $2n^2$ gives 2 (as $2(1)^2 = 2$) and 8 (as as $2(2)^2 = 8$)

9 18 : $9n$

Listing what needs to be added to the sequence of $2n^2$ to get the original sequence. The sequence 9, 18 goes up in 9s so must involve 9n, so b is 9. The 0th term would be 0 (tracing the sequence backward, it goes down in 9s and 9 - 9 = 0), so c is 0

Answer $2n^2 + 9n$

Adding the sequences of $2n^2$ and 9n gives the original sequence

Turn over for the next question

7

Do not write outside the box

22 Solve $\frac{x}{x+4} + \frac{7}{x-2} = 1$

You **must** show your working.

[4 marks]

$x + \frac{7(x+4)}{x-2} = x+4$ ← Multiplying all terms by (x + 4) to eliminate it as a denominator

$x(x-2) + 7(x+4) = (x+4)(x-2)$ ← Multiplying all terms by (x - 2) to eliminate it as a denominator

$x^2 - 2x + 7x + 28 = x^2 - 2x + 4x - 8$ ← Expanding all the brackets

$3x = -36$ ← Subtracting x^2 from both sides gets rid of the x^2 terms.
Bringing all the x terms to the left side: -2x + 7x + 2x - 4x = 3x.
Bringing the constants to the right side: -8 - 28 = -36

$x =$ -12

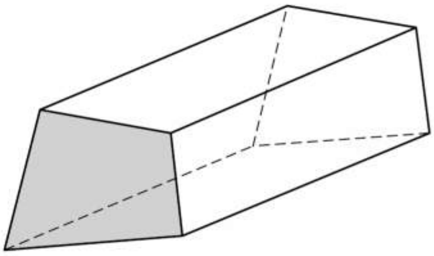
Dividing both sides by 3 gets x on its own

23 Prisms A and B are similar.

The cross sections are shaded.

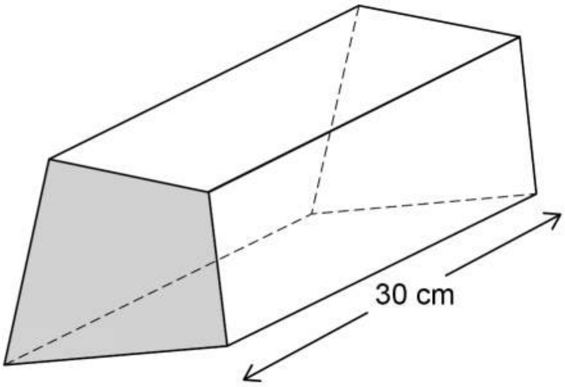
Prism A

volume = 480 cm^3



Prism B

length = 30 cm



area of the cross section of A : area of the cross section of B = 4 : 9

Work out the area of the cross section of B.

[5 marks]

2 : 3

Square rooting both sides of 4 : 9 gives the ratio of the lengths

8 : 27

Cubing both sides of 2 : 3 gives the ratio of the volumes

480 ÷ 8 = 60

8 parts of the volume ratio represent 480 cm^3 so dividing the 480 cm^3 by 8 works out 1 part of of the volume ratio. 48 ÷ 8 = 6 so 480 ÷ 8 = 60

$$\begin{array}{r} 27 \\ \times 60 \\ \hline 1\underset{4}{6}20 \end{array}$$

Multiplying the value of 1 part of the volume ratio by 27 works out what the 27 parts which represent the volume of B are worth. So the volume of B is 1620 cm^3

$$3\overline{)16^{1}2}\quad 54$$

Volume of a prism = (cross-sectional area) x length.
Cross-sectional area = (volume of a prism)/length.
So dividing the volume of prism B by the length of prism B gives the area of the cross section of B.
1620/30 simplifies to 162/3

Answer 54 cm^2

9

24 Show that $\frac{2\sqrt{6}}{\sqrt{5}} - \frac{\sqrt{3}}{\sqrt{10}}$ can be written in the form $\frac{c\sqrt{d}}{10}$

where c and d are integers.

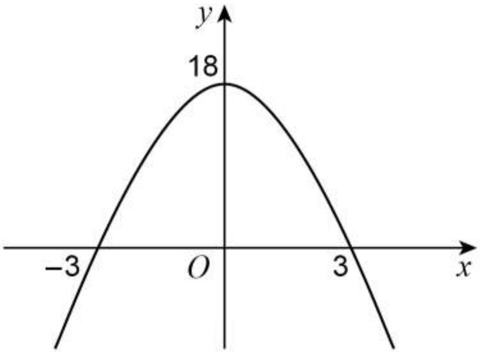
[3 marks]

$\frac{2\sqrt{30}}{5} - \frac{\sqrt{30}}{10}$ ← Rationalising both of the denominators by multiplying both the numerator and denominator of the first fraction by √5 and the second fraction by √10

$\frac{4\sqrt{30} - \sqrt{30}}{10}$ ← Multiplying the numerator and denominator of the first fraction by 2 to make the denominators the same then combining the fractions with the numerators subtracting

$\frac{3\sqrt{30}}{10}$ ← $4x - x = 3x$, so $4\sqrt{30} - \sqrt{30} = 3\sqrt{30}$

25 A quadratic curve intersects the axes at (–3, 0), (3, 0) and (0, 18)



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Work out the equation of the curve.

[3 marks]

$18 = a(0)^2 + b(0) + c$

The equation is a quadratic so must be in the form $y = ax^2 + bx + c$. Substituting in the coordinates of (0 , 18) gives this. So c = 18

$0 = a(3)^2 + b(3) + 18 = 9a + 3b + 18$

Substituting in the coordinates of (3 , 0) and 18 for c. Simplifying to form the first equation

$0 = a(-3)^2 + b(-3) + 18 = 9a - 3b + 18$

Substituting in the coordinates of (-3 , 0) and 18 for c. Simplifying to form the second equation

$6b = 0$

Solving the equations simultaneously by subtracting the second equation from the first equation to eliminate the a terms

Dividing both sides by 6 finds that b = 0

$9a = -18$

Substituting 0 for b in the first equation gives 9a = -18

Dividing both sides by 9 finds that a = -2

Answer $y = -2x^2 + 18$

$y = ax^2 + bx + c$. Substituting -2 for a, 0 for b and 18 for c

Turn over for the next question

6

Turn over ▶

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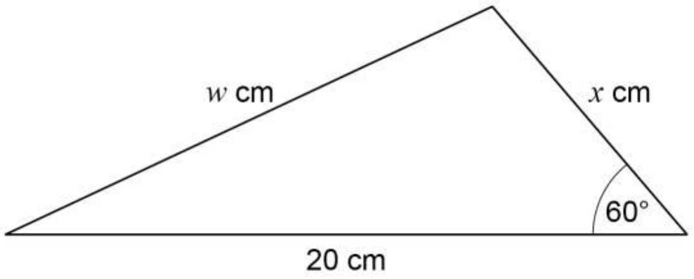
26 The area of this triangle is $25\sqrt{3}$ cm^2

Not drawn accurately



Work out the value of w.

Give your answer in the form $a\sqrt{b}$ where a and b are integers greater than 1

[5 marks]

0	30	45	60	90
0	1	2	3	4
4	3	2	1	0

Listing the angles of 0, 30, 45, 60, 90 degrees. Listing 0, 1, 2, 3, 4 under these for the sin values. Listing 4, 3, 2, 1, 0 under these for the cos values. Square rooting the 3 and putting it over 2 finds that sin60 = √3/2. Square rooting the 1 and putting it over 2 finds that cos60 = 1/2

$\frac{1}{2} \times 20 \times x \times \frac{\sqrt{3}}{2}$

Area of triangle = 1/2 absinC. Substituting 20 for a, x for b and √3/2 for sinC

$5\sqrt{3}x = 25\sqrt{3}$

Simplifying the expression and setting it equal to the actual area of 25√3

$x = 5$

Dividing both sides by 5√3 to get x on its own

$w^2 = 20^2 + 5^2 - 2 \times 20 \times 5 \times \frac{1}{2}$

The cosine rule can be used to find side w (the sine rule can't be used as there aren't two opposite pairs of sides and angles). $a^2 = b^2 + c^2 - 2bc\cos A$. Substituting w for a, 20 for b, 5 for c and 1/2 for cosA (as it is cos60)

$= 400 + 25 - 100$

Simplifying

$= 325$

Simplifying

$w = \sqrt{325}$

Square rooting both sides undoes the power of 2 on the left and finds w

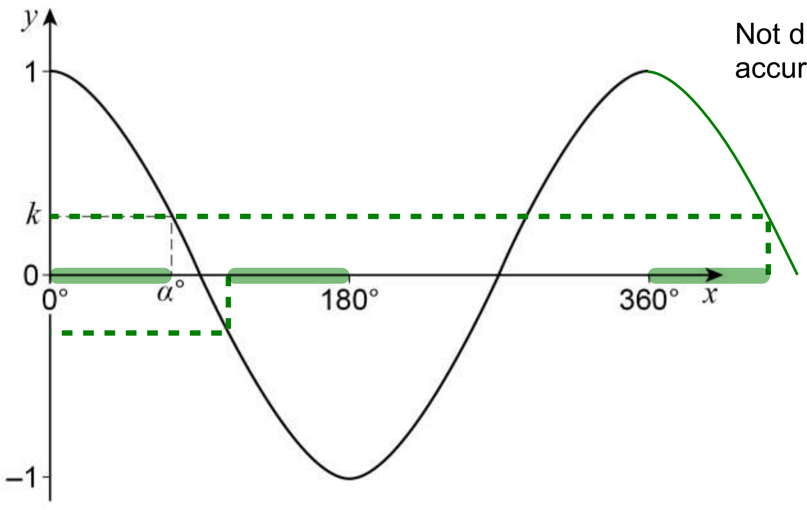
$$\begin{array}{r} 1\;3 \\ 25\overline{)3\;2\;{}^{7}5} \end{array}$$

Splitting √325 into √25 and another surd. Dividing √325 by √25 to work out what the other surd is

Answer $5\sqrt{13}$

√325 = √25 x √13 = 5√13

27 Here is a sketch of $y = \cos x$ for values of x from 0° to 360°



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α° is an acute angle.

$\cos \alpha° = k$

27 (a) Circle the value of $\cos(180° - \alpha°)$

[1 mark]

$1 - k$ k [✓ $-k$] $-1 - k$

All the distances highlighted in green are the same. The curve is the same from 0 to 90 degrees as 90 to 180 degrees, except it is negative

27 (b) Circle the value of $\cos(360° + \alpha°)$

[1 mark]

$k - 1$ $k + 1$ $-k$ [✓ k]

All the distances highlighted in green are the same. The curve repeats after 0 to 360 in the same way

END OF QUESTIONS

7